Defence Science Journal, Vol. 66, No. 3, May 2016, pp. 236-241, DOI : 10.14429/dsj.66.9366

RESEARCH PAPER

Error Analysis in Spin Measurement using Synchro-ballistic Method and its Improvement

Prem Sahani*, S. Mohapatra, H.S. Panda, S. Sarkar, and M. Alam

Proof and Experimental Establishment, Balasore - 756 025, India

**Correspondence e-mail: p.sahani@pxe.drdo.in*

ABSTRACT

Error analysis of spin measurement using synchro-ballistic method is presented in this paper. Error in the spin measurement varies as the projections of marker move away from axis of symmetry, and the reason for variation of error is explained. An improvement to synchro-ballistic method is proposed for spin measurement, which reduces the error in measured spin, and an algorithm for implementation of same is provided. Results of improved synchro-ballistic method are compared with the theoretical estimation of spin from muzzle velocity.

Keywords: Error analysis, spin measurement, synchro-ballistic, barrel rifling, optical flow, rectangle fitting

1. INTRODUCTION

The projectiles fired from different weapons move through subsonic, transonic or supersonic flow regimes during its flight. The complex interaction of the flow field with the projectile affects the stability of the projectile[1]. Spin imparts aerodynamic stability to the projectile[2-4] and hence is an important aspect from the perspectives of its design and development. The analysis and prediction of trajectory of spinning projectiles has been a subject of investigation since decades[5,6]. It is very essential to measure the spin rate of projectile moving in air. Methods used for spin measurement are cameras[7-9] and Doppler radar[10-12]. Another way to measure spin of projectiles is using the principle of induced electro-motive force (emf) in which the pickup coil generates a sinusoidal wave as the projectile passes over it[13]. Synchro-ballistic method has been used to find motion parameters of projectiles in flight[7-9,14]. But, none of the previous works have dealt with analysing the error in measured spin, or developing a method of spin measurement so as to reduce the error in measured spin.

In synchro-ballistic method[7], spin is measured by taking the projection of a particular marker on the projectile surface, onto image plane, in a particular time interval. In this method a bright marker is put on projectile surface. The high speed camera is placed orthogonal to the line of fire of the projectile at an offset from muzzle of the gun; keeping in mind that projectile is well illuminated by ambient light or artificial light as seen from the camera, as shown in Fig. 1. When the projectile is fired from the gun, the camera is triggered using an external trigger device and records the desired images in the field of view (FOV) of the camera. After recording is done, the desired images are extracted and analysed for spin measurement. For a gun having rifling, spin can be obtained from muzzle velocity using the following equation[15]

$$Spin(rpm) = \frac{60\,V}{l/n} = \frac{60\,V}{pitch} \tag{1}$$

where V, n, l are muzzle velocity (m/s), number of start and axial distance travelled by projectile in one complete rotation inside the barrel (m), respectively. The l/n ratio is also known as pitch of rifling or barrel twist rate (BTR).

2. SYNCHRO-BALLISTIC METHOD

The projection of the marker on the projectile surface is taken on object plane as shown in Fig. 2. The angle θ_1, which the marker makes with the axis of symmetry at time t_1 is calculated thereof. Similarly angle θ_2 at time t_2 is calculated. The arc PQ is the distance travelled by the marker in time t s. Thus we have

$$\theta_1 = \sin^{-1}\frac{D_1}{R}$$

$$\theta_2 = \sin^{-1}\frac{D_2}{R}$$

$$\Delta\theta = \theta_1 - \theta_2$$

$$Spin(rpm) = \frac{60\,\Delta\theta}{2\pi\,t} = \frac{30\,\Delta\theta}{\pi\,t} \tag{2}$$

where $t = t_2 - t_1$ is the time considered for calculating spin.

The projection of marker on the vertical axis follows a simple harmonic path.

The application of spin measurement using Eqn. (1) is suitable only to projectiles for rifled guns at the muzzle exit. But the improved synchro-ballistic method is valid for projectiles

Received : 08 December 2015, Revised : 24 March 2016
Accepted : 28 March 2016, Online : 25 April 2016

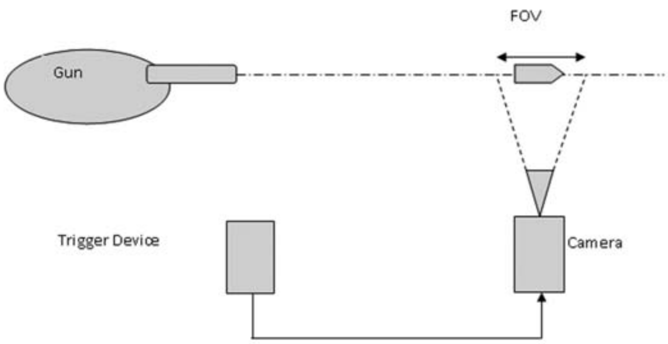

Figure 1. Camera setup for image acquisition for spin measurement.

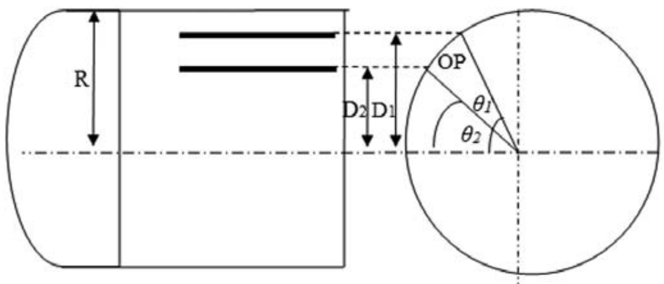

Figure 2. Marker movement.

for all types of guns. When considered for rifled guns for calculation of spin of projectile at muzzle exit, the method in Eqn. (1) performs better due to its simplicity in application, while the proposed method employs a complex setup of high speed cameras aligned parallel to the trajectory of projectile.

3. ERROR ANALYSIS IN SYNCHRO-BALLISTIC METHOD

As the frame rate chosen for image acquisition is very high (typically >10000 fps), the time for acquisition of image sequence is too small (<100 μs). Thus it is assumed that spin of the projectile doesn't vary over time in the frames of the acquired video. The image plane is assumed to be parallel to the trajectory of the projectile. Thus, the distance Ds_i between the projections of one edge of marker onto the image plane within i^{th} inter-frame time decreases as the marker moves away from the axis of symmetry in the image as shown in Fig. 3.

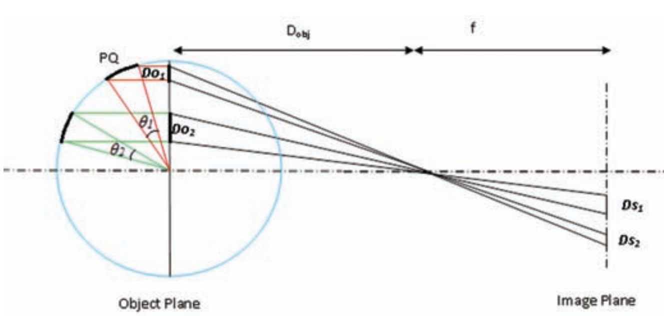

Figure 3. Object plane to image plane correspondence.

The arc PQ (Fig. 3) is the distance travelled by one edge of marker. Let the displacement of marker along the circumference of projectile be Do_i in i^{th} inter-frame time t_i, and the corresponding angular displacement of marker is θ. As the angular velocity is assumed to be constant for the duration of captured frames, thus θ is same for all four cases (Fig. 3). But the projection of arc is different ($Do_1<Do_2<Do_3<Do_4$). For simplicity, Do_3 and Do_4 are not shown in figure.

Proposition-1: The error in spin measurement increases as the projection of markers move away from axis of symmetry.

Proof:

Let Δt_i be the projection of distance of marker on object plane in $(i\text{-}1)^{th}$ frame to i^{th} frame. Magnification is given by[16]

$$M = I/O \tag{3}$$

where I is distance in image plane, and O is distance in object plane.

Distance Do_i in object plane maps to distance Ds_i in image plane, where i is a positive integer. Thus minimum error in distance measurement is the error due to least count of resolution of camera sensor i.e. one pixel. Thus error in linear displacement measurement is given by

$$\varepsilon_{dti} = \frac{Do_i}{Ds_i} = \frac{Do_i}{p\,N_x} = \frac{1}{M} \tag{4}$$

where p is pixel pitch of the image sensor, and N_x is number of pixels corresponding to Ds_i distance.

The error in linear displacement ε_{dti}, corresponds to error in measurement of angular displacement $\Delta\theta = (\theta_2 - \theta_1)$ due to distance Ds_i. Thus, maximum error in spin measurement due to the contribution of one pixel error is given by

$$\varepsilon spin = \max\left(\left|\frac{30}{\pi t}\left[\sin^{-1}\frac{D_1\pm 1}{R} - \sin^{-1}\frac{D_2\pm 1}{R} - \sin^{-1}\frac{D_1}{R} + \sin^{-1}\frac{D_2}{R}\right]\right|\right) \tag{5}$$

Using Taylor series expansion of $\sin^{-1}x$ into Eqn. (5) we get

$$\varepsilon spin = \max\left(\left|\frac{30}{\pi t}\left[\begin{array}{l}\displaystyle\sum_{m=0}^{\infty}\frac{(2m)!\left(\frac{D_1\pm 1}{R}\right)^{2m+1}}{2^{2m}\,(m!)^2\,(2m+1)} - \sum_{m=0}^{\infty}\frac{(2m)!\left(\frac{D_2\pm 1}{R}\right)^{2m+1}}{2^{2m}\,(m!)^2\,(2m+1)} - \\ \displaystyle\sum_{m=0}^{\infty}\frac{(2m)!\left(\frac{D_1}{R}\right)^{2m+1}}{2^{2m}\,(m!)^2\,(2m+1)} + \sum_{m=0}^{\infty}\frac{(2m)!\left(\frac{D_2}{R}\right)^{2m+1}}{2^{2m}\,(m!)^2\,(2m+1)}\end{array}\right]\right|\right) \tag{6}$$

Taking common the terms of m in Eqn (5), we can write

$$\varepsilon spin = \max\left(\left|\frac{30}{\pi t}\left[\sum_{m=0}^{\infty}\frac{(2m)!}{4^m\,(m!)^2\,(2m+1)}\left\{\begin{array}{l}\left(\frac{D_1\pm 1}{R}\right)^{2m+1} - \left(\frac{D_2\pm 1}{R}\right)^{2m+1} \\ -\left(\frac{D_1}{R}\right)^{2m+1} + \left(\frac{D_2\pm 1}{R}\right)^{2m+1}\end{array}\right\}\right]\right|\right) \tag{7}$$

For m=0, the Eqn. (7) reduces to

$$\varepsilon spin = \max\left(\left|\frac{30}{\pi\, t\, R}\left\{D_1 \pm 1 - D_2 \mp 1 - D_1 + D_2\right\}\right|\right) \tag{8}$$

Equation (8) is maximum when

$D_1 \pm 1 - D_2 \mp 1 - D_1 + D_2 = +2 \; or \; -2$, and thus can be written as

$$\varepsilon_{spin} = \frac{60}{\pi \, t \, R} \tag{9}$$

As Do_i is reduced gradually, Ds_i also reduces. Do_i is positive real number, but Ds_i is positive integer. Thus Ds_i corresponding to Do_i is obtained as $Ds_i = M^*Do_i$ by rounding off to nearest integer value. Thus, as Doi decreases, rounding-off error in Do_i increases, due to which error in spin measurement increases. Thus proposition-1 is proved.

Proposition-2 : The error incorporated in spin measurement is due to the limitation in resolution of displacement of marker position, in pixels, accounted by the fixed image sensor resolution, as markers move away from the axis of symmetry and/or due to lower angular velocity of projectile.

Proof:

From Eqns (3) and (4), we have

$$N_x = \frac{M * Do_i}{p} \tag{8}$$

Case-I: When M^*Do_i is ∞, $N_x = \infty$.

This is the ideal case, which implies any Do_i on object plane is always resolvable on image plane, if M is too large.

Case-II: M^*Do_i is reduced such that $\frac{M * Do_i}{p} < 1$, i.e. $N_x < 1$, it appears that on image plane, there is no movement of the marker (ignoring sub pixel estimation). In this case, synchro-ballistic method fails.

This implies that the synchro-ballistic method fails in either or both of the following two cases:

(i) M is reduced below $M_{critical} = \frac{p}{Do_i}$, so that $N_x < 1$.

(ii) Do_i is reduced to $Do_{i(critical)} = \frac{p}{M}$, so that $N_x < 1$.

This is the case when markers are considered at farthest distance from the axis of symmetry, and/or angular velocity of projectile is low.

Case-III: M^*Do_i is reduced gradually up to a value such that $N_x > 1$. In this case Do_i is resolvable in N_x number of pixels and spin can be obtained from the synchro-ballistic method with an error due to limitation of N_x to resolve Do_i on image plane. Hence, proposition-2 is proved.

4. IMPROVED SYNCHRO-BALLISTIC METHOD

The error incorporated in spin measurement as mentioned in proposition-2 can't be ignored when the error is significant. Spin is calculated over a large number of consecutive time intervals, assuming that the total time is very small such that spin variation over that time is negligible. In principle, spin measured for all the time intervals should be same, but due to error incorporated in measurement as proposed in proposition-1 and proposition-2, the spin measured are not same.

The average of any number of independent, identically distributed random variable follows normal distribution[17]. So, we can approximate all spin measurement values obtained, with normal distribution. The error can be minimised by averaging the spin values measured over a large number of consecutive time intervals.

Thus the spin can be obtained as

$$Spin(rpm) = \frac{\left(\sum_{i=-k}^{k} \frac{30 \times \delta\theta_i}{\pi (t_{i+1} - t_i)} \right)}{2k+1} \tag{10}$$

where t_i and t_{i+1} are initial and final time at which projection of a particular marker is considered, and number of observation taken is $2k + 1$. The limit of the counter i is taken from $-k$ to k where $i = 0$ represents the frame interval with marker nearest to the axis of symmetry.

Equation (9) can be viewed as the ensemble average value of spin values measured at the desired instant of time.

5. IMPLEMENTATION ALGORITHM FOR IMPROVED SYNCHRO-BALLISTIC METHOD OF SPIN MEASUREMENT

Optical flow is the distribution of apparent velocities of movement of brightness patterns in an image[18]. Techniques for estimating motion field are divided into two major classes: differential techniques[19], and matching techniques[20]. A widely used differential algorithm[21] that gives good results is chosen for our implementation.

The implementation algorithm is proposed as follows:

- Edges are extracted in projectile image using canny edge detector[22]. The largest edge segment gives the outer edges of the projectile.
- The minimum bounding rectangle fitting method[23] is applied onto the desired edge segment and thus diameter of the projectile can be obtained as the length of the minor axis of the projectile.
- To obtain the desired marker, optical flow technique is used. Using Harris Corner detection[24], one marker point is obtained in a defined region of interest (ROI), and then using optical flow, track of the marker is obtained within the selected ROI in an image. The perpendicular distance from selected marker to the axis of symmetry is obtained as D_i in a particular frame. In the next frame, the distance D_{i+1} from the tracked marker to the axis of symmetry is obtained.

  Thus, we can calculate

$$\delta\theta_i = abs(\theta_{i+1} - \theta_i) = abs\left(\sin^{-1}\frac{D_{i+1}}{R} - \sin^{-1}\frac{D_i}{R} \right) \tag{10}$$

- Spin can be calculated thereafter using modified synchro-ballistic method.

The images obtained using the above algorithm is shown in Fig. 4 for three consecutive instance of time for a projectile.

6. RESULTS AND DISCUSSION

The instantaneous spin is computed using Eqn (1), for six numbers of projectiles, as the projectile exits completely from the muzzle. The projectiles taken for experimentation are of 120 mm diameter. The pitch of the rifling of the barrel is 2160 mm. The setup used high speed camera with make-Photron, model- Fastcam SA-5. The camera was setup with a

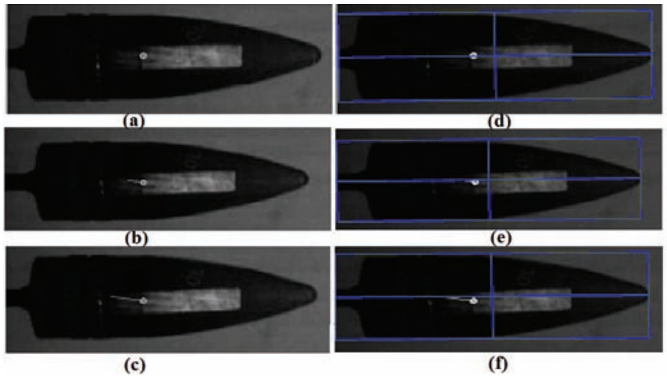

Figure 4. (a)-(c) are the images processed with optical flow algorithm. (d)-(f) are the images with rectangle fitting algorithm applied on images (a)-(c).

frame rate of 15000 fps, exposure time of 1 μs (with artificial illumination), and resolution of 896 x 512. High speed image sequence of projectile at the muzzle exit is captured. The translational motion of the points on the projectile is tracked using optical flow algorithm[21]. The Euclidian distance D (in pixels) between the tracked points is calculated. The velocity is obtained using the formula

$$Velocity = \frac{D}{T*M} \tag{11}$$

where *T* is time between the considered consecutive images.

Considering movement of marker point in two consecutive frames, spin is calculated for all the six rounds using the improved synchro-ballistic method. The method is repeated for a series of consecutive frames and all the spin values are averaged to give desired spin. As the average spin is calculated over a very short duration of time, it can be approximated as the instantaneous spin of projectile. Both the spin results are compared and shown in Tables 1-2, and Fig. 5, from which it is validated that the improved synchro-ballistic method gives nearly same spin of projectile as obtained by spin measurement using barrel rifling. For simplicity, data for round number 4 is only provided in Table 2.

The error bounds for spin are calculated by subtracting the spin values after deliberately introducing error of one pixel in measurement as shown in Eqn. (5). The results are shown in Fig. 6, which validates the proposition-1.

From Table 1, it can be observed that the spin values have an error characterised by proposition-1 and propostion-2. The error can be minimised by averaging the spin values over a series of frames, which can be observed from Table 1 and Table 2. From proposition-2, it can also be observed that the spin has increased percentage of error for lower spin projectiles.

For very high spin projectile, frame rate of image capture should be kept sufficiently high in order to keep the time interval appreciably small so that a particular marker will be available in a number of frames and that the marker will not be lost from image field of view within a few frames.

Table 1. Comparison of spin using the two methods

Round no	Spin using Eqn (1)	Spin by improved synchro-ballistic method	Error (%)
1	10634	10682	0.451382
2	11000	11340	3.090909
3	10997	11139	1.291261
4	18883	19016	0.704337
5	19503	19771	1.374148
6	15853	15919	0.416325

Table 2. Data for round number-4 from Table 1 using improved synchro-ballistic method

Time (us)	Spin using synchro ballistic method (rpm)	Spin using our proposed method (rpm)
66.67	9302	
133.34	16535	
200.01	17028	
266.68	25864	
333.35	17723	
400.02	25965	
466.69	21437	
533.36	14150	
600.03	21371	
666.70	20124	
733.37	17192	
800.04	29055	
866.71	11463	19016.08

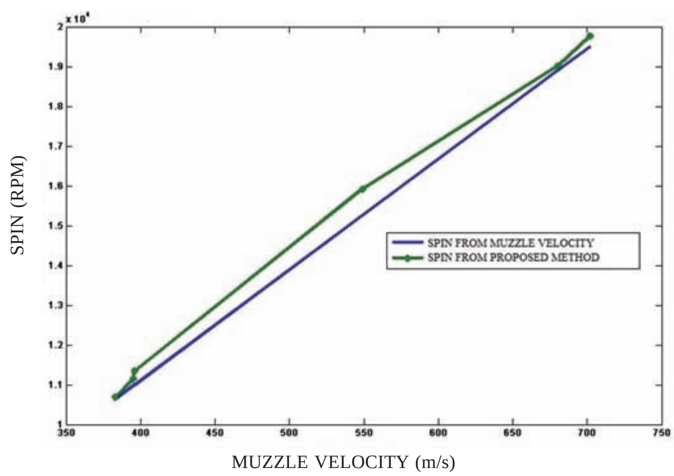

Figure 5. Plot of spin *vs* muzzle velocity.

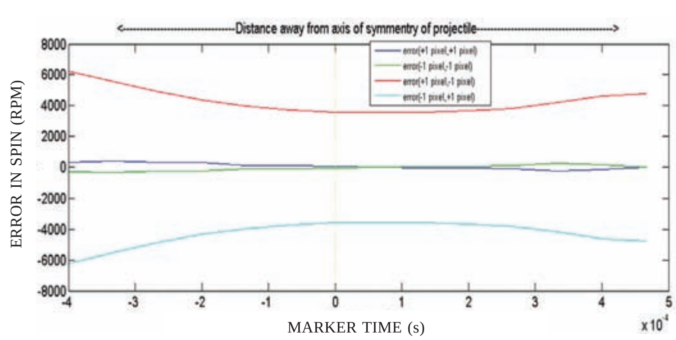

Figure 6. Error in spin *vs* time.

7. CONCLUSION

The findings of error analysis in spin measurement using synchro-ballistic method are presented. It has been shown that the error in measured spin can vary significantly depending on the positions of the markers considered for spin measurement. It also explains the variation in error in spin measurement due to limitation in image sensor resolution and lower angular velocity of the projectile. The proposed method minimises the error in spin measurement and is validated with the spin estimated from muzzle velocity.

REFERENCES

1. Kumar, A.; Panda, H.S.; Biswal, T.K. & Appavuraj, R. Flow around a conical nose with rounded tail projectile for subsonic, transonic, and supersonic flow regimes: A numerical study. *Def. Sci. J.*, 2014, **64**(6), 509-516. doi: 10.14429/dsj.64.8111
2. Milinovic M.P.; Jerkovi, D.D.; Jeremic, O.M. & Kova, M. Experimental and simulation testing of flight spin stability for small caliber cannon projectile. *Strojniški vestnik – J. Mech. Eng.*, 2012, **58**(6), 394-402. doi: 10.5545/sv-jme.2011.277
3. Baranowski, L. Numerical testing of flight stability of spin-stabilized artillery projectiles. *J. Theoretical Appl. Mech.*, 2013, **51**(2), 375-385.
4. Moss, G.M.; Farar, C.L. & Leeming, D.W. Military ballistics-a basic manual. Brassey's. (UK), 1995.
5. Khalil, M; Abdalla, H. & Kamal, O. Dispersion analysis of spinning artillery projectile. *In* Proceedings, 13th International Conference on Aerospace Science and Aviation Technology, ASAT-13-FM-03, Military Technical College, Cairo, Egypt, 2009.
6. Chand, K.K. & Panda H.S. Mathematical model to simulate the trajectory elements of an artillery projectile proof shot. *Def. Sci. J.*, 2007, **57**(1), 139-148. doi: 10.14429/dsj.57.1741
7. Verma, S.R.; Mukherjee, R.N. & Balakrishnan, M. Measurement of spin of projectiles. *Def. Sci. J.*, 1989, **39**(1), 73-77. doi: 10.14429/dsj.39.4750
8. Hughett, P. Projectile velocity and spin rate by image processing of synchro-ballistic photography. *In* Ultra high- and High-Speed Photography, Videography, Photonics, and Velocimetry. San Diego, 1990. pp. 237-248. doi:10.1117/12.23354
9. Kadowaki, T; Kobayashi, K & Kajiro W. Rotation angle measurement of high-speed flying object. *In* Proceedings of the International Joint Conference on SICE-ICASE, Busan, Korea, 2006. pp. 5256–5259. doi: 10.1109/SICE.2006.315833
10. Ruoyu, Z.; Yuanqin, W. & Wendong, G. Radar reflected signal process of high spinning rate projectiles. *In* 8th International Conference on Electronic Measurement and Instruments, 2007. pp. 3-982. doi: 10.1109/ICEMI.2007.4351084
11. Zhang, R; Yuanqin W & Jing S. A Method of projectile spinning rate processing based on acceleration compensation. *In* International Symposium on Precision Engineering Measurement and Instrumentation 2012. International Society for Optics and Photonics, 2013. pp. 87590J-87590J. doi:10.1117/12.2015524
12. Wei, Y.; Ping, H.; Zhaokun, Q.; Xiaohong, H. & Zengping, C. A new method for spin estimation using the data of Doppler radar. *In* IEEE Proceeding of ICSP2000. pp. 1911-1914. doi: 10.1109/ICOSP.2000.893478
13. Goldser, M. E. Projectile spin rate measurement using induced electromotive force techniques. User Report No. FA-TR-75031. Frankford Arsenal Philadelphia PA, 1975.
14. Zhuxin, Z.; Bingwei, H.; Gongjian, W. & Deren, L.A method for the motion parameters estimation in incomplete synchro-ballistic photography. *In* IEEE the 3rd International Symposium on Photonics and Optoelectronics, 2011. pp. 1-4. doi: 10.1109/SOPO.2011.5780524
15. Panda, H. S. & Mohapatra, S. Spin measurement of canon lunched high caliber supersonic projectile using high speed video. User Report. Proof and Experimental Establishment, Chandipur, India. May, 2015.
16. Ray, S. F. Scientific Photography and Applied Imaging. Focal Press, 1999, pp. 45-46.
17. Papoulis, A. & Pillai, S.U. Probability, random variables and stochastic processes. Ed. 4. Tata Mc-Graw Hill Education, 2002.
18. Horn, B. Robot vision. MIT press, 1986.
19. Horn, B.K. & Schunck, B.G. Determining Optical flow. *Artificial Intell.*, 1981, **17**, 185-203. doi:10.1117/12.965761
20. Barron, J.; Fleet, D. & Beauchemin, S. Performance of optical flow techniques. *Int. J. Comput. Vision*, 1994, **12**(1), 43–77. doi: 10.1007/BF01420984
21. Lucas, Bruce D. & Kanade, T. An iterative image registration technique with an application to stereo vision. *In* Proceedings of the International Joint Conference on Artificial Intelligence, 1981. pp. 674-679.
22. Canny, J. A computational approach to edge detection. *IEEE Trans. Pattern Anal. Mach. Intell.*, 1986, **8**(6), 679-698. doi: 10.1109/TPAMI.1986.4767851
23. Chaudhuri, D. & Samal, A. A Simple Method for fitting of bounding rectangle to closed regions. *Pattern Recogntion*, 2007, **40**(7), 1981-1989. doi:10.1016/j.patcog.2006.08.003
24. Harris, C. & Stephens, M. A combined corner and edge detector. *In* Proceedings of the 4th Alvey Vision Conference, 1988. pp. 147–15.

ACKNOWLEDGEMENTS

The authors are thankful to Mr R Appavuraj, Director, PXE, Chandipur, and Mr T. K. Biswal, Associate Director and head, Imaging Wing, PXE, Chandipur for constant support and encouragement for the work. The authors are thankful to the staffs of Imaging Wing for supporting the authors in field trials and data generation for the work.

CONTRIBUTORS

Mr Prem Kumar Sahani received his BTech (Electronics & Telecommunication Engineering) from University College of Engineering, Burla, India, in 2008. Presently working as Scientist 'C' in PXE, Chandipur, India. His research interests include : Image processing, pattern recognition, and computer vision.

In current study, he has conceptualised the work and contributed to the error analysis using synchro-ballistic method and the proposed method. He conceptualised and contributed to the programming aspects on implementation algorithm, and analysed the data and results.

Mrs Sudeshna Mohapatra received her BE (Mechanical Engineering) from Indira Gandhi Institute of Technology, Sarang, India in 2000. Presently working as Scientist 'C' in PXE, Chandipur, India. Her research interests include : Modelling and simulation in the field of ballistics.

In current study, she has contributed to spin measurement using Muzzle velocity method and data analysis throughout the text, and refinement and of the manuscript.

Dr H.S. Panda received his MTech and PhD (Mechanical Engineering) from IIT Kharagpur. Presently working as Scientist 'E' in PXE, India. His research areas include : Experimental ballistics, computational fluid dynamics, spectral methods for scientific computing, microchannel flows in MEMS devices. Currently involved in high speed flow visualisation and analysis and experimental impact dynamics study.

In current study, he has guided throughout the work and has contributed to the analysis of spin measurement using Muzzle velocity method, and manuscript preparation.

Mr Sumit Sarkar received his BE (Electronics and Telecommunication Engineering) from BESU, Shibpur, India, in 2008. Presently working as Scientist'C' in PXE, Chandipur, India. His research interests include : Image processing, pattern recognition, and computer vision.

In current study, he has contributed to the majority of the programming aspects on implementation algorithm.

Md. Maqsood Alam received his BTech (Electronics and Telecommunication Engineering) from NIT Patna, India, in 2009. Presently working as Scientist 'C' in PXE, Chandipur, India. His research interests include : Image processing, pattern recognition and computer vision.

In current study, he has major role in data generation and conducting the field work.