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Design of Frequency Invariant Wideband Beamformer with Real and Symmetric FIR Filters

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ABSTRACT

An approach to wideband beamformer with frequency invariant property is proposed by optimising the space-time two-dimensional finite impulse response (FIR) filters. Frequency invariant beam pattern brings some restrictions on frequency bandwith and filter length. Using the Landau-Pollak theorem to estimate the rank of wideband signals, we give the lower bound of filter length respect to array element numbers and the relative bandwidth of wideband signal. With the expression of beam pattern using real and symmetric FIR filters, we develop corresponding optimisation formation considering the robustness of beamformer as well, which can be easily solved by the least mean square (LMS) algorithm. A design example is provided to show the effectiveness of the proposed method.

Keywords: Wideband beamformer, frequency invariant, finite impulse response filters, Landau-Pollak theorem

1. INTRODUCTION

Wideband beamforming has been studied extensively due to its potential application in radar, sonar, and communication[1,2]. For wideband signals, the inter-element phase shift is frequency dependent. If using traditional beamforming techniques for narrowband signals, significant degradation of performance will occur. According to previous research, three main techniques are proposed for wideband beamforming, subband processing[3,4,5], direct two-dimensional discrete Fourier transformation (2D-IDFT) method[6,7,8], and FIR filters structure[9,10,11].

Using non-overlapped band-pass filters, the wideband signal can be decomposed into several narrowband signals. Performing narrowband beamforming technique to each subband and summing up the outputs, the resulting wideband beamformer is achieved. Subband processing provides an easy way to deal with the wideband signal, but also introduces some problems, such as the non-continuity in phase of the output signal.

By exploiting the Fourier transform relationship between the array's spatial and temporal parameters and its beam pattern, L. Wei[7,8] proposed approaches for frequency invariant beamformer. Starting from the desired frequency invariant beam pattern, using a series of substitutions and IDFT, the desired frequency response of each array element are obtained. In the direct IDFT structure, the frequency sampling mode will influence the system performance.

In FIR filters structure, a bank of filters appending to each array elements are used to form frequency dependent response to compensate the inter-element phase shift. The coefficients of each filterbanks can be achieved by optimisation methods such that the resulting beam pattern approximates the desired one. However, to get frequency invariant beamformer, there are some restrictions on these filters. On the other hand, the number of coefficients to be optimised will be extremely large for large arrays, which brings difficulty in computation.

2. WIDEBAND BEAMFORMING

For clarity in this article, a uniformed linear array (ULA) with isotropic antenna elements is considered. The extensions to other array configurations such as sparse linear array, two or three dimensional arrays could be gotten by similar idea.

2.1 Wideband Array Pattern Response

The ULA has N array elements aligned with the x-axis with inter-element distance d. The angle θ is measured with respect to the x-axis, with zero degrees lying perpendicular to the axis. The far-field beam pattern of the ULA is given as[1]

$$P(f,\theta) = \sum_{n=0}^{N-1} a_n e^{-j2\pi fnd\sin\theta/c} \tag{1}$$

where c is the propagation speed in free space, f is the radiating frequency and a_n are complex weight coefficients which are chosen to steer the beam and to control sidelobes.

For wideband arrays, the radiating frequency f will cover a range of finite bandwidth, i.e., $f \in [f_l, f_u]$, where f_l and f_u are the lower and upper bound frequency respectively. The bandwidth $B = f_u - f_l$. From Eqn (1), we can see that the beam pattern of wideband array changes with frequency.

2.2 Wideband Beamforming with FIR Filterbanks

In order to compensate the frequency dependence of wideband beam pattern, a solution[2] is replacing the complex coefficients a_n with frequency responses $H_n(f)$, i.e.

$$P(f,\theta) = \sum_{n=0}^{N-1} H_n(f) e^{-j2\pi fnd\sin\theta/c}, \tag{2}$$

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A special realization of frequency responses $H_n(f)$ is the space-time processor[12] as shown in Fig. 1.

In Fig. 1, the desired frequency response $H_n(f)$ of the n^{th} array element is obtained by FIR filters with length M, i.e.

$$H_n(f) = \sum_{m=0}^{M-1} h_{m,n} e^{-j2\pi f m T_s}, n = 0,1,...,N-1 \quad (3)$$

where T_s is the sampling period. The wideband beam pattern with FIR filterbanks can be expressed as

$$P(f,\theta) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} h_{m,n} e^{-j2\pi f m T_s} e^{-j2\pi f n d \sin\theta / c} \quad (4)$$

In the common case, the inter-element distance d is set to be half-wavelength of the maximum working frequency to avoid grating lobes, i.e., $d = \lambda_{min}/2 = c/(2f_u)$. The sampling frequency $f_s = 1/T_s$ is usually twice of the maximum working frequency, i.e., $f_s = 2f_u$. Then we can get $d/c = 1/(2f_u) = 1/f_s = T_s$. Substitute $d/c = T_s$ into Eqn (4), it yields

$$P(f,\theta) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} h_{m,n} e^{-j2\pi f (m+n\sin\theta) T_s} \quad (5)$$

Now, the wideband beamforming problem is transformed into designing the filterbank coefficients $h_{m,n}$ to insure that the resulting beam pattern approximate the desired one over the working frequency band.

2.3 Restrictions of Frequency Invariant Wideband Beamforming

To be frequency invariant, the beam pattern $P(f, \theta)$ must be a function of only θ, or $\sin\theta$. Let $F(\sin\theta)$ be such a frequency invariant beam pattern. From Eqn (5), it seems that the filter coefficients $h_{m,n}$ could be computed directly and easily by applying 2D-IDFT on $F(\sin\theta)$. However, there are some restrictions between $F(\sin\theta)$ and $h_{m,n}$.

2.3.1 Restriction on Bandwidth

Frequency invariant beam pattern could only be obtained over limited working frequency band. Otherwise, if $P(f_1,\theta) = P(f_2,\theta)$ for arbitrary frequency f_1 and f_2, we can get

$$\sum_{n=0}^{N-1} \sum_{m=0}^{M-1} h_{m,n} (e^{-j2\pi f_1 x_{m,n}} - e^{-j2\pi f_2 x_{m,n}}) = 0, \forall f_1, f_2 \quad (6)$$

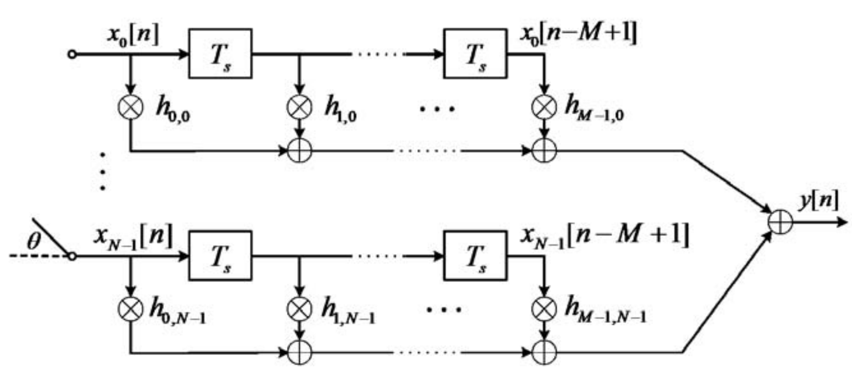

Figure 1. Wideband beamforming by space-time processor.

where $x_{m,n} = (m + n\sin\theta)T_s$. By properly choosing a series of discrete frequencies $f_{1,k}$ and $f_{2,k}$, k=1,2,...,K, the Eqn (6) can be written as

$$\mathbf{Ah} = 0 \quad (7)$$

where

$$[\mathbf{A}]_{k,m+nM} = e^{-j2\pi f_{1,k} x_{m,n}} - e^{-j2\pi f_{2,k} x_{m,n}}, \ [\mathbf{h}]_{m+nM} = h_{m,n}$$

For a lot of choices of $f_{1,k}$ and $f_{2,k}$, the matrix **A** is non-singularity and **h**=0. Therefore, to get non-zeros FIR filterbanks, the working frequency band must be limited.

2.3.2. Restriction on Filterbanks

For given frequency bandwidth $B = f_u - f_l$, the length of filterbanks M must be long enough to ensure that the space-time processor in Fig. 1 has sufficient degrees of freedom (DOF) to form the desiring beam pattern. With sufficient large M, the equations in (7) could have non-zero solutions. On the other hand, filterbanks with real coefficients are easier to be implemented than those with complex coefficients. Further, if the filterbank coefficients are symmetric, the implementation costs and computation requirements can be reduced. Therefore, we limit the coefficients $h_{m,n}$ to be real and symmetric, i.e.

$$h_{m,n} = h_{m,N-1-n}, h_{m,n} = h_{M-1-m,n} \quad (8)$$

3. DESIGN APPROACH

3.1 Determination of Filterbank Length

For the space-time processor in Fig. 1, the system DOF r_s is defined as the number of free parameters. Based on the adaptive signal processing theory, the system DOF r_s should to larger than the DOF of the received wideband signal to form effective beam pattern[12]. If the space-time processor has real and symmetric filterbank coefficients, the system DOF r_s can be computed directly by counting the number of free parameters and given as

$$r_s = \begin{cases} MN/4 & M, N\ even \\ (M+1)(N+1)/4 & M, N\ odd \end{cases} \quad (9)$$

We denote r_w as the DOF of the received wideband signal. It can be estimated by the Landau-Pollak theorem which is expressed as follows.

Theorem[13,14]: A signal with frequency bandwidth B and time duration T has its energy concentrated in its largest r_w eigenvalues, $r_w = 2BT + 1$.

For wideband beamforming problem in Eqn (5), the equivalent space-time two-dimensional signal has the following form,

$$s_{m,n} = e^{-j2\pi f (m+n\sin\theta) T_s} \quad (10)$$

where $m = 0,1,...,M-1$, $n = 0,1,...,N-1$ and $\theta \in (\theta_{min}, \theta_{max})$. According to the space time equivalence[12,14], the two-dimensional signal above can be viewed as the equivalent one-dimensional signal with bandwidth $B = f_u - f_l$ and a maximal time duration of $(M-1+2(N-1))T_s$. Applying the Landau-Pollak theorem, the DOF of the received wideband signal can be easily obtained, i.e.,

$$\begin{aligned} r_w &= 2BT+1 \\ &= 2(f_u - f_l)(M-1+2(N-1))T_s+1 \\ &= 2(f_u - f_l)(M+2N-3)/(2f_u)+1 \\ &= (2N+M-3)B_r+1 \end{aligned} \quad (11)$$

where $B_r = (f_u - f_l)/f_u$ is the relative bandwidth of interesting signal frequency.

Therefore, we get the conditions for filterbank length M as follow

$$r_s > r_w \Rightarrow \begin{cases} M > \dfrac{(2N-3)B_r+1}{N/4-B_r} & N \text{ even} \\ M > \dfrac{(2N-3)B_r - N/4+3/4}{(N+1)/4-B_r} & N \text{ odd} \end{cases} \quad (12)$$

3.2 Optimisation of Filterbank Coefficients

The optimisation of filterbanks is to insure that the resulting wideband beam pattern approximate the reference pattern over the interesting frequency band. For real and symmetric filterbanks, after substituting Eqn (8) into Eqn (5), the resulting beam pattern is real and could be expressed as follows. When M and N are even numbers,

$$P(f,\theta) = \sum_{n=0}^{N/2-1}\sum_{m=0}^{M/2-1}\left\{h_{m,n}\cos[\pi f(M-2m-1)T_r] \times\cos[\pi f(N-2n-1)\sin(\theta)T_r]\right\} \quad (13)$$

When M and N are odd numbers,

$$P(f,\theta) = \sum_{n=0}^{(N-1)/2}\sum_{m=0}^{(M-1)/2}\left\{h_{m,n}\cos[\pi f(M-2m-1)T_r] \times\cos[\pi f(N-2n-1)\sin(\theta)T_r]\right\} \quad (14)$$

Let $F(\sin\theta)$ be the desired frequency invariant pattern at angle θ, which is usually given by applications. The optimisation problem is to find $\{h_{m,n}\}$ such that $P(f,\theta)$ approximates $F(\theta)$ over the working frequency band $[f_l, f_u]$. At the same time, to improve the robustness of wideband beamformer against random errors, we need to constrain the norm of $h_{m,n}$ under some known level δ. D. P. Scholnik and J. O. Coleman[9] firstly introduced this constraint to avoid a large noise gain. One can get more details in this reference. Therefore, the optimisation problem can be formulated as

$$\begin{aligned} & min_{\mathbf{h}} \quad \|P(f,\theta)-F(\sin\theta)\|_2^2,\ f\in[f_l, f_u] \\ & s.t. \quad \|\mathbf{h}\|_2 \le \delta \end{aligned} \quad (15)$$

where $\|\cdot\|_2$ is the Euclidean norm.

By discretizing the frequency band and angle range with a finite number of samples, $f_k, k=0,1...,K-1$ and $\theta_l, l=0,1,...,L-1$, the problem (15) can be reformulated as

$$\begin{aligned} & min_{\mathbf{h}} \quad \|\mathbf{Ah}-\mathbf{F}\|_2^2 \\ & s.t. \quad \|\mathbf{h}\|_2 \le \delta \end{aligned} \quad (16)$$

where

$$\begin{cases} [\mathbf{A}]_{l+kL,m+nM} = \cos[\pi f_k(M-2m-1)T_r] \times\cos[\pi f_k(N-2n-1)\sin\theta_l\cdot T_r] \\ [\mathbf{F}]_{l+kL} = F(\sin\theta_l) \\ [\mathbf{h}]_{m+nM} = h_{m,n} \\ k=0,1,...,K-1; l=0,1,...,L-1 \\ m=0,1,...,\lfloor M/2-1\rfloor; n=0,1,...,\lfloor N/2-1\rfloor \end{cases}$$

By introducing a new non-negative variable λ, Eqn (16) can be converted to an equivalent optimisation without constraints as

$$\min_{\mathbf{h}}\left(\|\mathbf{Ah}-\mathbf{F}\|_2^2+\lambda\|\mathbf{h}\|_2^2\right) \quad (17)$$

Let $L(\mathbf{h}) = \|\mathbf{Ah}-\mathbf{F}\|_2^2+\lambda\|\mathbf{h}\|_2^2$. The gradient of $L(\mathbf{h})$ is given by

$$\frac{\partial L(\mathbf{h})}{\partial\mathbf{h}} = 2\mathbf{A}^T(\mathbf{Ah}-\mathbf{F})+2\lambda\mathbf{h}$$

where T denotes the transpose operator. Let $\dfrac{\partial L(\mathbf{h})}{\partial\mathbf{h}}=0$ and it yields

$$\mathbf{h} = (\mathbf{A}^T\mathbf{A}+\lambda\mathbf{I})^{-1}\mathbf{A}^T\mathbf{F} \quad (18)$$

which is also known as the least mean square (LMS) solution.

4. SIMULATIONS

Simulations are performed to show the effectiveness of the proposed method. The frequency range of interest is set to be $1 < f \le 3$ GHz. The uniformed linear array has $N = 21$ elements with an adjacent element spacing $d = \lambda_{min}/2 = c/(2f_u) = 5cm$. The desired beam pattern is set to be the response for a narrow-band signal with frequency 1.5 GHz and using Taylor weights of -30 dB sidelobes. The Taylor weights[15] can provide a near optimum beamwidth for a given peak-sidelobe level. The expression of the desired beam pattern is given by

$$\begin{aligned} F(\sin\theta) &= \sum_{n=-10}^{10} w_n e^{-j2\pi nfd\sin\theta/c} \\ &= \sum_{n=-10}^{10} w_n e^{-j\pi nf/f_u\sin\theta} \\ &= \sum_{n=-10}^{10} w_n e^{-j\pi n\sin\theta/2} \end{aligned} \quad (19)$$

where w_n are Taylor weights of –30 dB sidelobes, which can be computed approximately by the following formula

$$\begin{cases} B = 0.9067\sqrt{(R+9.7)^2/(13.26+9.7)^2-1} \\ w_n = Besseli\left(0, \pi B\sqrt{1-(2n/\tilde{N}-1)^2/2\pi}\right) \end{cases} \quad (20)$$

where R=30, $\tilde{N} = N-1$. The desired beam pattern is shown in Fig. 2.

After discretizing the angle interval [0°,180°] and the frequency band [1,3] GHz into uniform grids, 100 angle samples $\theta_1, \theta_2, ..., \theta_{100}$ and 100 frequency points $f_1, f_2, ..., f_{100}$ are obtained to form the matrix **A** and the desired pattern **F**. The robust parameter δ is set to be 0.1. Using the formula (18), we get the impulse responses of FIR filters. The frequency magnitude of $H_1(f), H_2(f), \cdots, H_{21}(f)$ are given in Fig. 3. The achieved wideband beam pattern is shown in Fig. 4.

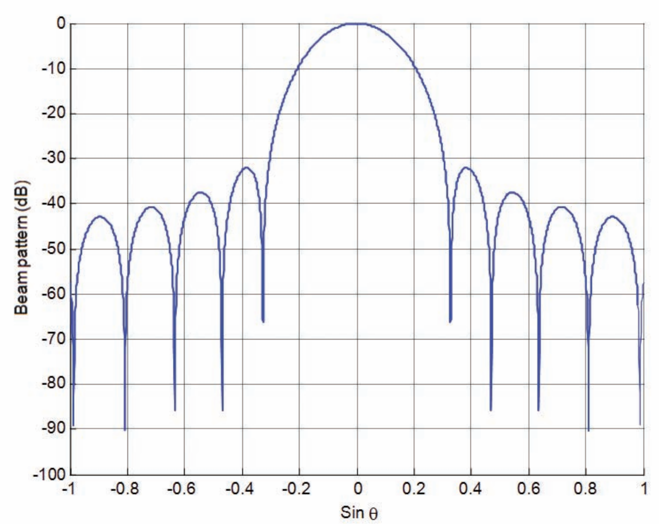

Figure 2. Desired beam pattern with Taylor weights.

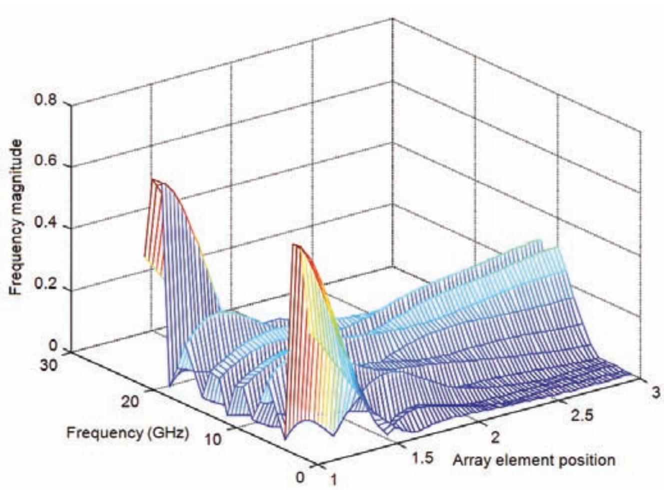

Figure 3. Frequency magnitude of FIR filters.

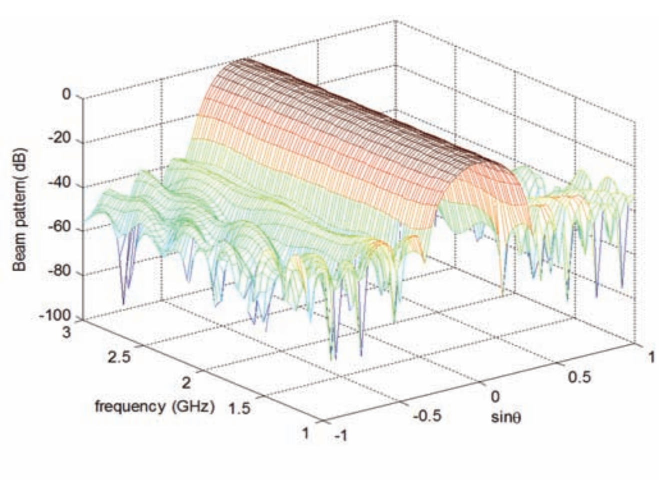

Figure 4. Resultant wideband beam pattern.

The beam pattern has a clear frequency invariant property which shows the effectiveness of the proposed beamforming method. One can set the robust parameter δ to be other values other than 0.1. With a smaller δ, the resulting frequency invariant beam pattern can approximate the desired pattern better. But it will be more sensitive to random noise. On the other hand, if δ is larger, the resulting beam pattern will be more robust.

5. CONCLUSION

In this article, we proposed a new wideband beamformer with frequency invariant property by optimising the coefficients of real and symmetric FIR filters. The lower bound of filter length is given by applying the Landau-Pollak theorem to the received wideband signals. The filter coefficients are solved by the LMS algorithm. The proposed method can also be generalized to sparse linear array, two or three dimensional arrays easily.

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