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Date: 19 December 2015

Subject: Submission of Manuscript

Dear Mr Gopal Bhushan,

I am submitting a manuscript of a research paper jointly authored with my research scholar whose details are given below. Kindly consider it for publication in Defence Science Journal.

Article Title: **Tracking Control Design for Quadrotor UAV**

Name of Journal: Defence Science Journal

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Statement of Relevance: The paper proposed control method for Quadrotor a flying robot

With warm regards,

A. Swarup

Tracking Control Design for Quadrotor UAV

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Abstract: The model of a quadrotor unmanned aerial vehicle (UAV) is nonlinear and dynamically unstable. A flight controller design is proposed on the basis of Lyapunov stability theory which guarantees that all the states remain and reach on the sliding surfaces. The control strategy uses sliding mode with a backstepping control to perform the position and attitude tracking control. This proposed controller is simple and effectively enhance the performance of quadrotor UAV. In order to demonstrate the robustness of the proposed control method, the aerodynamic forces, moments and air drag taken as external disturbance are taken into account. The performance of the nonlinear control method is evaluated by comparing the performance with developed linear quadratic regulator (LQR) and existing backstepping control technique[20]. The comparative performance results demonstrate the superiority and effectiveness of the proposed control strategy for the quadrotor UAV.

Keywords: Quadrotor UAV, Backstepping Control, Sliding Mode control, Lyapunov

1. INTRODUCTION

The applications of unmanned aerial vehicle (UAV) are becoming very popular in civil, dangerous environment, military and scientific research domains. The quadrotor is an important class of UAV which is vertical take-off and landing (VTOL) aircraft and having four lift generating propellers. This system is under actuated with six outputs, four inputs and highly coupled states. The dynamics of quadrotor is nonlinear and unstable. Developing control for such complex dynamics has become an active area of recent research interest.

The linear proportional-integral-derivative (PID) and pole placement controls have been applied to the quadrotor platform[1-2] providing smooth stabilization without any large overshoot and oscillations. These techniques are not very effective when the positional angles are not near to zero. LQR control technique[3] has been shown to improve the response. The linear controllers were very sensitive to its parameter and even small changes in the parameterization could lead to an unstable response. To solve the stabilization problem of quadrotor, different linearization control algorithms[4-7] have been proposed. These control techniques are restricted to control the certain condition like the hover flight condition. Therefore, the nonlinear control methods have been developed to improve the performance of quadrotor. Backstepping and sliding mode control have received attention in literature due to its ability for disturbance rejection, stability and robustness[9-21]. Backstepping control techniques[10-13] are based on Lyapunov stability theory to follow the desired trajectory and stabilize the whole system. Saif and Dhaifullah et.al[14] have proposed a modified backstepping control technique to reduce the control parameters by half as compared to the classical backstepping approach. The feedback linearization[15] coupled with a PD controller for a translational subsystem and backstepping-based PID controller for rotational subsystem has been used to improve the performance of quadrotor. The sliding mode control has been shown[17-22] to stabilize the quadrotor helicopter which can move it to any position with any yaw angle. An adaptive sliding mode controller has been developed[23] to improve performance and reliability, for handling aerodynamic parameter uncertainties and external disturbance. The main advantage of the nonlinear controllers is that, its parameters are very easy to tune.

A model of quadrotor[20] has been considered for developing the control. An attempt has been made in this paper to extend this model by including the disturbance term for a comprehensive dynamics similar to[24]. A control has been proposed using sliding mode with backstepping technique for obtaining position and attitude control of quadrotor. The proposed control provides very good response on quadrotor dynamics. Further a control has been developed using LQR and the performance of same model is compared with the proposed control technique. The effectiveness of proposed control has also been compared with an existing control technique[20]. The responses demonstrate that the proposed control method provides superior performance.

The paper is organized in the following sections. The dynamical modeling of micro quadrotor is presented in section 2. The development of sliding mode control based on backstepping control and LQR control techniques are discussed in section 3. The simulation results are presented in section 4. The performance of proposed controller is concluded in section 5.

2. QUADROTOR MODELLING

The dynamics of quadrotor derived using Euler-Lagrange Formalism[7] has been considered in this paper. Let the p, q and r denote its angular velocities in the body-frame; the outputs of the system are x, y and z, which are the positions of the center of gravity of the quadrotor; $\emptyset, \theta$ and φ represent the Euler angles and F_i $(i = 1,2,3,4)$ is the thrust force produced by each propeller[24]. Simultaneous increase or decrease in speed of two pairs of rotor (1, 3) and (2, 4) turn in opposite direction in order to balance the moments and produce vertical motion as shown in fig.1. Pitch angle θ, is obtained by increasing/decreasing the speed of motor pair (3, 1) independently about the y axis, which can be controlled with the indirect control of motion along the same axis. Similarly, the roll angle $\emptyset$ is obtained by increasing/decreasing the speed of motor pair (2, 4) independently about x axis, which can be controlled with the indirect control of motion along the same axis. Finally the yaw angle φ is obtained by counter clockwise rotation of motor pair (1, 3) and motor pair (2, 4), which can be controlled by z axis. Defining $E = [e_x, e_y, e_z]$, the Earth fixed frame and $B = [x, y, z]$, the Body fixed frame.

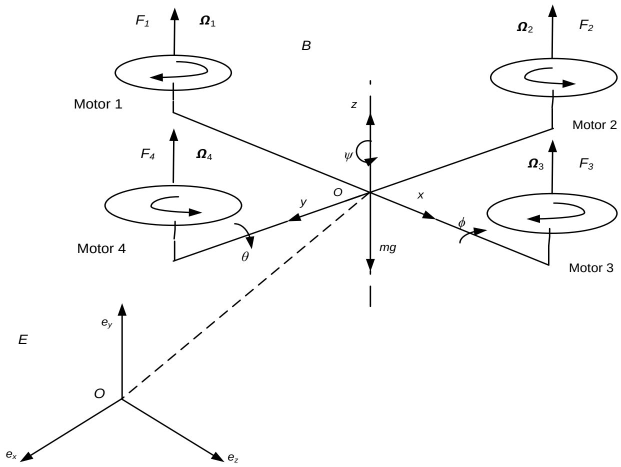


Fig: 1 Quadrotor UAV

The equation of motion for the quadrotor can be obtained using Lagrangian function L.

$$L = Kinetic\ Energy - Potential\ Energy$$

The dynamics in terms of L can be expressed as

$$\Gamma_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}$$

where $\dot{q}_i$ are the generalized coordinates and Γ_i are the generalized forces. Then the three equations of motion for roll, pitch and yaw are[7]

$$\begin{aligned} I_{xx}\ddot{\emptyset} &= \dot{\theta}\dot{\varphi}\left(I_{yy} - I_{zz}\right) \\ I_{yy}\ddot{\theta} &= \dot{\emptyset}\dot{\varphi}(I_{zz} - I_{xx}) \\ I_{zz}\ddot{\varphi} &= \dot{\emptyset}\dot{\theta}\left(I_{xx} - I_{yy}\right) \end{aligned} \tag{1}$$

where, I is the linear inertia, I_{xx}, I_{yy} and I_{zz} are the cross inertia resulted by interaction of two angular velocity. The torque applied on quadrotor along an axis depends on the difference between the torques generated by each propeller (Ω) on the other axis as

$$\begin{aligned} \tau_x &= bl(\Omega_4^2 - \Omega_2^2) \\ \tau_y &= bl(\Omega_3^2 - \Omega_1^2) \\ \tau_z &= d(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \end{aligned}$$

where, b, d and l are physical parameter and defined in the table 1.1.

Further, the actuator actions are described as

$$\begin{aligned} I_{xx}\ddot{\emptyset} &= \dot{\theta}\dot{\varphi}\left(I_{yy} - I_{zz}\right) - J\dot{\theta}\Omega_r + \tau_x \\ I_{yy}\ddot{\theta} &= \dot{\emptyset}\dot{\varphi}(I_{zz} - I_{xx}) - J\dot{\emptyset}\Omega_r + \tau_y \\ I_{zz}\ddot{\varphi} &= \dot{\emptyset}\dot{\theta}\left(I_{xx} - I_{yy}\right) + \tau_z \end{aligned} \tag{2}$$

where, Ω_r is the overall residual propeller angular speed and J is the rotational inertia.

Following the above development, complete quadrotor dynamical model[24] with x, y and z motions as a consequence of roll, pitch and yaw rotation, can be expressed as:

$$\begin{aligned} \ddot{\emptyset} &= \dot{\theta}\dot{\varphi}\left(\frac{I_y - I_z}{I_x}\right) - \frac{J_r}{I_x}\dot{\theta}\Omega + \frac{l}{I_x}U_2 - \frac{K_1 l}{I_x}\dot{\emptyset} \\ \ddot{\theta} &= \dot{\emptyset}\dot{\varphi}\left(\frac{I_z - I_x}{I_y}\right) - \frac{J_r}{I_y}\dot{\emptyset}\Omega + \frac{l}{I_y}U_3 - \frac{K_2 l}{I_y}\dot{\theta} \\ \ddot{\varphi} &= \dot{\emptyset}\dot{\theta}\left(\frac{I_x - I_y}{I_z}\right) + \frac{l}{I_z}U_4 - \frac{K_3 l}{I_z}\dot{\varphi} \\ \ddot{z} &= -g + (cos\emptyset\ cos\theta)\frac{1}{m}U_1 - \frac{K_4 l}{m}\dot{z} \\ \ddot{x} &= (cos\emptyset\ sin\theta\ cos\varphi + sin\emptyset\ sin\varphi)\frac{1}{m}U_1 - \frac{K_5 l}{m}\dot{x} \\ \ddot{y} &= (cos\emptyset\ sin\theta\ sin\varphi - sin\emptyset\ sin\varphi)\frac{1}{m}U_1 - \frac{K_6 l}{m}\dot{y} \end{aligned} \tag{3}$$

where, I_x, I_y, I_z, g and m are physical parameters, K_i denote the positive drag coefficients, defined in table 1.1. Now, considering the external disturbances and including them in the dynamics, in this paper, to modify the controller design.

The inputs to the quadrotor[9] namely the vertical forces U_1, the roll actuator input U_2, the pitch actuator input U_3 and the yaw moment input U_4 are defined as

$$U = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ b(\Omega_4^2 - \Omega_2^2) \\ b(\Omega_3^2 - \Omega_1^2) \\ d(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \end{bmatrix} \quad (4)$$

The physical parameters of the model have been taken from[24], for simulation exercise.

Table 1. Physical Parameters

Symbol	Definition	Value
m	Mass	.23kg
I_x	Inertia on x axis	7.5e-3 kgm^2
I_y	Inertia on y axis	7.5e-3 kgm^2
I_z	Inertia on z axis	1.3e-2 kgm^2
b	Thrust coefficient	3.13e-5 Ns^2
d	Drag coefficient	7.5e-7 Nms^2
J_r	Rotor inertia	6e-5 kgm^2
l	Arm length	0.23 m
g	Acceleration of gravity	9.8 m/s^2
$K_1 = K_2 = K_3$	Positive constants	0.01Ns/m
$K_4 = K_5 = K_6$		0.012Ns/m

3. FLIGHT CONTROLLER DESIGN

Sliding mode control based on backstepping and linear quadratic regulator control methods for 6DOF of quadrotor UAV are presented in this section.

3.1. Sliding Mode Control with Backstepping: The dynamic model developed in equation set (3) can be expressed in state space form $\dot{X} = f(X, U)$ by introducing state vector as

$$X^T = [\emptyset, \dot{\emptyset}, \theta, \dot{\theta}, \varphi, \dot{\varphi}, z, \dot{z}, x, \dot{x}, y, \dot{y}].$$

$x_1 = \emptyset$, $x_2 = \dot{x}_1 = \dot{\emptyset}$, $x_3 = \theta$, $x_4 = \dot{x}_3 = \dot{\theta}$, $x_5 = \varphi, x_6 = \dot{x}_5 = \dot{\varphi}, x_7 = z$, $x_8 = \dot{x}_7 = \dot{z}, x_9 = x, x_{10} = \dot{x}_9 = \dot{x}$, $x_{11} = y$, $x_{12} = \dot{x}_{11} = \dot{y}$ (5)

From equations (3) and (5) the dynamics is formulated as

$$\dot{x} = f(X, U) = \begin{pmatrix} x_2 \\ x_4 x_6 a_1 + x_4 a_2 \Omega + b_1 U_2 - \frac{K_1 l}{I_x} x_2 \\ x_4 \\ x_2 x_6 a_3 + x_2 a_4 \Omega + b_2 U_3 - \frac{K_2 l}{I_y} x_4 \\ x_6 \\ x_4 x_6 a_5 + b_3 U_4 - \frac{K_3 l}{I_z} x_6 \\ x_8 \\ -g + (\cos x_1 \cos x_3) \frac{1}{m} U_1 - \frac{K_4 l}{m} x_8 \\ x_{10} \\ u_x \frac{1}{m} U_1 - \frac{K_5 l}{m} x_{10} \\ x_{12} \\ u_y \frac{1}{m} U_1 - \frac{K_6 l}{m} x_{12} \end{pmatrix} \quad (6)$$

where $a_1 = (I_y - I_z)/I_x$, $a_2 = -J_R/I_x$, $a_3 = (I_z - I_x)/I_y$, $a_4 = -J_R/I_y$ $a_5 = (I_x - I_y)/I_z$

and $b_1 = l/I_x$, $b_2 = l/I_y$, $b_3 = l/I_z$

and $u_x = \cos x_1 \sin x_3 \cos x_5 + \sin x_1 \sin x_5$, $u_y = \cos x_1 \sin x_3 \cos x_5 - \sin x_1 \cos x_5$

Classical sliding mode control has the property to keep the system state trajectory on a chosen surface called the sliding surface by using the discontinuous control. In this paper a sliding mode control algorithm is developed based on backstepping for the flight controller design of the quadrotor unmanned aerial vehicle.

3.1.1 Attitude Control: The choice of sliding surface[17] calculation concerning the tracking error is defined as:

$$z_i = x_{id} - x_i \quad i \in \{1,3,5,7,9,11\}$$
$$z_i = x_i - \dot{x}_{(i-1)d} - \alpha_{(i-1)} z_{(i-1)} \quad i \in \{2,4,6,8,10,12\}$$

with $\alpha_i > 0 \;\; \forall\, i \in [1,12]$

Considering the Lyapunov function,

$$V_i = \frac{1}{2} z_i^2, \quad i \in \{1,3,5,7,9,11\}$$

and

$$V_i = \frac{1}{2}(V_{i-1} + z_i^2), i \in \{2,4,6,8,10,12\}$$

These satisfy the necessary sliding condition $S\dot{S} < 0$.
Let $i = 1{,}2$ surfaces are

$$z_1 = x_{1d} - x_1$$
$$s_2 = z_2 = x_2 - \dot{x}_{1d} - \alpha_1 z_1 \quad (7)$$

and Lyapunov function is

$$V(z_1, s_2) = \frac{1}{2}(z_1^2 + s_2^2)$$

Applying the condition $S\dot{S} < 0$ for attractive surface and simplifying, it makes the following:

$$\dot{s}_2 = -k_1 sign(s_2) - k_2 s_2$$
$$= \dot{x}_2 - \ddot{x}_{1d} - \alpha_1 \dot{z}_1$$
$$= a_1 x_4 x_6 + a_2 x_4 \Omega + b_1 U_2 - \frac{K_1 l}{I_x}\dot{\emptyset} - \ddot{x}_{1d} - \alpha_1(z_2 + \alpha_1 z_1)$$

The control input U_2 is formulated using the backstepping approach[9] as:

$$U_2 = \frac{1}{b_1}\left(-a_1 x_4 x_6 - a_2 x_4 \Omega - \alpha_1^2 z_1 - k_1 sign(s_2) - k_2 s_2 + \frac{K_1 l}{I_x} x_2\right)$$

Chattering occurs due to the sign function present in the above equation, to avoid this drawback which affect the overall performance, this discontinuous function is replaced by a saturation function defined as:

$$sat(s_k) = \begin{cases} s_k & if\ |s_k| \le 1 \\ sign\ (s_k) & if\ |s_k| > 1 \end{cases}$$

k = 2,3,…7.

Then modified control law $U_2$is,

$$U_2 = \frac{1}{b_1}\left(-a_1 x_4 x_6 - a_2 x_4 \Omega - \alpha_1^2 z_1 - k_1 sat(s_2) - k_2 s_2 + \frac{K_1 l}{I_x} x_2\right) \quad (8)$$

The sliding mode control based on backstepping for pitch and yaw subsystem has been designed to obtain U_3 and U_4, following the steps above similar to roll subsystem.
The control inputs U_3 and U_4 are calculated as:

$$U_3 = \frac{1}{b_2}\left(-a_3 x_2 x_6 - a_4 x_2 \Omega - \alpha_2^2 z_3 - k_3 sat(s_3) - k_4 s_3 + \frac{K_2 l}{I_y} x_4\right) \quad (9)$$

$$U_4 = \frac{1}{b_2}\left(-a_5 x_2 x_4 - \alpha_3^2 z_5 - k_5 sat(s_4) - k_6 s_4 + \frac{K_3 l}{I_z} x_6\right) \quad (10)$$

where
$z_3 = x_{3d} - x_3$, $s_3 = z_4 = x_4 - \dot{x}_{3d} - \alpha_2 z_3$, $z_5 = x_{5d} - x_5$, $s_4 = z_6 = x_6 - \dot{x}_{5d} - \alpha_3 z_5$

3.1.2 Altitude control: Further, the altitude control U_1 is obtained using the same approach as above.

$$U_1 = \frac{m}{cosx_1 cosx_3}\left(g - k_7 sat(s_5) - k_8 s_5 - \alpha_4^2 z_7 + \frac{K_4 l}{m} x_8\right) \quad (11)$$

with
$z_7 = x_{7d} - x_7$, $s_5 = z_8 = x_8 - \dot{x}_{7d} - \alpha_4 z_7$

3.1.3 Position Control: From the nonlinear model (6), it is clear that the motion through the axes x and y depends on input U_1. Therefore it is necessary to compute the control U_x and U_y, satisfying the condition $S\dot{S}$ <0.
Then

$$u_x = (m/U_1)(k_9 sat(s_6) - k_{10} s_6 - \alpha_5^2 z_9 + \frac{K_5 l}{m} x_{10})$$
$$u_y = (m/U_1)(k_{11} sat(s_7) - k_{12} s_7 - \alpha_6^2 z_{11} + \frac{K_6 l}{m} x_{12}) \quad (12)$$

where$z_9 = x_{9d} - x_9$,$s_6 = z_{10} = x_{10} - \dot{x}_{9d} - \alpha_5 z_9$, $z_{11} = x_{11d} - x_{11}$,$s_7 = z_{12} = x_{12} - \dot{x}_{11d} - \alpha_6 z_{11}$
All the control inputs required for the dynamics have been derived above and given in the equations from (8-12).

3.1.4 Stability: Lyapunov stability approach is used to prove and evaluate the state convergence property of nonlinear flight controller equations (8-12). Considering the Lyapunov function as[14, 15]

$$V_i = \frac{1}{2} s_i^2 \qquad i = 2,3 \ldots 7$$

with $V(0) = 0$ and $V(t) > 0$ for $s(t) \neq 0$. A sufficient condition for the stability is guaranteed if the derivative of the Lyapunov function is negative definite:

$$\dot{V}_i = s_i \dot{s}_i$$
$$= s_i(-k_i sign(s_i) - k_{i+1} s_i)\ \forall\ k_i, i = 1, 3, 5, 7, 9, 11$$
$$= -k_i sign|s_i| - k_{i+1} {s_i}^2$$
$$\le 0$$

Hence, $\dot{V}_i$ is negative definite and all the system state trajectories can reach and stay on the corresponding sliding surfaces, under the control laws.

3.2. LINEAR QUADRATIC REGULATOR CONTROL: Reformulating the dynamical model (6) of the quadrotor in the following form.

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$

The matrices $A(t)$, $B(t)$ and $u(t)$ are expressed as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & f_1 & 0 & g_1 & 0 & g_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_3 & 0 & f_2 & 0 & g_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_5 & 0 & f_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_6 \end{bmatrix}$$

where $f_1 = -\frac{K_1 l}{I_x}$, $f_2 = -\frac{K_2 l}{I_y}$, $f_3 = -\frac{K_3 l}{I_z}$, $f_4 = -\frac{K_4 l}{m}$, $f_5 = -\frac{K_5 l}{m}$, $f_6 = -\frac{K_6 l}{m}$
and $g_1 = a_2 \Omega$, $g_2 = a_1 x_4$, $g_3 = a_4 \Omega$, $g_4 = a_3 x_2$, $g_5 = a_5 x_4$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ b_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & b_3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_3 \end{bmatrix}$$

where $d_1 = -g + (cosx_1\ cosx_3)\frac{1}{m}$, $d_2 = (cosx_1\ sinx_3\ cosx_5 + sinx_1\ sinx_5)\frac{1}{m}$, $d_3 = (cosx_1\ sinx_3\ cosx_5 - sinx_1\ cosx_5)\frac{1}{m}$

and $u = \begin{bmatrix} U_2 \\ U_3 \\ U_4 \\ U_1 \end{bmatrix}$

The state feedback control $u(t)$ is obtained for the above system by minimising the standard quadratic performance index using suitable values of Q and R. LQR control has been commonly used for optimal control of dynamic systems. Here, this control is developed to compare the performance of proposed control.

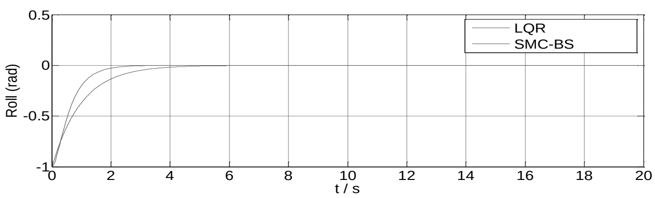
4. APPLICATION OF PROPOSED CONTROL

In order to implement the proposed control design, the position and attitude tracking of a quadrotor have been obtained through simulation in MATLAB. The model dynamics of quadrotor[24] has been considered in this paper for the application of proposed control method.

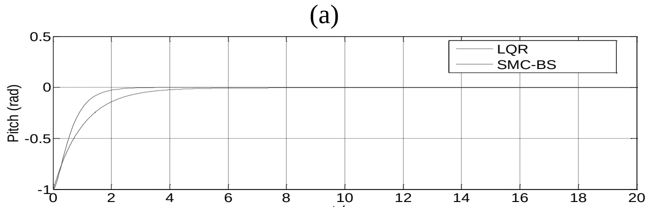
4.1 EXERCISE: This exercise presents the application of (i) proposed control (Section 3.1) and (ii) LQR control (Section 3.2) to the quadrotor and the responses are compared.

The following conditions have been used in simulation. Initial states $\emptyset_0 = -1,\ \theta_0 = -1,\ \varphi_0 = -1$ and $x = y = z = 1$. Reference values for angles = [0, 0, 0] rad and positions = [0, 0, 0] meter.

The roll, pitch and yaw angular motion responses for both the controls have been given in figures 2(a-c). The responses of positions in x, y, and z directions for both the controls have been obtained and shown in figures 2(d-f).



(a)



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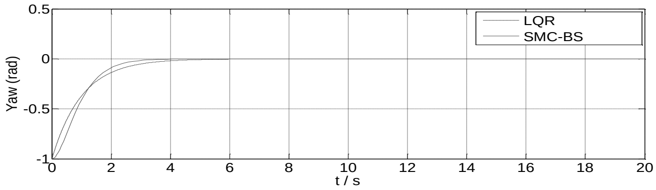

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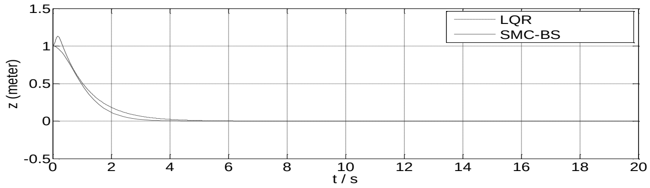

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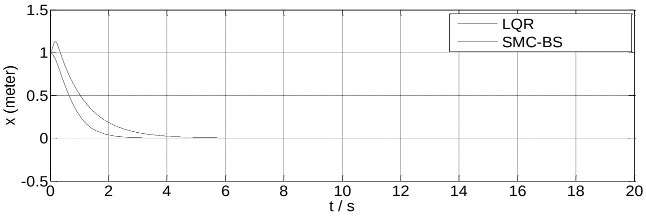
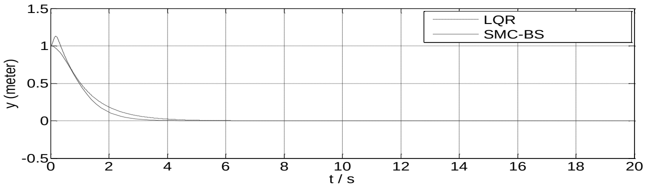


(e)



(f)

Fig 2: Angle and Position responses using (i) proposed control and (ii) LQR control

The angular motions and the positions settle faster on application of proposed control. Also, no overshoot is observed in position responses on application of proposed control, whereas LQR control produces little overshoot. The settling time and overshoot values for positions and angles under both the controls have been mentioned in Table 1.2.

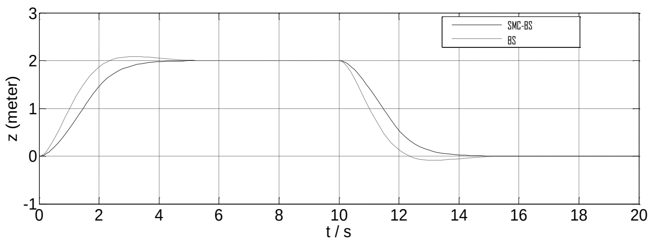
Table 1.2: Time Domain performance

Figure 2		a	b	c	d	e	f
Settling time (s)	LQR	5.9	6.5	6.9	7.6	5.5	4.8
	SMC-BS	3.7	3.8	4.5	4.2	3.0	3.5
Overshoot (%)	LQR	0.0	0.0	0.0	1.8	1.9	1.8
	SMC-BS	0.0	0.0	0.0	0.0	0.0	0.0

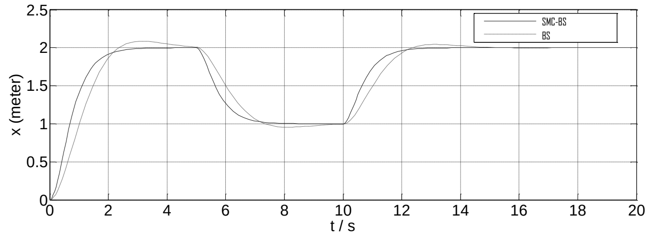
4.2 EXERCISE: This exercise presents the performance comparison of proposed control with conventional backstepping control[20]. The initial position and angle values of the quadrotor for the simulation test are $[0, 0, 0]$ m and $[-1, -1, 0]$rad respectively. The different desired/reference position and angle values are listed in Table 1.3.

Table 1.3: Reference positions and angles

Variables	Values	Time (s)
$[x_d, y_d, z_d]$	[2, 2, 2] m	0
	[1, 1, 2] m	5
	[2, 2, 0] m	10
$[\emptyset_d, \theta_d, \varphi_d]$	[0, 0, 2] rad	0
	[0, 0, 0] rad	10



(a)



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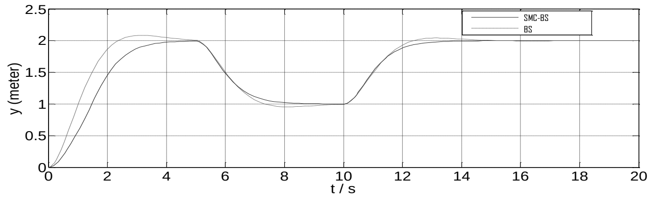
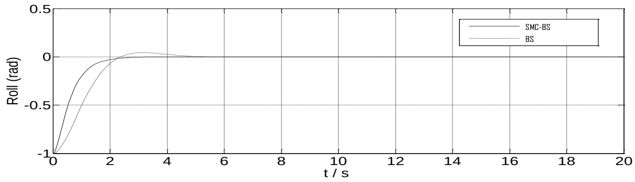


(c)

Fig 3: The positions (z, x and y) from Exercise 2



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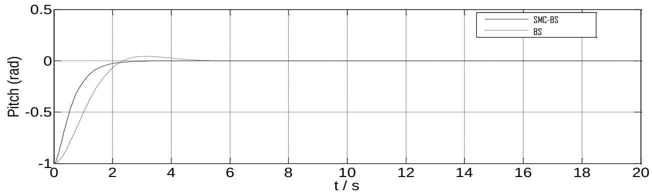

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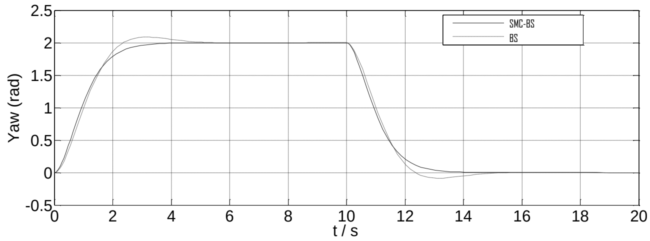
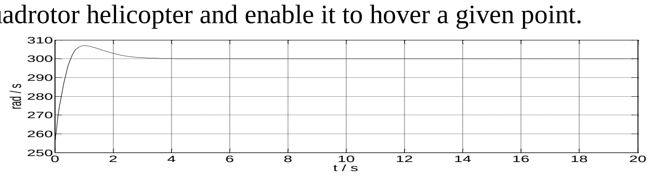


(c)

Fig. 4: The angles Roll, Pitch and Yaw from Exercise 4.2.

The position and angle responses under proposed and conventional BS control have been obtained for the quadrotor and are shown in fig. 3 and 4. It is clear from the responses that when the reference values of angles and positions are abruptly changed, the proposed controller is able to move all positions and angles to new reference values very quickly and effectively hold the quadrotor position and attitude in finite time. Fig. 5 shows the rotor speed response of the quadrotor during hovering. Two pair of propeller (1,3) and (2,4) rotating in opposite direction, as responses show that the rotor speed is able to produce sufficient lift to overcome the weight of the quadrotor helicopter and enable it to hover a given point.



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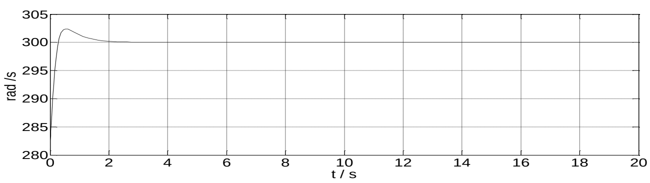

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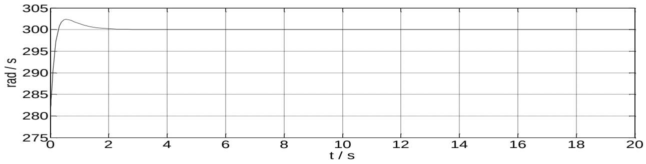

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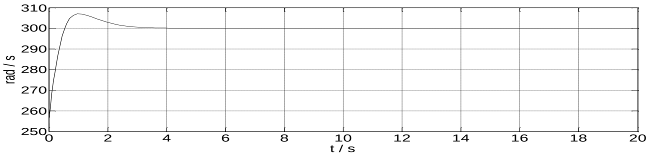
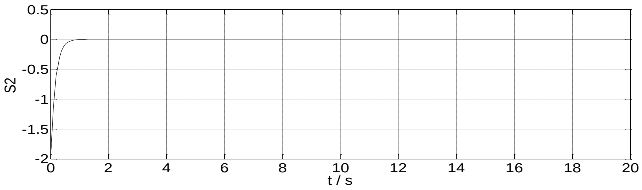


(d)

Fig. 5: Control response of quadrotor rotor speeds ($\Omega_1, \Omega_2, \Omega_3$ and Ω_4)

The behaviour of the sliding variables is shown in fig. 6. All the variables converge to their sliding surface as expected. The finite time convergence of s_2 and s_3 is faster than s_4 and s_5. This exhibits the same behaviour as shown in fig. 3 and 4.



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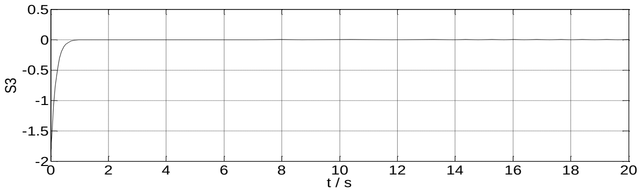

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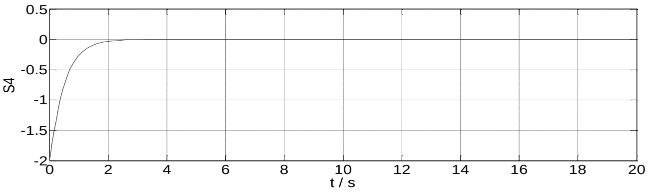

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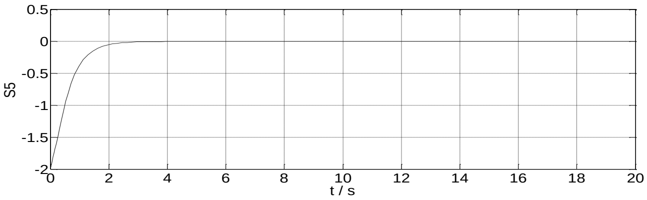
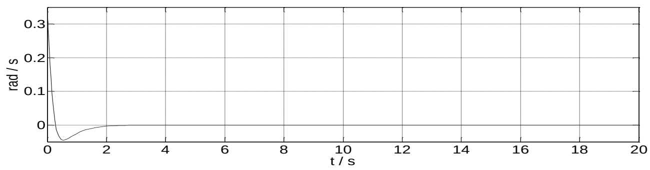
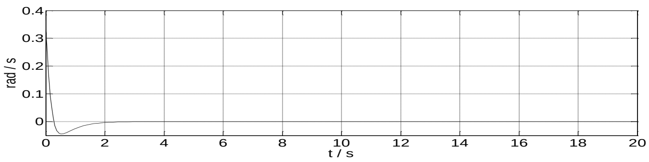


(d)

Fig. 6: Response of Sliding variables (s_2, s_3, s_4 and s_5)



(a)



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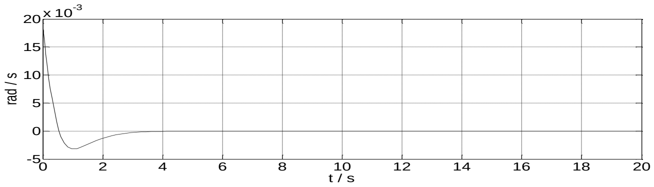

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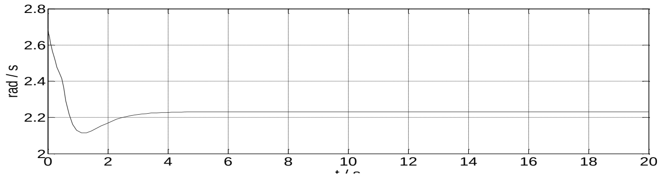
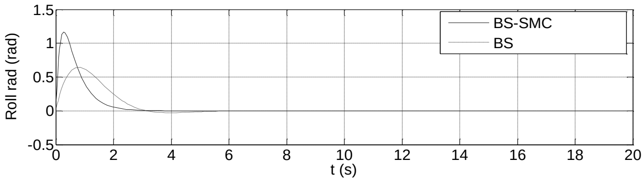


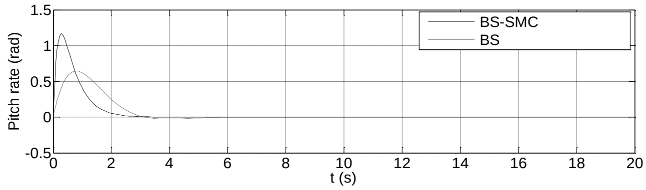
(d)

Fig. 7: The controller (U_2, U_3, U_4 and U_1)

The plots of control inputs obtained from proposed method are shown in fig. 7. It is observed from these plots that all the control inputs are continuous as desired and converge to their steady state value in finite time. The attitude angles rate and positions rate responses of the quadrotor with BS-SMC and conventional BS control are presented in figs. 8- 9.



(a)



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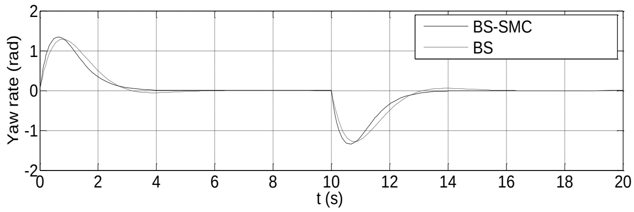
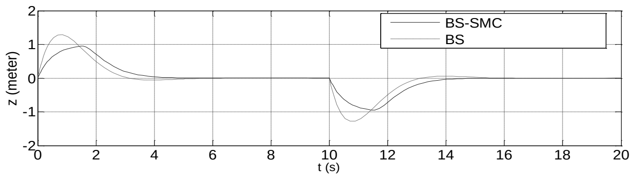
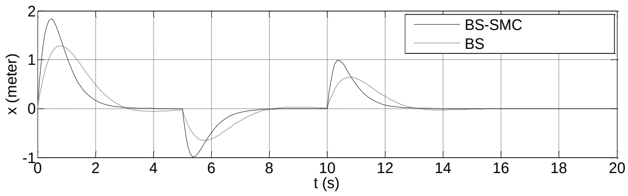


(c)

Fig. 8: Attitude rate response of a quadrotor



(a)



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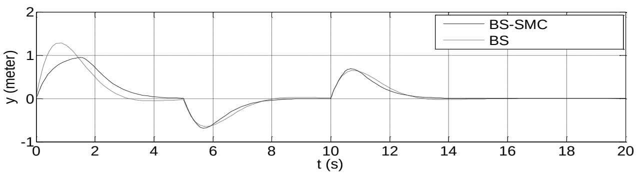

(c)

Fig. 9 Altitude and position rate response of a quadrotor

It is obvious from the responses in fig. 8 and 9 that the position controller and attitude controller effectively keep the quadrotor at a given point. It can be noted that, the proposed controller has better tracking and robustness performance than the conventional controller[20].

4.2.1 EXERCISE: Control responses with external disturbances: - White Gaussian noise disturbance is introduced to test the robustness of proposed controller. Fig. 10 shows the response of quadrotor, when external disturbance as white Gaussian noise is added in Euler angles. The initial angle values of the quadrotor for the simulation test are $[-1, -1, -1]$ rad and desired values are $[0, 0, 0]$ rad.

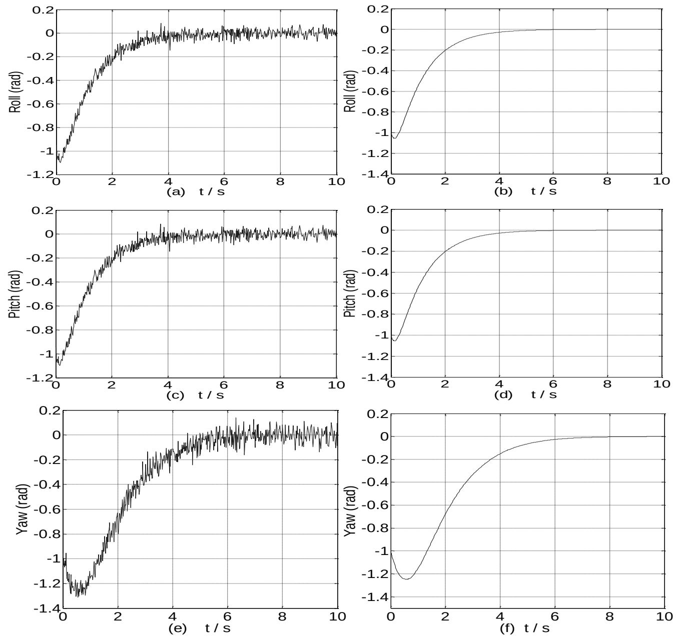

Fig. 10: Fig. a, c, e are state with Gaussian noise and fig. b, d, f are respective outputs

It reflects good performance and robustness of the proposed control algorithm. It has been obviously observed from the above responses that smooth output is obtained even the input is noisy. Therefore the proposed controller performance is good and robust.

5. CONCLUSION

There are nonlinear control methods for tracking of UAV in the literature but their performances were not satisfactory. In this paper a position and attitude tracking nonlinear controller is developed for a quadrotor UAV including the disturbance terms in the model. The design method is based on Lyapunov stability theory, combining sliding mode control with backstepping. The control implementation has been exercised for varying the positions and angles in a flight. The tracking performance and robustness of the proposed control method has been demonstrated and compared with (i) standard LQR control and (ii) conventional backstepping control[20] From the simulation results it has been concluded that the proposed approach is effectively promising for both the position and attitude tracking control of the quadrotor to their desired/reference values in finite time. Also results show that, the effect of aerodynamic forces, moments and external disturbances are invisible on all the states variables, controller and sliding variables. The tracking capability of the controller can further be improved by some modifications as future work.

REFERENCES

1 Bouabdallah S, Noth A & Siegwart R. PID vs LQ control techniques applies to an indoor micro quadrotor. Int Conf on Intelligent Robots and Syst. Vol.3, 2004; pp. 2451 – 2456.

2 Younes Y M, Jarrah M A A & Jhemi A A. Linear vs nonlinear control technology for a quadrotor vehicle. Proc of the 7th Int Symposium on Mechatronics and its Application. Shajah, UAE, April 20-22, 2010; pp. 1-10.

3 Yacef F, Boudjedir H, Bouhali O, Khebbache H & Boudjema F. Quadrotor attitude stabilization using state feedback controller: an LMI Approach. 4th Int Conf on Electrical Eng. Algeria, May 07-09, 2012, pp. 478-483.

4 Belkheiri M, Rabhi A, Hajjaji A E & Pegard C, Different linearization control techniques for a quadrotor system. 2nd Int Conf on Communication, Computing and Control Applications. Marseilles, Dec 6-8, 2012; pp. 1-6.

5 Lee D, Kim H.J & Sastry S. Feedback linearization vs adaptive sliding mode control for a quadrotor helicopter. Int Journal of Control Autom and Syst. 2009, 7(3), 419-428.

6 Voos H. Nonlinear control of a quadrotor micro-UAV using feedback-linearization. IEEE Int Conf on Mechatronics. Spain April 14-16, 2009; pp. 1-6.

7 Dikmen I C, Arisoy A & Temeltas H. Attitude control of a quadrotor. Proc of Recent Advances in Space Technologies. Istanbul June 11-13, 2009; pp. 722-727.

8 Slotine J J E & Li W. Applied nonlinear control. Prentice-Hall, Inc, USA, 1991.

9 Fang Z, Wang X Y & Sun J. Design and nonlinear control of an indoor quadrotor flying robot. Proc of the 8th World Congress on Intelligent Control and Autom. China, July 6-9, 2010; pp. 429-434.

10 Madani T & Benallegue A. Backstepping control for a quadrotor helicopter. Proc of Int Conf on Intelligent Robots and Syst. Beijing, China, October 9-15, 2006; pp. 3255-3260.
11 Bouabdallah S & Siegwart R. Full control of a quadrotor. IEEE Int Conf on Intelligent Robots and Syst. San Diego, USA, Oct 29-Nov 02, 2007; pp. 153-158.
12 Yali Y, Feng S & Yuanxi W. Controller design of quadrotor aerial robot. Int Conf on Medical Physics and Biomedical Eng. Vol. 33, 2012; pp. 1254-1260.
13 Bouabdallah S, Murrieri P & Siegwart R. Design and control of an indoor micro quadrotor. Proc of IEEE Int Conf on Robotics and Autom. New Orleans, 26 April - 1 May, 2004; pp. 4393-4398.
14 Saif A A, Dhaifullah M, Al-Malki M & Shafie M E. Modified backstepping control of quadrotor. IEEE Int Multi-Conf on Syst Signals and Devices. Chemnitz, March 20-23, 2012; pp. 1-6.
15 Ahmad M A & Daobo W. Modeling and backstepping-base nonlinear control strategy for a 6 DOF quadrotor helicopter. Chinese Journal of Aeronautics. 2008, 21(1), 261-268.
16 Zheng F & Gao W. Adaptive integral backstepping control of a micro-quadrotor. Int Conf on Intelligent Control and Information Processing (ICICIP). July 25-28 2011; Vol.2, pp. 910-915.
17 Runcharoon K & Srichatrapimuk V. Sliding mode control of quadrotor. Int Conf of Technological Advances in Electrical Electronics and Computer Eng. Konya, May 9-11, 2013; pp. 552-556.
18 Xu R & Ozguner U. Sliding mode control of a quadrotor helicopter. Proc of the 45th IEEE Conf on Decision and Control. USA, Dec. 13-15, 2006; pp. 4957-4960.
19 Patel A R, Patel M A & Vyas D R. Modeling and analysis of quadrotor using sliding mode control. 44th South-eastern Symposium on Syst Theory. Jacksonville, March 11-13, 2012; pp.111-114.
20 Boubdallah S & Siegwart R. Backstepping and sliding mode techniques applied to an indoor micro quadrotor. IEEE Int Conf on Robotics and Auto. Baecelona, Spain, April 18-22, 2005; pp. 2259-2264.
21 Mohamed, H A F, Yang S S & Moghavvemi M. Sliding mode controller design for a flying quadrotor with simplified action planner. ICCAS-SICE. Aug 18-21, 2009; pp. 1279-1283.
22 Khelfi, M F & Kacimi A. Robust control with sliding mode for a quadrotor unmanned aerial vehicle. IEEE Int Symposium on Industrial Electronics (ISIE). Hangzhou, May 28-31, 2012; pp. 886,892.
23 Sudhir and A. Swarup, “An Overview of Unmanned Aerial Vehicle Control”, Proceeding of 8th National Seminar & Exhibition on Aerospace and Related Mechanisms, ARMS-MCE-04, 2012 DRDO, Pune, India.
24 Xiong J J & Zheng E. H. Position and attitude tracking control for a quadrotor UAV. IAS Transactions. 2014, 53(3), 725–731.
25 Bouadi H & Tadjine M. Nonlinear observer design and sliding mode control of four rotor helicopter. Int Journal of Eng and Applied Science. 2007, 3(6), 333-338.

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