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Tracking Control Design for Quadrotor Unmanned Aerial Vehicle

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ABSTRACT

The model of a quadrotor unmanned aerial vehicle (UAV) is nonlinear and dynamically unstable. A flight controller design is proposed on the basis of Lyapunov stability theory which guarantees that all the states remain and reach on the sliding surfaces. The control strategy uses sliding mode with a backstepping control to perform the position and attitude tracking control. This proposed controller is simple and effectively enhance the performance of quadrotor UAV. In order to demonstrate the robustness of the proposed control method, White Gaussian Noise and aerodynamic moment disturbances are taken into account. The performance of the nonlinear control method is evaluated by comparing the performance with developed linear quadratic regulator and existing backstepping control technique and proportional-integral-derivative from the literature. The comparative performance results demonstrate the superiority and effectiveness of the proposed control strategy for the quadrotor UAV.

Keywords: Quadrotor UAV, Backstepping control; Sliding Mode control; Lyapunov Stability

1. INTRODUCTION

The applications of unmanned aerial vehicle (UAV) are becoming very popular in civil, dangerous environment, military and scientific research domains. The quadrotor is an important class of UAV which is vertical take-off and landing (VTOL) aircraft and having four lift generating propellers. The dynamical model of quadrotor has 6 degree of freedom (6DOF) with four actuators inputs and highly coupled states. The dynamics of quadrotor is nonlinear and unstable. Developing control for such complex dynamics has become an active area of recent research interest.

The proportional-integral-derivative (PID) and pole placement controls have been applied to the quadrotor platform[1-2] providing smooth stabilisation without any large overshoot and oscillations. These techniques are not very effective when the positional angles are not near to zero. Linear quadratic regulator (LQR) control technique[3] has been shown to improve the response. The linear controllers were very sensitive to its parameter and even small changes in the parameterisation could lead to an unstable response. To solve the stabilisation problem of quadrotor, different linearisation control algorithms[4-7] have been proposed. These control techniques are restricted to control the certain condition like the hover flight condition. Therefore, the nonlinear control methods have been developed to improve the performance of quadrotor. Backstepping and sliding mode control have received attention in literature due to its ability for disturbance rejection, stability and robustness[9-21]. Backstepping control techniques[10-13] are based on Lyapunov stability theory to follow the desired trajectory and stabilize the whole system. Saif and Dhaifullah[14], *et al.* have proposed a modified backstepping control technique to reduce the control gain parameters by half as compared to the classical backstepping approach. The feedback linearization[15] coupled with a PD controller for a translational subsystem and backstepping-based PID controller for rotational subsystem has been used to improve the performance of quadrotor. The sliding mode control has been shown[17-22] to stabilize the quadrotor helicopter which can move it to any position with any yaw angle. An adaptive sliding mode controller has been developed[23] to improve performance and reliability, for handling aerodynamic parameter uncertainties and external disturbance. The main advantage of the nonlinear controllers is low sensitivity to plant variations and disturbances. However the sliding mode control has some drawbacks such as chattering effect, limited design freedom for designer with sliding function. The chattering effect[18-20] occurs due to the inclusion of the sign function in the switching control and due to the non-ideal behaviour of system. Extended sliding mode control method has been proposed[24-28], to reduce the chattering effect, improve the performance, reliability for handling external disturbance and aerodynamic parameter uncertainties.

A model of quadrotor[20] has been considered for developing the control. An attempt has been made in this paper to extend this model by including the disturbance term for a comprehensive dynamics similar to[24]. A control has been proposed using sliding mode with backstepping technique for obtaining position and attitude control of quadrotor. The proposed control provides very good response on quadrotor dynamics. Further a control has been developed using LQR and the performance of same model is compared with the proposed control technique. The effectiveness of proposed control has also been compared with

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an existing control technique[20]. The responses demonstrate that the proposed control method provides superior performance.

2. QUADROTOR MODELLING

The dynamics of quadrotor derived using Euler-Lagrange Formalism[7] has been considered in this paper. Let the p, q and r denote its angular velocities in the body-frame; the outputs of the system are x, y and z, which are the positions of the center of gravity of the quadrotor; ϕ, θ and φ represent the Euler angles and $F_i\,(i = 1,2,3,4)$ is the thrust force produced by each propeller[24]. Simultaneous increase or decrease in speed of two pairs of rotor (1, 3) and (2, 4) turn in opposite direction in order to balance the moments and produce vertical motion as shown in Fig. 1. Pitch angle θ, is obtained by increasing/decreasing the speed of motor pair (3, 1) independently about the y axis, which can be controlled with the indirect control of motion along the same axis. Similarly, the roll angle ϕ is obtained by increasing/decreasing the speed of motor pair (2, 4) independently about x axis, which can be controlled with the indirect control of motion along the same axis. Finally the yaw angle φ is obtained by counter clockwise rotation of

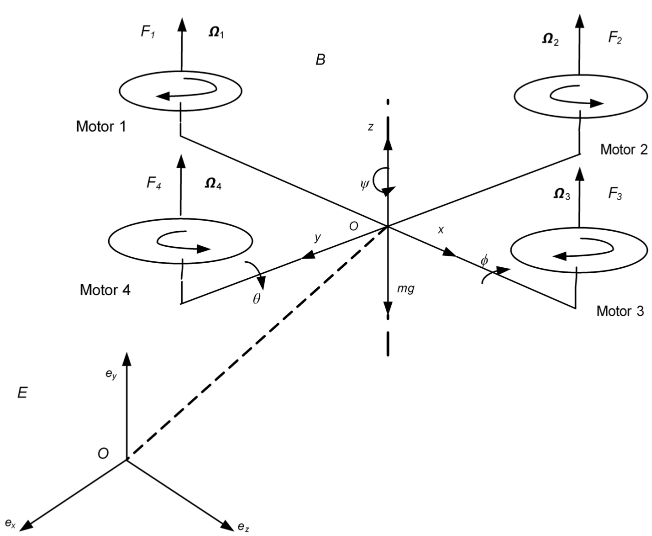

Figure 1. Quadrotor UAV.

motor pair (1, 3) and motor pair (2, 4), which can be controlled by z axis. Defining $E = [e_x, e_y, e_z]$ the Earth fixed frame and $B = [x, y, z]$, the Body fixed frame.

For any point of the airframe expressed in the earth fixed frame, the rotation matrix (R) and transformation matrix (T) are needed to change the axis of equation from the body fixed frame to the earth fixed frame[17], which can be expressed as

$$R = \begin{bmatrix} \cos(\theta)\cos(\varphi) & \sin(\phi)\sin(\theta)\cos(\varphi) - \cos(\phi)\sin(\theta)\cos(\varphi) & \sin(\phi)\sin(\theta)\cos(\varphi) - \cos(\phi)\sin(\theta)\cos(\varphi) \\ \cos(\theta)\sin(\varphi) & \sin(\phi)\sin(\theta)\sin(\varphi) - \cos(\phi)\cos(\varphi) & \cos(\phi)\sin(\theta)\cos(\varphi) - \sin(\phi)\cos(\varphi) \\ -\sin(\theta) & \sin(\phi)\cos(\theta) & \cos(\phi)\cos(\theta) \end{bmatrix}$$

and $$T = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)\sec(\theta) & \cos(\phi)\sec(\theta) \end{bmatrix}.$$

The equation of motion for the quadrotor can be obtained using Lagrangian function.

$L = \textit{Kinetic Energy}(\mathrm{T}) - \textit{Potential Energy}(\mathrm{V})$

The dynamics in terms of L can be expressed as

$$\Gamma_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}$$

where $\dot{q}_i$ are the generalised coordinates and Γ_i are the generalised forces.

The quadrotor dynamic model describing the roll, pitch and yaw rotations contains then, three terms which are the gyroscopic effect resulting from the rigid body rotation, the gyroscopic effect resulting from the propeller rotation coupled with the body rotation and finally the actuators action on roll, pitch and yaw are expressed as[7]

$$I_{xx}\ddot{\phi} = \dot{\theta}\dot{\varphi}\left(I_{yy} - I_{zz}\right)$$
$$I_{yy}\ddot{\theta} = \dot{\phi}\dot{\varphi}\left(I_{zz} - I_{xx}\right)$$
$$I_{zz}\ddot{\varphi} = \dot{\phi}\dot{\theta}\left(I_{xx} - I_{yy}\right) \tag{1}$$

where I is the linear inertia, I_{xx}, I_{yy} and I_{zz} are the cross inertia resulted by interaction of two angular velocity. The torque applied on quadrotor along an axis depends on the difference between the torques generated by each propeller (Ω) on the other axis as

$$\tau_x = bl\left(\Omega_4^2 - \Omega_2^2\right)$$
$$\tau_y = bl\left(\Omega_3^2 - \Omega_1^2\right)$$
$$\tau_z = d\left(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2\right)$$

where b, d and l are physical parameter and defined in the Table 1.

Further, the actuator actions are described as

$$I_{xx}\ddot{\phi} = \dot{\theta}\dot{\varphi}\left(I_{yy} - I_{zz}\right) - J\dot{\theta}\Omega_r + \tau_x$$
$$I_{yy}\ddot{\theta} = \dot{\phi}\dot{\varphi}\left(I_{zz} - I_{xx}\right) - J\dot{\phi}\Omega_r + \tau_y$$
$$I_{zz}\ddot{\varphi} = \dot{\phi}\dot{\theta}\left(I_{xx} - I_{yy}\right) + \tau_z \tag{2}$$

where Ω_r is the overall residual propeller angular speed i.e. $\Omega_r = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4$ and is the rotational inertia.

Following the above development, complete quadrotor dynamical model[24] with x, y and z motions as a consequence of roll, pitch and yaw rotation, can be expressed as:

$$\ddot{\phi} = \dot{\theta}\dot{\varphi}\left(\frac{I_y - I_z}{I_x}\right) - \frac{J_r}{I_x}\dot{\theta}\Omega + \frac{l}{I_x}U_2 - \frac{K_1 l}{I_x}\dot{\phi}$$

$$\ddot{\theta} = \dot{\phi}\dot{\varphi}\left(\frac{I_z - I_x}{I_y}\right) - \frac{J_r}{I_y}\dot{\phi}\Omega + \frac{l}{I_y}U_3 - \frac{K_2 l}{I_y}\dot{\theta}$$

$$\ddot{\varphi} = \dot{\phi}\dot{\theta}\left(\frac{I_x - I_y}{I_z}\right) + \frac{l}{I_z}U_4 - \frac{K_3 l}{I_z}\dot{\varphi}$$

$$\ddot{z} = -g + \left(\cos\phi\cos\theta\right)\frac{1}{m}U_1 - \frac{K_4 l}{m}\dot{z}$$

$$\ddot{x} = \left(\cos\phi\sin\theta\cos\varphi + \sin\phi\sin\varphi\right)\frac{1}{m}U_1 - \frac{K_5 l}{m}\dot{x}$$

$$\ddot{y} = \left(\cos\phi\sin\theta\sin\varphi - \sin\phi\sin\varphi\right)\frac{1}{m}U_1 - \frac{K_6 l}{m}\dot{y} \quad (3)$$

where I_x, I_y, I_z, g and m are physical parameters, J_r is the rotor's inertia, K_i denote the positive drag coefficients, defined in Table 1. Now, considering the external disturbances and including them in the dynamics, in this paper, to modify the controller design.

The U_1 is the total thrust on the body in the z-axis; U_2 and U_3 are the roll and pitch inputs; U_4 is the yawing moment and Ω is a disturbance[9]. The four inputs can be written in the following manner

$$U = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b\left(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2\right) \\ b\left(\Omega_4^2 - \Omega_2^2\right) \\ b\left(\Omega_3^2 - \Omega_1^2\right) \\ d\left(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 + \Omega_3^2\right) \end{bmatrix} \quad (4)$$

The physical parameters of the model have been taken from[24], for simulation exercise.

Table 1. Physical Parameters

Symbol	Definition	Value
m	Mass	.23 kg
I_x	Inertia on axis	7.5e-3 kgm^2
I_y	Inertia on axis	7.5e-3 kgm^2
I_z	Inertia on axis	1.3e-2 kgm^2
b	Thrust coefficient	3.13e-5 Ns^2
d	Drag coefficient (assume constant)[24]	7.5e-7 Nms^2
J_r	Rotor inertia	6e-5 kgm^2
l	Arm length	0.23 m
g	Acceleration of gravity	9.8 m/s^2
$K_1 = K_2 = K_3$	Positive constants	0.01 Ns/m
$K_4 = K_5 = K_6$		0.012 Ns/m

3. FLIGHT CONTROLLER DESIGN

Sliding mode control based on backstepping and linear quadratic regulator control methods for 6DOF of quadrotor UAV are presented in this section.

3.1 Sliding Mode Control with Backstepping

The dynamic model developed in equation set (3) can be expressed in state space form $\dot{X} = f(\mathrm{X}, \mathrm{U})$ by introducing state vector as

$$X^T = \left[\phi, \dot{\phi}, \theta, \dot{\theta}, \varphi, \dot{\varphi}, z, \dot{z}, x, \dot{x}, y, \dot{y}\right].$$

$$x_1 = \phi,\ x_2 = \dot{x}_1 = \dot{\phi},\ x_3 = \theta,\ x_5 = \varphi,\ x_6 = \dot{x}_5 = \dot{\varphi},\ x_7 = z,$$

$$x_8 = \dot{x}_7 = \dot{z},\ x_9 = x,\ x_{10} = \dot{x}_9 = \dot{x},\ x_{11} = y,\ x_{12} = \dot{x}_{11} = \dot{y} \quad (5)$$

From Eqns. (3) and (5) the dynamics is formulated as

$$\dot{X} = f\left(\mathrm{X}, U\right)$$

$$= \begin{bmatrix} x_2 \\ x_4 x_6 a_1 + x_4 a_2 \Omega + b_1 U_2 - \frac{K_1 l}{l_x} x_2 \\ x_4 \\ x_2 x_6 a_3 + x_2 a_4 \Omega + b_2 U_3 - \frac{K_2 l}{l_y} x_4 \\ x_6 \\ x_4 x_6 a_5 + b_3 U_4 - \frac{K_3 l}{l_x} x_6 \\ x_8 \\ -g + \left(\cos x_1 \cos x_3\right)\frac{1}{m}U_1 - \frac{K_4 l}{m} x_8 \\ x_{10} \\ u_x \frac{1}{m}U_1 - \frac{K_5 l}{m} x_{10} \\ x_{12} \\ u_y \frac{1}{m}U_1 - \frac{K_6 l}{m} x_{12} \end{bmatrix} \quad (6)$$

where $a_1 = \left(I_y - I_z\right)/I_x$, $a_2 = -J_R/I_x$, $a_3 = \left(I_z - I_x\right)/I_y$, $a_4 = -J_R/I_y$, $a_5 = \left(I_x - I_y\right)/I_z$ and $b_1 = l/l_x$, $b_2 = l/l_y$, $b_3 = l/l_z$ and $u_x = \cos x_1 \sin x_3 \cos x_5 + \sin x_1 \sin x_5$, $u_y = \cos x_1 \sin x_3 \cos x_5 - \sin x_1 \cos x_5$

Classical sliding mode control has the property to keep the system state trajectory on a chosen surface called the sliding surface by using the discontinuous control. In this paper a sliding mode control algorithm is developed based on backstepping for the flight controller design of the quadrotor unmanned aerial vehicle.

3.1.1 Attitude Control

The choice of sliding surface[17]calculation concerning the tracking error is defined as:

$$z_i = x_{id} - x_i \quad i \in \{1, 3, 5, 7, 9, 11\}$$

$$z_i = x_i - \dot{x}_{(i-1)d} - \alpha_{(i-1)} z_{(i-1)} \quad i \in \left(2, 4, 6, 8, 10, 12\right)$$

with $\alpha_i > 0 \ \forall i \in [1, 12]$

Considering the Lyapunov function,

$$V_i = \frac{1}{2} z_i^2 \quad i \in \{1, 3, 5, 7, 9, 11\}$$

$$V_i = \frac{1}{2}\left(V_{i-1} + z_i^2\right) \quad i \in \{2, 4, 6, 8, 10, 12\}$$

These satisfy the necessary sliding condition $S\dot{S} < 0$.

Let $i = 1,2$ surfaces are

$$z_1 = x_{1d} - x_1$$
$$S_2 = z_2 = x_2 - \dot{x}_1 d - \alpha_1 z_1 \quad (7)$$

and Lyapunov function is

$$V(z_1, s_2) = \frac{1}{2}\left(z_1^2 + s_2^2\right)$$

Applying the condition $S\dot{S} < 0$ for attractive surface and simplifying, it makes the following:

$$\dot{s}_2 = -k_1 sign(s_2) - k_2 s_2$$
$$= \dot{x}_2 - \ddot{x}_{1d} - \alpha_1 \dot{z}_1$$
$$= a_1 x_4 x_6 + a_2 x_4 \Omega + b_1 U_2 - \frac{K_1 l}{l_x}\dot{\phi} - \ddot{x}_{1d} - \alpha_1 (z_2 + \alpha_1 z_1)$$

The control input U_2 is formulated using the backstepping approach[9] as:

$$U_2 = \frac{1}{b_1}\left(-a_1 x_4 x_6 - a_2 x_4 \Omega - \alpha_1^2 z_1 - k_1 sign(s_2) - k_2 s_2 + \frac{k_1 l}{l_x} x_2\right)$$

Chattering occurs due to the sign function present in the above equation, to avoid this drawback which affect the overall performance, this discontinuous function is replaced by a saturation function defined as:

$$sat(s_k) = \begin{cases} s_k & if\,|s_k| \le 1 \\ sign(s_k) & if\,|s_k| > 1 \end{cases} \text{for k = 2,3,…7.}$$

Then modified control law U_2 is,

$$U_2 = \frac{1}{b_1}\left(-a_1 x_4 x_6 - a_2 x_4 \Omega - \alpha_1^2 z_1 - k_1 sat(s_2) - k_2 s_2 + \frac{K_1 l}{l_x} x_2\right) \quad (8)$$

The sliding mode control based on backstepping for pitch and yaw subsystem has been designed to obtain U_3 and U_4 following the steps above similar to roll subsystem.

The control inputs U_3 and U_4 are calculated as:

$$U_3 = \frac{1}{b_2}\left(-a_3 x_2 x_6 - a_4 x_2 \Omega - \alpha_2^2 z_3 - k_3 sat(s_3) - k_4 s_3 + \frac{K_2 l}{l_y} x_4\right) \quad (9)$$

$$U_4 = \frac{1}{b_2}\left(-a_5 x_2 x_4 - \alpha_3^2 z_5 - k_5 sat(s_4) - k_6 s_4 + \frac{K_3 l}{l_z} x_6\right) \quad (10)$$

where

$z_3 = x_{3d} - x_3$, $s_3 = z_4 = x_4 - \dot{x}_{3d} - \alpha_2 z_3$, $z_5 = x_{5d} - x_5$, $s_4 = z_6 = x_6 - \dot{x}_{5d} - \alpha_3 z_5$

3.1.2 Altitude Control

Further, the altitude control U_1 is obtained using the same approach as above.

$$U_1 = \frac{m}{\cos x_1 \cos x_3}\left(g - k_7 sat(s_5) - k_8 s_5 - \alpha_4^2 z_7 + \frac{K_4 l}{m} x_8\right) \quad (11)$$

with

$z_7 = x_{7d} - x_7$, $s_5 = z_8 = x_8 - \dot{x}_{7d} - \alpha_4 z_7$

3.1.3 Position Control

From the nonlinear model (6), it is clear that the motion through the axes x and y depends on input U_1. Therefore it is necessary to compute the control U_x and U_y, satisfying the condition $S\dot{S} < 0$.

Then

$$u_x = (m/U_1)\left(k_9 sat(s_6) - k_{10} s_6 - \alpha_5^2 z_9 + \frac{K_5 l}{m} x_{10}\right)$$
$$u_y = (m/U_1)\left(k_{11} sat(s_7) - k_{12} s_7 - \alpha_6^2 z_{11} + \frac{K_6 l}{m} x_{12}\right) \quad (12)$$

where $z_9 = x_{9d} - x_9$, $s_6 = z_{10} = x_{10} - \dot{x}_{9d} - \alpha_5 z_9$,

$z_{11} = x_{11d} - x_{11}$, $s_7 = z_{12} = x_{12} - \dot{x}_{11d} - \alpha_6 z_{11}$

All the control inputs required for the dynamics have been derived above and given in the Eqns. (8)-(12).

3.1.4 Stability

Lyapunov stability approach is used to prove and evaluate the state convergence property of nonlinear flight controller equations (8-12). Considering the Lyapunov function as[14,15]

$V_i = \frac{1}{2}s_i^2 \;\; i = 2,3,....7$ with $V(0) = 0$ and $V(t) > 0$ for $s(t) \ne 0$. A sufficient condition for the stability is guaranteed if the derivative of the Lyapunov function is negative definite:

$$\dot{V}_i = s_i \dot{s}_i$$
$$= s_i\left(-k_i sign(s_i) - k_{i+1} s_i\right) \forall k_i, i = 1,3,5,7,9,11$$
$$= k_i sign|s_i| - k_{i+1} s_i^2$$
$$\le 0$$

Hence $\dot{V}_i$ is negative definite and all the system state trajectories can reach and stay on the corresponding sliding surfaces, under the control laws.

3.2 Linear Quadratic Regulator Control

In order to compare the tracking performance on application of proposed backstepping sliding mode control, the conventional linear quadratic control has also been developed. Reformulating the dynamical model (6) of the quadrotor in the following form.

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \quad (13)$$

The matrices $A(t)$, $B(t)$ and $u(t)$ are expressed as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & f_1 & 0 & g_1 & 0 & g_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_3 & 0 & f_2 & 0 & g_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_5 & 0 & f_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_6 \end{bmatrix}$$

where $f_1 = -\frac{K_1 l}{l_x}$, $f_2 = -\frac{K_2 l}{l_y}$, $f_3 = -\frac{K_3 l}{l_z}$, $f_4 = -\frac{K_4 l}{m}$,

$f_5 = -\frac{K_5 l}{m}$, $f_6 = \frac{-K_6 l}{m}$ and $g_1 = a_2 \Omega$, $g_2 = a_1 x_4$, $g_3 = a_4 \Omega$,

$g_4 = a_3 x_2$, $g_5 = a_5 x_4$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ b_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & b_3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_3 \end{bmatrix}$$

where $d_1 = -g + (\cos x_1 \cos x_3)\frac{1}{m}$,

$$d_2 = (\cos x_1 \sin x_3 \cos x_5 + \sin x_1 \sin x_5)\frac{1}{m},$$

$$d_3 = (\cos x_1 \sin x_3 \cos x_5 - \sin x_1 \cos x_5)\frac{1}{m} \text{ and } u = \begin{bmatrix} U_2 \\ U_3 \\ U_4 \\ U_1 \end{bmatrix}$$

The above model (13) is linearised and matrix A and B are becomes

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & f_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_6 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ b_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & b_3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

for LQR control to minimise the objective function $J = 1/2 \int_{t=0}^{\infty} \left[x^T Q x + u^T R u \right] dt$ with proper choice of the weighting matrix Q and R and thus the state feedback control u(t) is obtained. Then, this control is applied to model expressed in Eqn. (13). Here, the performance with this control is compared with the performance obtained from the proposed control.

4. APPLICATION OF PROPOSED CONTROL

To implement the proposed control design, the position and attitude tracking of a quadrotor have been obtained through simulation in MATLAB. The model dynamics of quadrotor[24] has been considered in this paper for the application of proposed control method.

4.1 Application Scenario I

This exercise presents the application of (i) proposed control (Section 3.1) and (ii) LQR control (Section 3.2) to the quadrotor and the responses are compared.

The following conditions have been used in simulation. Initial states $\phi_0 = -1rad, \theta_0 = -1rad, \varphi_0 = -1rad$ and $x = y = z = 1$. Reference values for angles = $[0,0,0]$ rad and positions = $[0,0,0]$ meter.

The roll, pitch and yaw angular motion responses for both the controls have been given in Figs. 2(a) - 2(c). The responses of positions in x, y and z directions for both the controls have been obtained and shown in Figs. 2(d) - 2(f).

The angular motions and the positions settle faster on application of proposed control. Also, no overshoot is observed in position responses on application of proposed control, whereas LQR control produces little overshoot. The settling time and overshoot values for positions and angles under both the controls have been mentioned in Table 2.

Table 2. Time domain performance

Figure 2		a	b	c	d	e	f
Settling time (s)	LQR	5.9	6.5	6.9	7.6	5.5	4.8
	SMC-BS	3.7	3.8	4.5	4.2	3.0	3.5
Overshoot (%)	LQR	0.0	0.0	0.0	1.8	1.9	1.8
	SMC-BS	0.0	0.0	0.0	0.0	0.0	0.0
Steady state error (%)	LQR	0.0041	0.0032	0.0054	0.0056	0.0071	0.0064
	SMC-BS	0.0014	0.0012	0.0023	0.0016	0.0027	0.0021

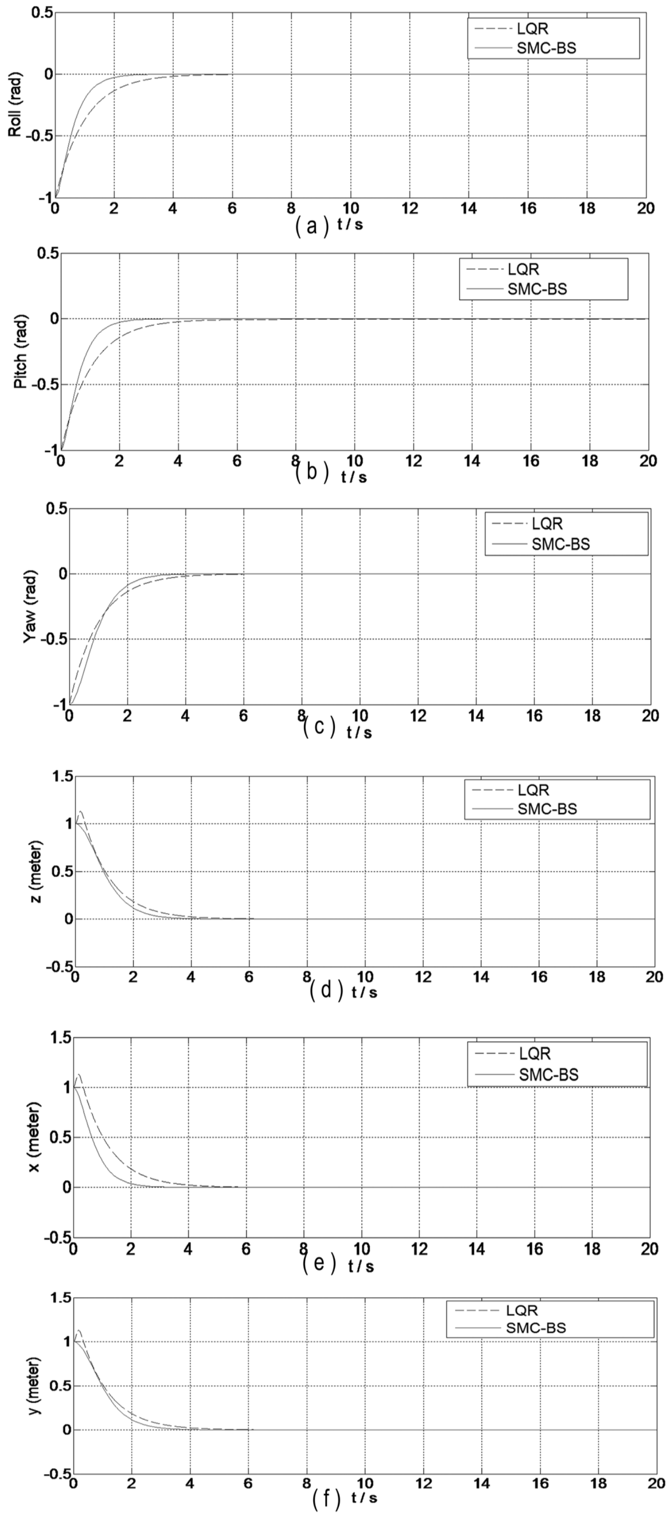

Figure 2. Angle and Position responses using (i) proposed control and (ii) LQR control.

4.2 Application Scenario II

This exercise presents the performance comparison of proposed control with conventional backstepping control[20]. The initial position and angle values of the quadrotor for the simulation test are $[0,0,0]$ m and $[-1,-1,0]$ rad respectively. The different desired/reference position and angle values are listed in Table 3.

The position and angle responses under proposed and conventional BS control have been obtained for the quadrotor and are shown in Figs. 3 and 4.

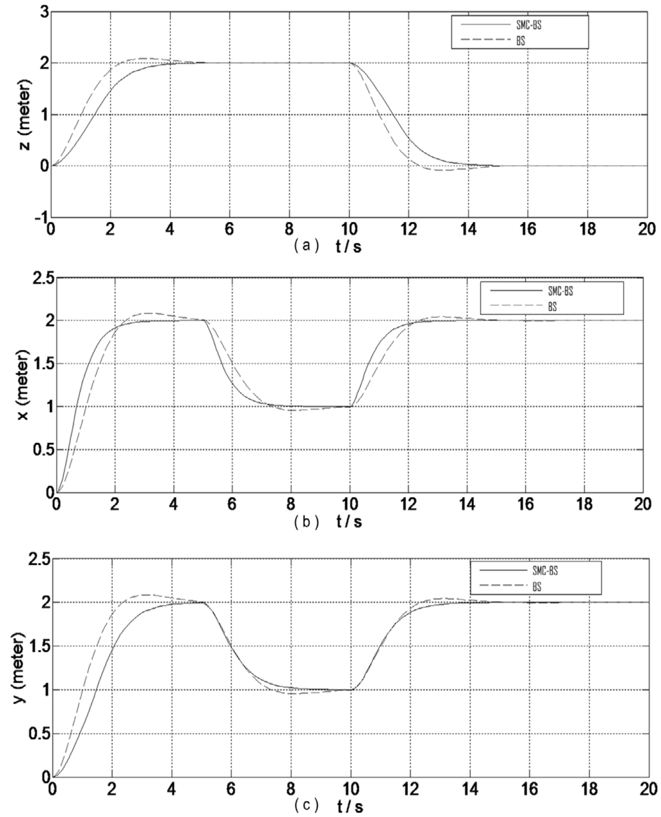

Figure 3. The positions (*z, x* and *y*) from application scenario II.

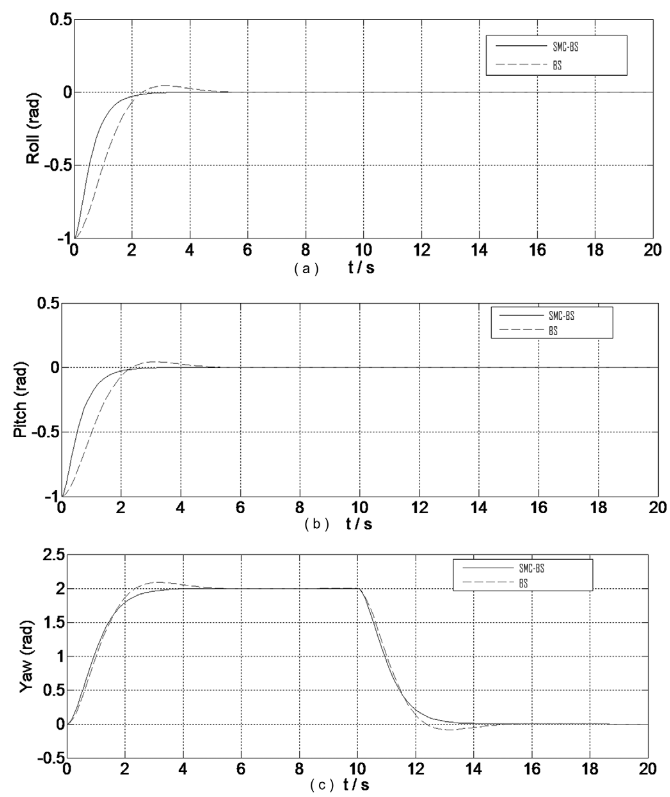

Figure 4. The angles Roll, Pitch and Yaw from application scenario II.

Table 3. Reference positions and angles

Variables	Values	Time (s)
$[x_d, y_d, z_d]$	[2, 2, 2] m	0
	[1, 1, 2] m	5
	[2, 2, 0] m	10
$[\phi_d, \theta_d, \varphi_d]$	[0, 0, 2] rad	0
	[0, 0, 0] rad	10

4.3 Application Scenario III

This exercise presents the performance comparison of proposed control with conventional PID control[2]. The simulation have been carried out to show the comparative response of proposed control technique with conventional PID control[2] of quadrotor, and a typical response of altitude is shown in Fig. 5.

From Fig. 5 it has been observed that the proposed control method take half time to reach the desired value as compare to conventional PID control with zero overshoot.

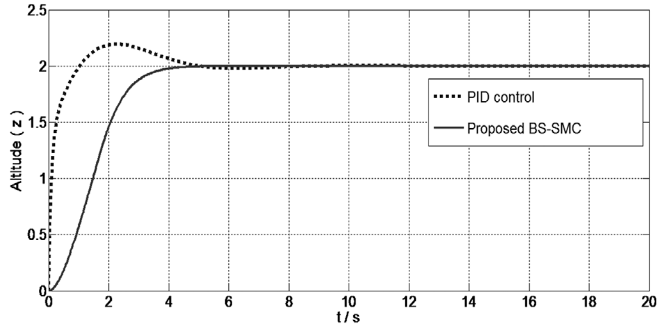

Figure 5. Comparison of proposed backstepping based sliding mode control and conventional PID control for altitude.

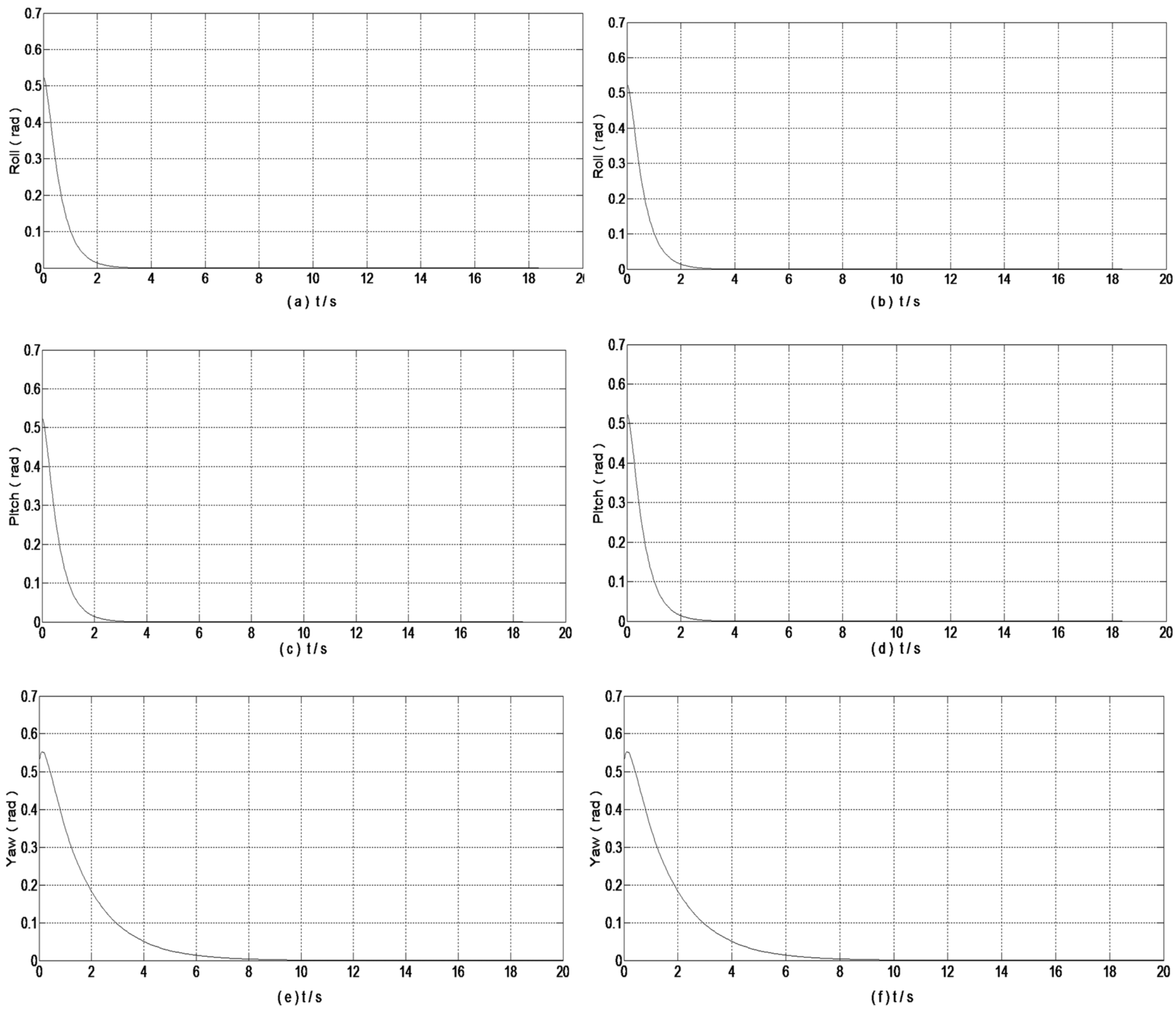

Figure 6. Response of rotational subsystem of a quadrotor with proposed backstepping based sliding mode control (a) Controlled response of roll angle without external disturbance (b) Controlled response of roll angle with external disturbance (c) Controlled response of pitch angle without external disturbance (d) Controlled response of pitch angle with external disturbance (e) Controlled response of yaw angle without external disturbance (f) Controlled response of yaw angle with external disturbance.

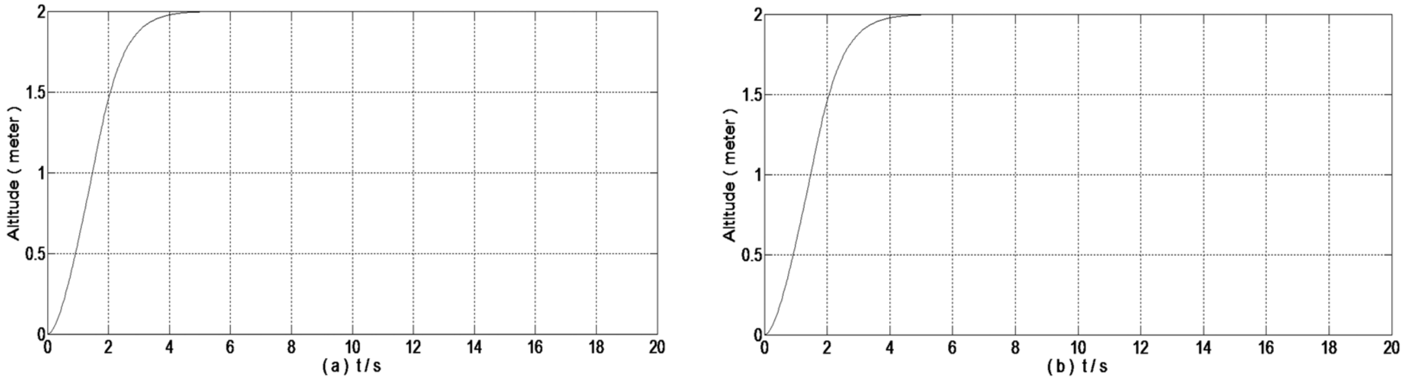

Figure 7. Response of altitude subsystem of a quadrotor with proposed backstepping based sliding mode control (a) Controlled response of altitude without external disturbances (b) Controlled response of altitude with external disturbance.

4.4 Application Scenario IV

Control responses with external disturbances:- White Gaussian noise disturbance is introduced in angular states and aerodynamic moment (i.e. sin(2t)) is added in altitude (z state) of quadrotor to test the robustness of proposed controller. The initial angle values of the quadrotor for the simulation test are $[0.524, 0.524, 0.524]$ rad and desired values are $[0, 0, 0]$ rad for attitude subsystem and initial value of altitude (z) is [0] meter and desired value altitude is [2] meter. The power of random white Gaussian noise generated in MATLAB for roll (7.90 mW), pitch (8.80 mW) and yaw (8.99 mW). Fig. 6 and Fig. 7 depicts the angular and altitude responseswith the proposed control without / with disturbance.

It reflects good performance and robustness of the proposed control algorithm. It has been obviously observed from the above responses that smooth output is obtained even the input is noisy. Therefore the proposed controller performance is good and robust.

5. CONCLUSIONS

There are nonlinear control methods for tracking of UAV in the literature but their performances were not satisfactory. In this study a position and attitude tracking nonlinear controller is developed for a quadrotor UAV including the disturbance terms in the model. The design method is based on Lyapunov stability theory, combining sliding mode control with backstepping. The control implementation has been exercised for varying the positions and angles in a flight. The tracking performance and robustness of the proposed control method has been demonstrated and compared with (i) standard LQR control, (ii) conventional backstepping control, and (iii) conventional PID control from the literature. From the simulation results it has been concluded that the proposed approach is effectively promising for both the position and attitude tracking control of the quadrotor to their desired/reference values in finite time. Also results show that, the effect of aerodynamic forces, moments and external disturbances are invisible on all the states variables, controller and sliding variables. The tracking capability of the controller can further be improved by some modifications as, by tuning parameters using optimisation technique, by using higher sliding mode control, using nonlinear sliding surface as future work.

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Contribution in the current study, he guided the discussion of the development of control algorithms, their implementation and performance characteristics. Also, providing future direction for improvement and further work.