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Analysis of Pisarenko Harmonic Decomposition-based subNyquist Rate Spectrum Sensing for Broadband Cognitive Radio

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ABSTRACT

The essential part of cognitive radio is spectrum sensing, so that the under utilised spectrum could be detected to improve the spectrum efficiency. For this purpose, a wide range of frequency bands are considered and locations of multiple occupied spectrum subbands are focused. A major challenge related with such broadband spectrum sensing is that it is either extravagant or impracticable to perform Nyquist sampling on the broadband signal. In this study, a broadband spectrum sensing method, that takes advantage of subNyquist sampling wherein the sampling rate is considerably reduced. The correlation matrix of a limited number of noisy samples is computed and is used to estimate the frequency function of the pisarenko harmonic decomposition method to detect the occupied and unoccupied channels. The salient feature of this approach as compared to other methods is that, no prior knowledge of signal properties (which would lead to uncertain problems) is necessary. Further, the efficiency of this method is assessed by calculating the detection probability of the occupied channel as a function of the limited number of samples and the signal to noise ratio of random input signals. The simulation results demonstrate a reliable detection, even with limited samples and a low SNR.

Keywords: Spectrum sensing; Multicoset sampling; Cognitive radio; Noise subspace

NOMENCLATURE

$x(t)$	Input signal
B	Channel bandwidth
B_{max}	Spectrum bandwidth
$X(f)$	Fourier transform of $x(t)$
L	Number of channels
N	Number of occupied channels
k	Spectral indexed vector
t	Time period
T	Time
Z	Set of integers
p	Number of samples in each block
γ	SubNyquist factor
σ	Noise eigenvalue
Λ	Diagonal matrix
f	Frequency
U	Full rank matrix
c_i	Sample pattern
E	Expected value
$\hat{R}$	Correlation matrix
q_{max}	Maximum number of occupied channels
q_{min}	Minimum number of occupied channels
q	Number of occupied channels
M	Number of samples
$h[n]$	Interpolation filter
$H(f)$	Frequency response
$w(t)$	White noise
$n(f)$	Gaussian complex noise
$A(k)$	Modulation matrix
$y(f)$	Observation vector
λ	Eigenvalues
E	Eigen vector
$g(r)$	Geometric mean
$a(r)$	Arithmetic mean
m	Samples
I	Identity matrix
Ω	Spectral occupancy ratio
f_{avg}	Average sampling rate
P_d	Probability of detection
P_f	Probability of false alarm

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1. INTRODUCTION

The rapid growth of communication systems and the ever growing number of applications and users has put a significant demand on the spectral resources. An intelligent system which is capable of surveying the spectrum, looking for an available frequency band, and adapting its operation to efficiently exploit the available environment, without interfering with the existing communication channels, is called as cognitive radio (CR). For such systems, the primary information needed to strategise its operation, is a quick survey of the spectrum to locate existing channels, and to estimate the white space (or free region) in between[1,2]. Such a CR system, in general, has no prior knowledge of the spectrum utilisation; it becomes necessary to analyse a wider spectrum, before a suitable white space is available for its use.

Currently, there exists several methods for sensing of the spectrum e.g., filter bank spectrum sensing, energy detection (ED), Multitaper spectrum estimation, etc., most of which use sampling the spectrum at Nyquist rate[3]. Such methods, when applied over wide bands necessitate very efficient, high speed, and accurate AD converters, thus increasing cost and forgoing speed and accuracy. To improve the efficiency and speed at a low cost, new technologies are evolved, which use less number of samples, or subNyquist sampling[4]. These technologies use a compressive sampling by initially using subNyquist samples to reconstruct the broadband spectrum and apply spectrum sensing on the reconstructed spectrum.

The methods available for recovery or reconstruction of the multiband spectrum are broadly classified into two classes – nonblind recovery and blind recovery. One of the nonblind methods proposed by Venkataramani & Bresler[5] employs multicoset subNyquist sampling, by dividing the multiband spectrum into a number of subbands, and uses multiple or complex AD converters[6] This needs prior knowledge of the subband locations. However, the blind methods proposed by Mishali[7], *et al.* and Feng[8], *et al.* needs no prior knowledge of the subband locations. In addition, other subNyquist sampling techniques using analog processing at the front end have also been proposed[9,10]. Blind methods involving different versions of multicoset samplings are also proposed[11-13]. Several other methods have been proposed using subNyquist sampling. While Edge-Detection Algorithm has been used by Tian and Giannakis[14], assuming that each of the subbands has a sharp edge, Pal and Vaidyanathan[15] have employed Coprime Sampling Algorithm which uses two branches of sampling, each of whose sampling rates are mutually coprimes.

In the current study, a well known method of broadband spectrum sensing is undertaken, which substantially brings down the sampling rate. A wide spectrum consisting a known number subbands is considered, each of which is tested by using its correlation matrix to detect the presence of an occupied signal. The efficacy of this method is evaluated by calculating the detection probability of the occupied channel as a function of the limited number of samples and the signal to noise ratio of random input signals.

2. PROBLEM STATEMENT

It is assumed that the received broadband analog signal $x(t)$ is sparse, and is band limited to $(0, B_{\max})$. The classical Nyquist rate is $B_{\max}$.

Let $X(f)$ be the Fourier transform of $x(t)$. Based on the methodology adopted, the given complete spectrum is divided into L narrow band channels. Let B the bandwidth of each of these narrow band channels, such that

$$B_{\max} = L \times B \tag{1}$$

The L narrow band channels, thus derived, are then labelled as 0 to $L-1$. The channels which contain the signals are called occupied channels, and the rest are called unoccupied channels. Let N be the number of such occupied channels. The indices of the detected occupied channels are organised into a vector k, called spectral indexed vector, whose length is q, and which satisfies $k = \{k_1, k_2, \cdots, k_q\}, \; q = |k|$.

Figure 1 shows the input spectrum of a typical multiband signal in frequency domain, with 32 narrow band channels (L = 32), each of which occupies a bandwidth of B = 10 MHz. In this study, N = 4 channels are occupied, and the occupied channel set $k = \{10, 20, 25, 30\}$. Thus, the problem is to detect the presence of the signals in each of the 32 spectral bands, at a subNyquist sampling rate, for a given $B_{\max}$ and B.

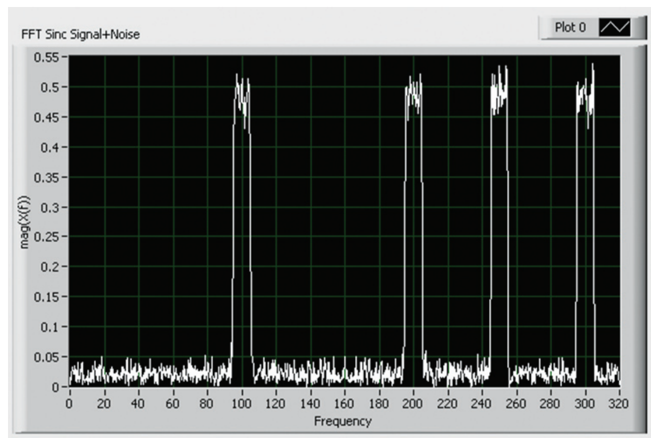


Figure 1. Input signal in frequency domain.

3. BROADBAND SPECTRUM SENSING

Figure 2 is the block diagram for broadband spectrum sensing system. Using a multicoset sampler, the input signal $x(t)$ is sampled, at a rate lesser than the classical Nyquist rate. Using a multirate system, the output of the multicoset sampler is partially shifted. The multirate system consists of an up sampling, interpolation stage, delay stage, and a down sampling stage. Using this data the sample correlation matrix is computed. This correlation matrix is analysed using subspace methods for estimating the number of occupied channels and

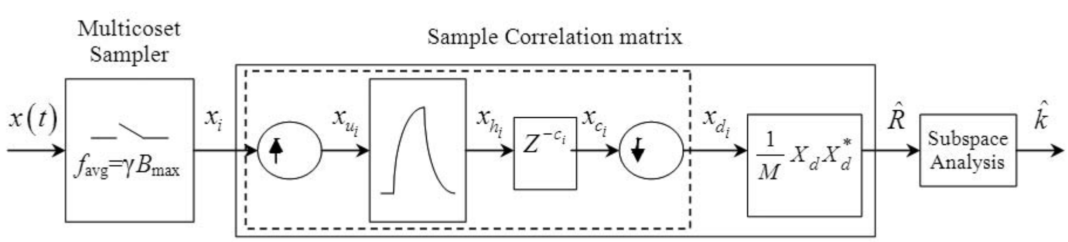


Figure 2. The block diagram for broadband spectrum sensing system.

occupied channel set recovery. Various stages of this block diagram are described in detail.

3.1 Multicoset Sampling

The multicoset sampler[5] is used to sample the given analog broadband signal $x(t)$. This broadband signal is considered to belong to the class of continuous multiband signals of limited energy. Assuming that the signal is sampled non uniformly at periods $t=(mL+c_i)T, m\in Z,\ 1\le i\le p$ and that the samples are categorised into p sequences satisfying the condition:

$$x_i(m)=x\left[(mL+c_i)/T\right],\quad m\in Z \tag{2}$$

where T represents time, L represents block length, p represents the number of samples in each of these L blocks, and $C=\{c_1,c_2,\ldots,c_p\}$ represents the sample pattern[7].

Choosing $T=1/B_{\max}$, the average sample rate of this scheme is $f_{avg}=\gamma.B_{\max}$ where, $\gamma=(p/L)$ is γ called the subNyquist factor. Given L the number of occupied slots are formulated using the relation $\left\lceil\frac{NBL}{B_{\max}}\right\rceil\le q\le N+N\left\lceil\frac{BL}{B_{\max}}\right\rceil$; $q_{\min}\le q\le q_{\max}$. The number of occupied slots which depends upon the band location is chosen between the above two bounds. Let $q=q_{\min}$ where $p>q_{\max}$ and choose $p=q_{\max}+1$ where $q_{\max}$ is the maximum number of occupied channels that can be detected. Though the contents or the information in the bands remains a constant, the carriers deviate as to fill a maximum number of occupied channels.

3.1.1 *Correlation Matrix*

For correlating the task of spectrum sensing and parameter estimation, the correlation matrix of the sampled data is computed. This is achieved by applying the following methodology on the sampled data:

It is assumed that $B_{\max}=1$, and $B=1/L$. Substituting for T and $B_{\max}$ in Eqn (2), the sampling sequence is given by

$$x_i(m)=x(mL+c_i) \tag{3}$$

Taking DFT of Eqn (3), results in:

$$X_i(f)=\frac{1}{L}\sum_{r=0}^{L-1}X\left(\frac{f}{L}+\frac{r}{L}\right)\,e^{\frac{j2\pi c_i f}{L}}\,e^{\frac{j2\pi c_i r}{L}} \tag{4}$$

Next, each $x_i(m)$ is oversampled by a factor L, such that

$$x_u[n]=\begin{cases} x_i\left(\frac{n}{L}\right), & n=mL, m\in Z\\ 0, & \text{otherwise}\end{cases} \tag{5}$$

In the frequency domain,

$$X_{u_I}(f)=X_i(Lf) \tag{6}$$

$$X_{u_I}(f)=B\sum_{r=0}^{L-1}X(f+rB)\,e^{j2\pi c_i f}\,e^{\frac{j2\pi c_i r}{L}} \tag{7}$$

where $X_{u_i}(f)$ represents the DFT of the sequence x_{u_I} that is oversampled.

Then it is sieved to get $x_{h_i}(n)=x_{u_i}[n]*h[n]$, where $h[n]$ is the interpolation mask. Its frequency response is

$$H(f)=\begin{cases}1, & f\in[0,B]\\ 0, & \text{otherwise}\end{cases} \tag{8}$$

Filtering $X_{ui}(f)$ with $H(f)$ clips the output frequency range such that

$$X_{h_I}(f)=X_{u_I}(f),\ f\in[0,B] \tag{9}$$

Further this output sequence is delayed with c_i samples satisfying,

$$x_{c_i}[n]=x_{h_i}[n-c_i] \tag{10}$$

Then delaying each signal with c_i samples gives

$$X_{c_i}(f)=X_{h_i}(f)e^{-j2\pi c_i f} \tag{11}$$

$$X_{c_I}(f)=B\sum_{r=0}^{L-1}X(f+rB)\,e^{\frac{j2\pi c_i r}{L}},\ f\in[0,B] \tag{12}$$

Assuming that the input signal $x(t)$ is distorted by the white noise $w(t)$ whose Fourier transform is $W(f)$, Eqn. (12) is re-written as

$$\begin{aligned}X_{c_i}(f)&=B\sum_{r=0}^{L-1}X(f+rB)\,e^{\frac{j2\pi c_i r}{L}}\\&+B\sum_{r=0}^{L-1}W(f+rB)\,e^{\frac{j2\pi c_i r}{L}},\ f\in[0,B]\end{aligned} \tag{13}$$

where the term $X(f+rB),\ 0\le r\le L-1$, represents frequency component of the signal in each channel. This term is zero for a unoccupied channel. Therefore the above equation takes the form:

$$\begin{aligned}X_{c_i}(f)&=B\sum_{r\in k}X(f+rB)\,e^{\frac{j2\pi c_i r}{L}}\\&+B\sum_{r=0}^{L-1}W(f+rB)\,e^{\frac{j2\pi c_i r}{L}},\ f\in[0,B]\end{aligned} \tag{14}$$

The matrix form of Eqn. (14) is

$$y(f)=A(k)x(f)+n(f),\quad f\in[0,B] \tag{15}$$

where $n(f)$ is equivalent to the noise component and $y(f)$ is the observation vector given by

$$y(f)=\left[X_1(f),\ldots,X_p(f)\right]' \tag{16}$$

where $x(f)$ represents the unknown vector of signal spectrum parameters, given by:

$$x(f)=\begin{bmatrix}X(f+k_1B)\\X(f+k_2B)\\ \cdot\\ \cdot\\ \cdot\\ X(f+k_qB)\end{bmatrix},\quad f\in[0,B] \tag{17}$$

where $X(f+k_iB)$, $f\in[0,B]$ are the frequency elements of the signal in the occupied channel, labelled by k_i $A(k)\in C^{p\times N}$ is the modulation matrix, expressed as

$$A(k)(i,l)=B\,e^{\frac{j2\pi c_i k_l}{L}} \tag{18}$$

Assume a Gaussian complex noise $n(f)$, which is also uncorrelated with the signal, with distribution of $N(0,\sigma^2\mathrm{I})$.

The correlation matrix using the observations is constructed and is given by

$$\begin{aligned}R&=E\left[y(f)y^*(f)\right]\\R&=A(k)UA^*(k)+\sigma^2 I\end{aligned} \tag{19}$$

where

$$U = E\left[x(f)\ x^{*}(f)\right] \tag{20}$$

represents the correlation matrix of the signal vector[16].

To compute the real correlation matrix R, the unknown distribution of the signal is required. Therefore R is estimated by integrating $E\left[y(f)y^{*}(f)\right]$ over the interval $[0, B]$. It can also be directly computed in time domain of the sequences, using Parseval's identity, at a Nyquist rate equal to. Because each sequence $x'_{c_i}(m)$ is the output of a narrowband filter, it can easily be sampled at a lower rate. Thus it is not necessary to compute at a higher sampling rate $B_{\max}$. Thus the sequences are down sampled by a factor L such that

$$x_{d_i}(m) = x_{c_i}[mL] \tag{21}$$

The cumulative process from $x_i(m)$ to $x_{d_i}(m)$ is considered as a partial shifting of $x_i(m)$. The snapshot vector $x_d(m)$ may be defined as:

$$X_d(m) = \begin{bmatrix} x_{d_1}(m) \\ x_{d_2}(m) \\ . \\ . \\ x_{d_p}(m) \end{bmatrix} \tag{22}$$

The correlation matrix is computed using Eqn. (16)

$$\hat{R} = \frac{1}{M}\sum_{m=1}^{M} X_d(m) X_d^{*}(m) \tag{23}$$

assuming that $\hat{R} \to R$ s $M \to \infty$.

3.1.2 Subspace Analysis

In each of the spectral band the signal is assumed to be uncorrelated and distinct with other bands. Therefore, the correlation matrix will have full-rank. The subspace methods used are as described as follows.

(a) Estimation of the Number of Occupied Channels

The eigenvectors of R, with respective eigenvalues σ^2, geometrically is orthogonal to the modulation matrix $A(k)$, as given by the relation (19). All the other eigenvectors are in the range space of $A(k)$ and thus they are called as signal eigenvectors. Decomposing R into signal and noise subspaces[16], gives -

$$R = E_s \Lambda_s E_s^{*} + E_n \Lambda_n E_n^{*} \tag{24}$$

where Λ_s is the diagonal matrix of the signal eigenvalue, Λ_n is the diagonal matrix of the noise Λ eigenvalue, and E_s is the matrix of the signal eigenvector and E_n is the matrix of the noise eigenvectors. The eigenvector matrix E_s, span the range space of $A(k)$, which denotes the signal subspace and the noise eigenvector is $E_n \perp A(k)$. Using the orthogonality property, the signal parameters are estimated by evaluating the dimension of the noise subspace.

The ordered eigenvalues of $\hat{R}$ are $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_q \geq \ldots \geq \lambda_p$, where q eigenvalues are significant, and the remaining are all equal to (σ^2)[17]. Thus, for a large M, the dimension of signal vector can be evaluated from the multiplicity of the least eigenvalues of $\hat{R}$. In practice the number of samples is a function of the sensing period. Thus, ideally the number of samples must be least possible, more so in case of time-varying channels. Therefore, the sample matrix $\hat{R} \neq R$.

Further, the separation between the signal and noise eigenvalues needs a threshold level. This threshold level depends on the number of samples and the corresponding noise power. For the present work the noise power is presumed to be a constant factor and the threshold is kept fixed throughout the analysis for the detection of the input frequencies. The upper half of the Fig. 3 shows the ordered eigenvalues that are bifurcated by a horizontal line representing the threshold level. It is observed that there are four eigenvalues greater than the threshold. This indicates the number of occupied channels. To overcome the difficulties of setting the threshold, theoretical information criteria, such as minimum description length procedures detailed below have been used.

In minimum description length, the number of occupied channels satisfying the criterion within the range $q_{\min} \leq r \leq q_{\max}$ can be evaluated using[17,18]

$$\hat{q} = \arg\min_{r}\ -M(p-r)\log\frac{g(r)}{a(r)} + \frac{1}{2}r(2p-r)\log M \tag{25}$$

where M is the number of samples; $g(r)$ is the geometric

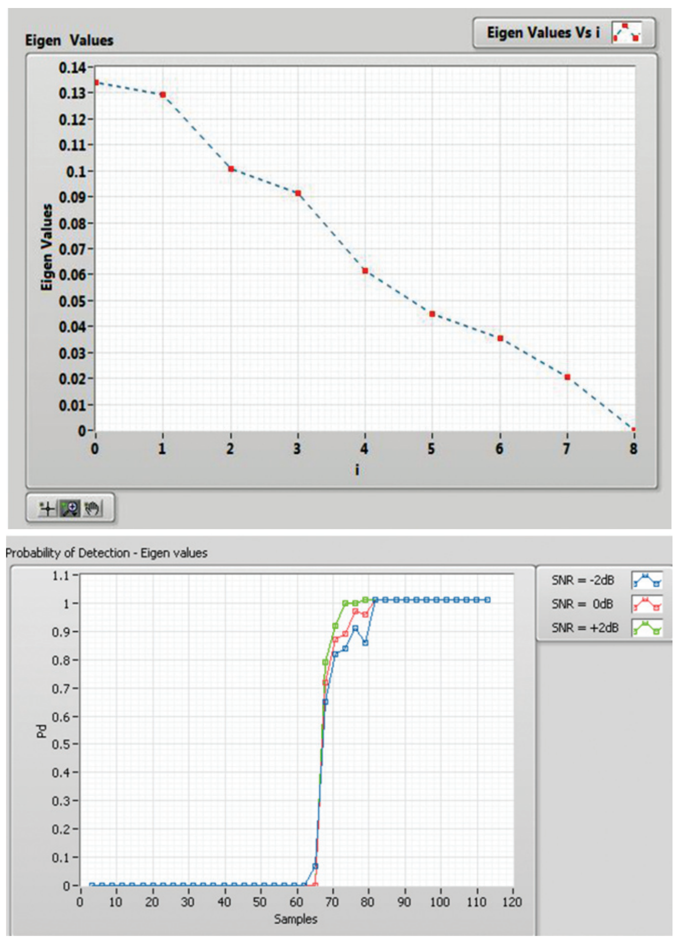


Figure 3. Ordered eigenvalues of the correlation matrix and Probability of detecting the number of occupied channels.

mean of the $(p-r)$ eigenvalues of the correlation matrix and is given by $g(r)=\prod_{i=r+1}^{p}\lambda_i^{\frac{1}{p-r}}$; $a(r)$ is the arithmetic mean of the $(p-r)$ eigenvalues of the correlation matrix and is given by $a(r)=\frac{1}{p-r}\sum_{i=r+1}^{p}\lambda_i$.

The probability of detection of the number of occupied channels, [i.e., $P_d=\text{Prob}[\hat{q}=q]$], is related to noise distribution, SNR and the number of samples, and is always not equal to unity. However, this can be improved by using the method of locating peaks, as done in Pisarenko harmonic decomposition (PHD) algorithm.

(b) Occupied Channel Set Recovery

$\hat{E}_n$ is denoted as a $p\times(p-\hat{q})$ matrix consisting of all noise eigenvectors in descending order. After estimating the occupied channels, the eigenvector corresponding to the minimum eigenvalue $\lambda_{\min}$ is chosen and is denoted by $v_{\min}$.

The location of occupied channels is recovered using the eigenspectrum of the Pisarenko harmonic decomposition (PHD) algorithm[19].

$$P_{PHD}(k)=\frac{1}{\left\|a_k v_{\min}\right\|_2^2} \quad , \quad 0\le k\le L-1 \tag{26}$$

where the notation $\|\ \|_2$ stands for the l_2-norm and k is the channel index. Also, a_k is a column of $A(k)$, given by

$$a_k=\frac{1}{LT}\left[e^{\frac{j2\pi kc_1}{L}}, e^{\frac{j2\pi kc_2}{L}}, \ldots, e^{\frac{j2\pi kc_p}{L}}\right]^T \tag{27}$$

In general, the algorithm generates as many values as there are number of channels (L). If k is the labelled as an occupied channel, $P_{PHD}(k)$ is significant at that point. The location of occupied channels is obtained by comparing the $P_{PHD}(k)$ values with a threshold value as follow:

$$\hat{k}=\{k_i\,|P_{PHD}(k)>threshold\} \tag{28}$$

Figure 4 shows the spectral index of the occupied channel set and it is observed that the presence of an occupied channel corresponds to significant values of $P_{PHD}(k)$. The remaining are obviously unoccupied channels available for the cognitive radio. After finding the spectral index, the procedure of reconstruction is the same as that of the known spectrum. The reconstructed signals in frequency domain are shown in the Fig. 5.

4. RESULTS AND ANALYSIS

The process of blind spectrum sampling and reconstruction using PHD algorithm is implemented in Lab VIEW™ and the performance analysis is carried out through MonteCarlo simulations.

The input signal by the CR is generated using

$$x(t)=\sum_{i=1}^{N}A_i\ \text{sin}\,c(B_i(t-t_i))\,e^{j2\pi f_i t} \tag{29}$$

where $\text{sin}\,c(x)=\sin(\pi x)/(\pi x)$ and N is the number of bands. The amplitude, the bandwidth, the time offset and the carrier frequency of the i th band are, respectively, A_i, B_i, t_i and f_i .

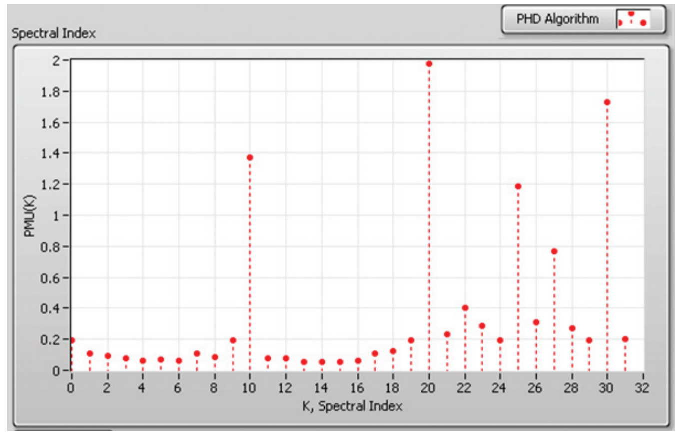


Figure 4. Spectral index of the occupied channel set.

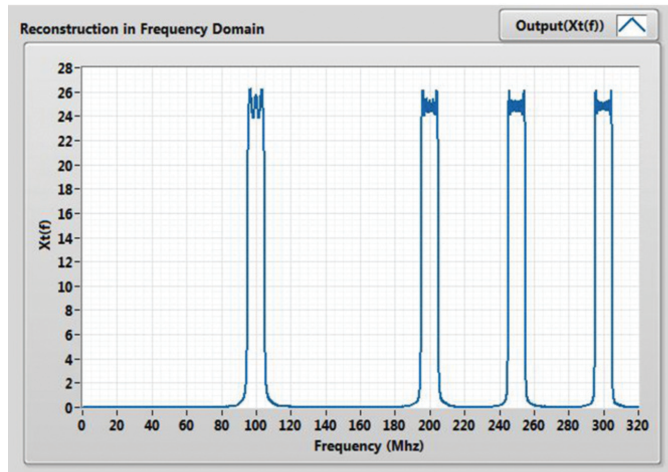


Figure 5. The reconstructed signals in frequency domain.

It is assumed that there are $N=4$ subbands, in the frequency range $F\in[0,B_{\max}]=(0,320)$ MHz . Hence the Nyquist rate for this signal is $B_{\max}=\frac{1}{T}=320$ MHz .

The bandwidths of the subbands are $B_1=B_2=B_3=B_4=10$ MHz .The central frequencies of the subbands are f_1 = 100 MHz, f_2 = 200 MHz, f_3 = 250 MHz, and f_4 = 300 MHz.

The Landau lower bound[20] for the set F, is defined as

$$\lambda(F)=\sum_{i=1}^{N}(b_i-a_i) \tag{30}$$

$$\lambda(F)=\sum_{i=1}^{4}B_i\ =\ 10+10+10+10=40\ .$$

To quantify the sampling efficiency for signals with a given spectral support F, the spectral occupancy ratio is given by

$$\Omega=\frac{\lambda(F)}{B_{\max}}=\frac{40}{320}=0.125 \tag{31}$$

The selection of parameters L, P, and set C are more important in the unknown spectral case. Discovering the number of occupied channels cannot be achieved exactly owing to unknown band locations.

L is calculated using the relation $L=\frac{B_{\max}}{B}=\frac{320}{10}=32$.

The spectral index is computed using the expression $k=\bigcup_{i=1}^{N}k_i$. Thus $k=\{10,20,25,30\}$ and $q=|k|$. Thus, selecting $p=q_{\max}+1=9$ is sufficient for a perfect reconstruction. The sampling pattern C can be obtained using the sequential forward selection algorithm as $C = \{0,1,2,8,10,16,18,24,26\}$. The corresponding condition number for this sampling pattern is $cond\left(A(k)\right) = 3.2$. The average sampling rate f_{avg} is given by $f_{avg} = \gamma * B_{\max}$, where $\gamma = p/L = 9/32 = 0.28125$ is termed as the subNyquist factor.

The process of simulation generated $M = 1024$ samples of $x(t)$, uniformly with $T = 1/320 = 0.003125$ and then the sequence of $x_i(n)$ for $i = 1,2,...,9$ is created by picking the c_i th sample and zero padding inter sample distance by $L-1 = 31$ zeroes. The sequences were filtered with a low-pass filter having the cut off frequency as $f_c = B_{\max} / L$. To create a real valued (nonideal) lowpass filter $h_r[n]$ of length $N_h = 383$, with normalised cut off frequency at $f_c = 1/L$, passband ripple of 0.02 and stopband ripple of 0.008, the frequency is shifted to obtain the complex filter $h[n] = h_r[n]\exp(j\pi n/L)$. The operation of filtering with $h[n]$ introduces a delay of t_d at the output that is equal to $t_d = (Nh+1)/2 = 192$ samples; hence the sample numbers are considered from $t_d + 1 = 193$ onwards.

The correlation matrix $\hat{R}_{9\times 9}$ is calculated using the Eqn (23). The ordered eigenvalues λ_1 to λ_9 and the corresponding eigenvectors of $\hat{R}$ are computed.

Using Eqn (25) the number of occupied channels are estimated to be $\hat{q} = 4$. The estimated spectral index from the Eqn (28) is found to be $\hat{k} = \{10,20,25,30\}$.

At low SNR values and for different number of samples (M), the probability of detecting the four occupied channels is empirically estimated. The PHD estimation function is expected to sense the occupied channels with a higher probability as M and SNR increases.

The lower half of the Fig. 3 depicts the computed probability of detecting the number of occupied channels $P_r(\hat{q} = 4)$ as a function of number of samples for different lower SNR values.

The number of occupied channels N is estimated using the eigenvalues of the correlation matrix, by using the 100 MonteCarlo simulations, along with the different values for M and SNR, using Eqn (25). It is expected that a high probability of detection can be achieved with increase in M and SNR.

It is observed from lower half of the Fig. 3 that for an SNR equal to +2 dB, P_r becomes 90 per cent after $M \geq 72$. This means that for an SNR equal to +2 dB, the number of occupied channels is detected with more than 72 samples. Also it is observed that for an SNR equal to 0 dB, the occupied channels can be detected with more than 74 samples and for SNR equal to 2 dB the number of samples required is more than 76. It indicates that for lower SNRs, more number of samples is required to sense the occupied channels, with a higher probability.

It is indicated from the lower half of the Fig. 3 that for an SNR equal to 0 dB the probability of detection curve indicated that one can achieve 90 per cent probability of detection for 74 samples. This is compared with the spectrum sensing techniques using music-like algorithm where the results of the analysis indicate for an SNR equal to +0 dB the achieved probability of detection of 90 per cent requires 41 samples using 1000 Monte-Carlo simulations. However the higher number of samples indicated by the proposed PHD technique can be attributed to the lower number of 200 Monte-Carlo simulations conducted.

Table 1 lists the results deduced from the lower half of the Fig. 3 which gives the subNyquist samples required for detecting the four occupied channels for different lower values of SNR.

Table 1. Results deduced from the P_d as a function of *m* and SNR

Frequency range (MHz)	No. of channels	Centre frequencies (MHz)	Channel bandwidth (MHz)	Total no. of samples	SubNyquist Sample (Pd=90 %)
0-320	4	100, 200, 250, 300	10, 10, 10, 10	320	70 (SNR = +2dB)
0-320	4	100, 200, 250, 300	10, 10, 10, 10	320	74 (SNR = 0dB)
0-320	4	100, 200, 250, 300	10, 10, 10, 10	320	76 (SNR = 2dB)

It is clear that the number of samples required for detecting the occupied channels with high probability increased from 70 samples to 76 samples as the SNR decreased from +2 dB to -2 dB (90 per cent). It therefore suggests that, the number of samples required to sense the occupied channels with higher probability increases with a decrease in SNR.

Further, the detection performance of the PHD algorithm is assessed by computing the probability of detection of the occupied channel as

$$P_{\mathrm{d}} = \frac{1}{N}\sum_{i=1}^{N} Pr\left(k_i \in \hat{\mathbf{k}} \middle| k_i \in \mathbf{k}\right) \tag{32}$$

and the probability of false alarm as

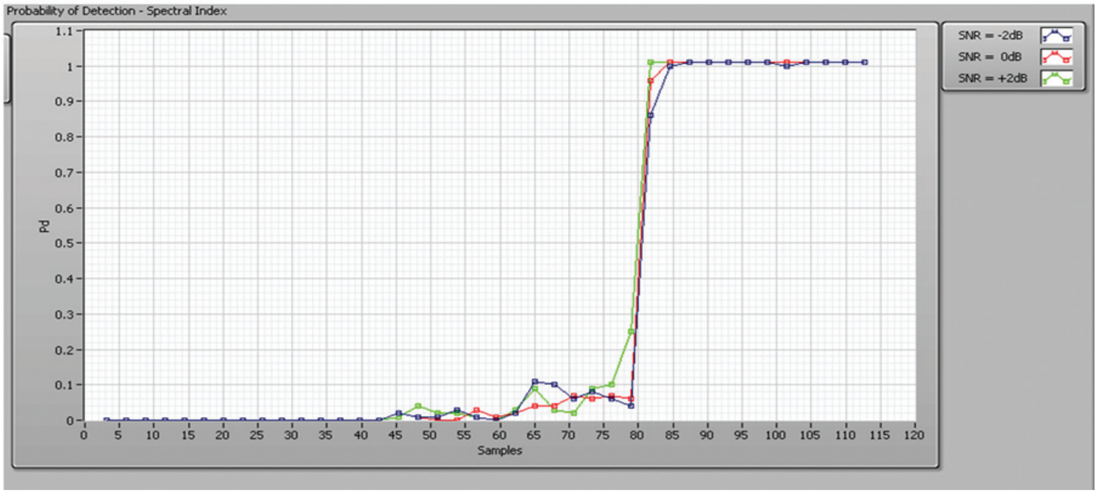
$$P_{\mathrm{f}} = \frac{1}{L-N}\sum_{i=1}^{L-N} Pr\left(k_i^c \in \hat{\mathbf{k}} \middle| k_i^c \in \mathbf{k}^c\right) \tag{33}$$

where $\mathbf{k}^c \in - \ \mathbf{k}$ is the complement set of $\mathbf{k}$.

P_d and P_f are computed from Eqn (32) and Eqn (33) for different M values and SNR. The results are displayed as in Fig. 6. The results show excellent detection performance even at low SNRs and smaller values of M. It is observed that at an SNR equal to+2 dB and the number of samples $M \geq 76$, the PHD algorithm senses the occupied channels with unit probability. The values of P_f drastically decreases with an increase in M. Thus for $M \geq 30$, it is near zero for all values of SNR. Figure 6 reveals that a perfect detection of channel occupancy is possible with a perfect estimation of N.

A comparison of the proposed PHD scheme with other existing schemes is shown in Table 2.

It is clearly seen in the Table 2 that the proposed PHD scheme has a good compression capability for detection of input frequencies with subNyquist samples alone, and has a low implementation complexity compared to other schemes. In view of this observation, it is suggested that the proposed PHD scheme is better than other schemes.



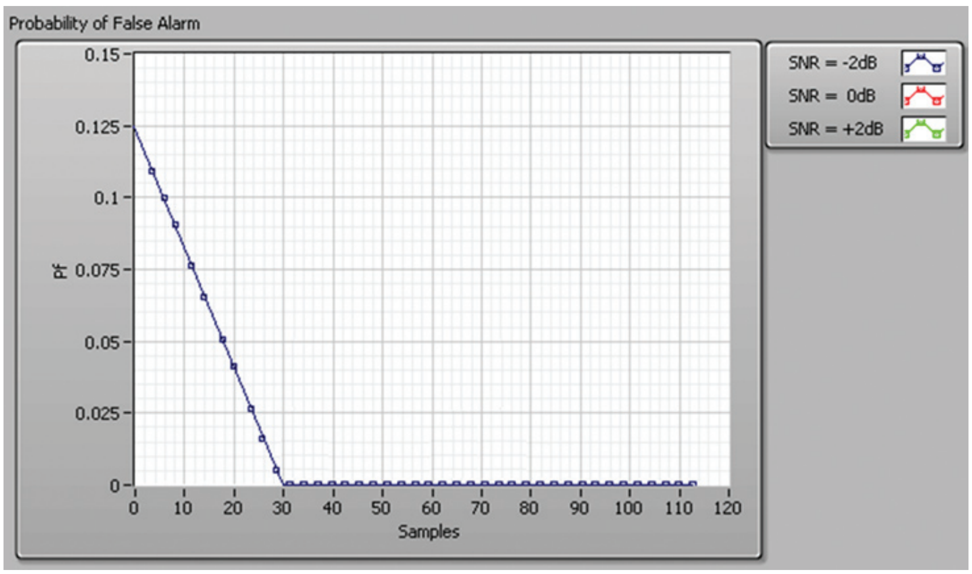


Figure 6. Probability of detection and probability of false alarm as a function of no. of samples for a given SNR.

Table 2. Comparisons between wideband spectrum sensing techniques

Techniques	Compression capability	Sampling scheme	Implementation complexity
Wavelet detection	No	Nyquist	Low
Multiband joint detection	No	Nyquist	High
Filter band detection	No	Nyquist	High
Music-like algorithm	Yes	subNyquist	Medium
Proposed PHD method	Yes	subNyquist	Low

5. CONCLUSION

The limitations of noise uncertainty, high complexity and the need for high sampling rate that exists can be overcome by the PHD method for broadband spectrum sensing. The multicoset sampling scheme has been used at a low sampling rate, near to the channel occupancy. The problem of spectrum sensing has been morphed into the problem of estimating the parameters which has been solved by subspace methods. The multicoset samples have been fractionally shifted and have been used to compute the correlation matrix of the sampled data. The computational complexity has been observed to be linearly proportional to the amount of data, under the assumption of low spectrum utilisation, and results in considerable savings in terms of the sampling rate.

For a broadband system with an SNR of +2 dB, it requires 82 samples, at a subNyquist rate (γ) of 28 per cent. The detection probability of 0.999 and the probability of the false alarm of 0.001 have been achieved.

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