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Free Vibration and Dynamic Stability of Functionally Graded Material Plates on Elastic Foundation

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ABSTRACT

The study of parametric resonance characteristics of functionally-graded material (FGM) plates on elastic foundation is proposed under biaxial in plane periodic load. Finite element method in conjunction with Hamilton's principle is utilised to establish the governing equations in a discrete form, Floquet's theory was applied to determine the instability regions of FGM plate resting on elastic foundation. The effects of power law index, temperature rise, and foundation coefficients on the natural frequencies and dynamic stability of the plate have been examined in detail through parametric studies. The first two natural frequencies decrease with increase in temperature and power law index values, on the contrary, these two frequencies increase with increase in the foundation constants. Increase in power law index enhances the instability of the FGM plate. Increased foundation stiffness enhances the stability of the plate. Influence of shear layer constant is more dominant compared to the Winkler foundation constant.

Keywords: Functionally graded material plates, thermal environment, elastic foundation, finite element method, free vibration, instability regions

1. INTRODUCTION

Functionally-graded material (FGM) plate structures resting on elastic foundation are extensively useful in many engineering applications. Due to smooth distribution of material constituents, there is no abrupt change of stresses. These structural components like plates supported on an elastic foundation often find applications in the construction of nuclear, mechanical, aerospace, and civil engineering structures. These FGM plates can be subjected to external in plane periodic excitations, which may cause parametric resonance. Parametric excitations refer to vibrational motion in a mechanical system due to periodic load that is parametric to the deformation of the system. The response of the system is perpendicular to the direction of external excitation. Parametric instability occurs when the external excitation is equal to as integral multiple of any of the natural frequencies of the system, the system is said to undergo parametric resonance. In parametric resonance, systems amplitude increases exponentially and may propagate without bound. This exponential increase of amplitude is potentially dangerous to the structure. Parametric resonance is also known as parametric instability or dynamic instability. The problem of dynamic stability in various structures was studied by Bolotin[2].

The bending, vibration, and buckling analysis of isotropic plates on various types of elastic foundations have attracted the attention of many researchers. Geoige and Voyiadjis[3] investigated the refined theory for bending of moderately-thick plates on elastic foundations. This theory includes the transverse normal strain effect in addition to the transverse shear and normal stress effects. The bending problem of rectangular plates with free edges on elastic foundations using Galerkin's variational method was presented by Cheng Xiang-sheng[4]. Ramesh and Sekhar[5] studied the behaviour of flexible rectangular plates resting on tensionless elastic foundations by finite-element method (FEM). Lucia and Paolo[6] developed finite element method for static analysis of functionally-graded Reissner–Mindlin plate. Canonical exact solutions using Green's functions approach was presented by Lam[7], *et al.* to study the bending, buckling and vibration analyses of levy-plates on two-parameter elastic foundations.

Shen[8], *et al.* have presented free and forced vibrations analyses of Reissner-Mindlin plates with free edges resting on a Pasternak-type elastic foundation. Their approach was based on Reissner-Mindlin first-order shear deformation plate theory and included the plate foundation interaction and thermal effects. Jha[9], *et al.* proposed the free vibration analysis of FGM plates with higher-order and normal shear deformation theory. Malekzadeh[10] used the three dimensional elasticity theories to study the free-vibration analysis of FG plates resting on elastic foundations and with simply-supported boundary conditions. Thai[11], *et al.* have proposed a refined shear deformation theory for bending, buckling, and vibration of plates on elastic foundation. This theory was based on the assumption that the in-plane and transverse displacements consist of bending and shear components. In addition, it did not require any shear correction factor. Mehdi and Gholam[12] investigated

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a coupled FE-DQ method for buckling and free vibration analyses of thick plate resting on elastic foundation. Here the proposed method benefits the ability of FEM in modelling of complicated geometry, and at the same time, advantages of the simplicity and accuracy of DQM. Ozdemir[13] developed a new fourth-order finite element for thick plates resting on a Winkler foundation, the element was free from shear-locking problem. This new fourth-order finite element gave excellent results for static and dynamic analyses. Ramu and Mohanty[14, 15] studied the buckling and free vibration of FGM thin plates using finite element method. Efraim and Eisenberger[16] presented free vibration analysis of annular FGM plates.

Kumar[17] studied a differential transform method (DTM) for free transverse vibration of isotropic rectangular plates resting on a Winkler foundation. Jahromi[18], *et al.* have studied the generalised differential quadrature method for free vibration analysis of moderately-thick plates resting on Pasternak foundation. An exact solution for free vibration analysis of simply supported rectangular plates on elastic foundation has been presented by Dehghany and Farajpour[19] employing the three dimensional elasticity theory. Seyedemad[20], *et al.* investigated a novel mathematical approach for free vibration of thin rectangular plates on Winkler and Pasternak elastic foundation. Here, closed-form solutions were developed through solving the governing differential equations of motion of plates. Few papers have reported the dynamic stability of plates on elastic foundation. Patel[21], *et al.* investigated the dynamic instability of laminated composite plates supported on elastic foundations, subjected to periodic in-plane loads using C^1 eight-noded shear-flexible plate element. Recent research on vibration and buckling analysis, focuses on the functionally-graded material structures. Hiroyuki[22] examined the two-dimensional higher-order theory for natural frequencies and buckling stresses of thick elastic plates resting on elastic foundations. A study of the literature reveals the existence of virtuous researches on buckling and free vibration analyses of FGM plates supported on elastic foundation. Sheikholeslami and Saidi[23] used higher-order shear and normal deformable plate theory for free-vibration analysis of FG rectangular plate resting on two-parameter elastic foundation. Baferani[1] *et al.* developed an accurate solution for free vibration analysis of functionally-graded thick rectangular plates resting on two parameter elastic foundation. Buckling analysis of thick FG plate was studied by Thai and Kim[24] using closed-form solution.

In the present work the dynamic stability of a FGM plate supported on Winkler and Pasternak foundations has been investigated. A four-noded rectangular finite element with five degrees of freedom per node has been adopted to model the plate. Finite element method in conjunction with Hamilton's principle has been used to establish the governing equation. Third-order shear deformation theory has been considered for theoretical formulation in the analysis. Floquet's theory has been used to establish the stability boundaries. Effects of different system parameters like foundation elastic constants, thickness ratio and power law index etc. on the frequencies and dynamic stability behaviour of the FGM plate have been investigated.

2. MATHEMATICAL FORMULATION

The FGM plate of length *L*, width *B*, and thickness *h*, resting on elastic foundation and subjected to in-plane dynamic load is shown in Fig. 1. The plate is assumed to be subjected to biaxial in-plane dynamic loading represented as $P(t) = P_s + P_t \cos\Omega t$. $P_{s,}\ P_t$ where are the static and dynamic load components, respectively and Ω is the dynamic load component excitation frequency.

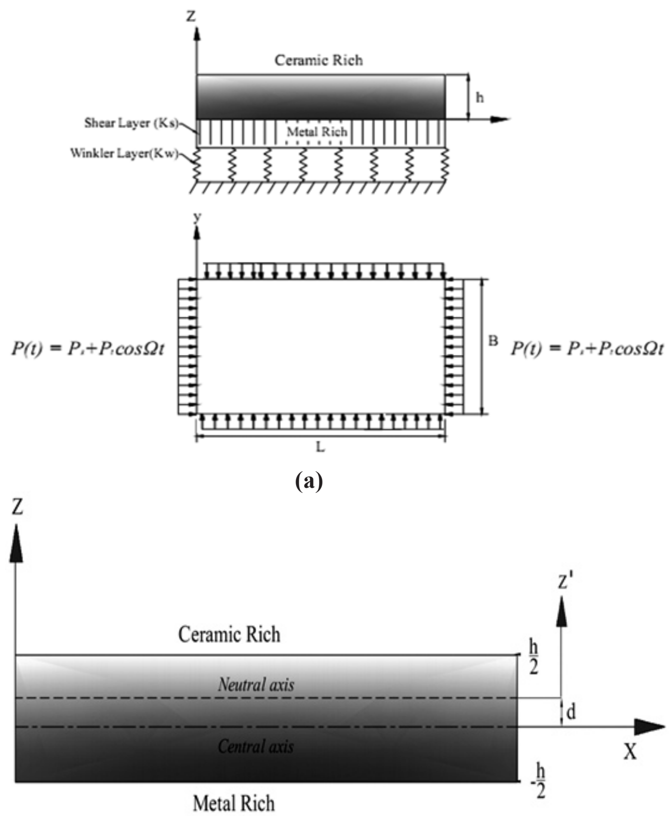


Figure 1. (a) FGM plate resting on elastic foundation and (b) Geometry of the FGM plate.

Assuming power law distribution in the thickness direction, the volume fraction of ceramic constituent may be written as

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^k \tag{1}$$

where *z* varies from metal surface -*h*/2 to ceramic surface +*h*/2.

The material property as a function of temperature is given as

$$P(T) = P_0\left(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3\right) \tag{2}$$

where P_0, P_{-1}, P_2, and P_3 are the coefficients of temperature *T* in Kelvin and are unique to each constituent.

$T = T_0 + T(z)$, where *T(z)* is temperature rise through the thickness direction and T_0 is room temperature.

Based on the volume fraction of the constituent materials, the effective material properties such as Young's modulus $E(z)$, Poison's ratio $\upsilon(z)$, mass density $\rho(z)$, and the coefficient of thermal expansion $\psi(z)$ of the temperature-dependent material properties are obtained using the following expressions:

$$E(z,T)=E_m(T)+[E_c(T)-E_m(T)]\left(\frac{2z+h}{2h}\right)^k$$
$$\psi(z,T)=\psi_m(T)+[\psi_c(T)-\psi_m(T)]\left(\frac{2z+h}{2h}\right)^k$$
$$\nu(z,T)=\nu_m(T)+[\nu_c(T)-\nu_m(T)]\left(\frac{2z+h}{2h}\right)^k \quad (3)$$
$$\rho(z)=\rho_m+[\rho_c-\rho_m]\left(\frac{2z+h}{2h}\right)^k$$
$$K(z)=K_m+[K_c-K_m]\left(\frac{2z+h}{2h}\right)^k$$

The temperature field is applied in the thickness direction only and the temperature field is assumed to be constant in the XY –plane of the plate for this analysis.

2.1 Physical Neutral Surface of the FGM Plate

For this analysis, the neutral plane concept has been employed. The FGM plate neutral plane does not coincide with the geometrical mid-plane due to the variation of the material properties along the thickness. The distance of the neutral surface (d) from the geometric mid-surface is expressed as

$$d=\frac{\int_{-h/2}^{h/2} zE(z,T)dz}{\int_{-h/2}^{h/2} E(z,T)dz} \quad (4)$$

2.2 Constitutive Relations

The displacements u, v and w at a point from the neutral surface of the plate can be expressed as

$$u=u_n+z'\theta_x-c_1z'^3\left(\theta_x+w_{n,x}\right),$$
$$v=v_n+z'\theta_y-c_1z'^3\left(\theta_y+w_{n,y}\right) \quad (5)$$
$$w=w_n$$

where u_n, v_n, w_n, θ_x and θ_y are functions *of x, y,* and *t* (time). u_n, v_n and w_n denote the displacements of a point on the neutral surface of the plate. θ_x and θ_y are the rotations of transverse normal about the *y* and *x* axes, respectively.

The general strain-displacement relations for small deformation are defined as

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^{(n)}\\ \varepsilon_y^{(n)}\\ \varepsilon_{xy}^{(n)}\end{Bmatrix}+z'\begin{Bmatrix}\varepsilon_x^{(1)}\\ \varepsilon_y^{(1)}\\ \varepsilon_{xy}^{(1)}\end{Bmatrix}-z'^3\begin{Bmatrix}\varepsilon_x^{(3)}\\ \varepsilon_y^{(3)}\\ \varepsilon_{xy}^{(3)}\end{Bmatrix}\quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=\begin{Bmatrix}\gamma_{yz}^{(n)}\\ \gamma_{xz}^{(n)}\end{Bmatrix}+z'^2\begin{Bmatrix}\gamma_{yz}^{(3)}\\ \gamma_{xz}^{(3)}\end{Bmatrix} \quad (6)$$

The stress-strain constituent relations of the FGM plate are represents as

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{yz}\\ \sigma_{xz}\\ \sigma_{xy}\end{Bmatrix}=\begin{bmatrix}Q_{11}&Q_{12}&0&0&0\\ Q_{21}&Q_{22}&0&0&0\\ 0&0&Q_{44}&0&0\\ 0&0&0&Q_{55}&0\\ 0&0&0&0&Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \varepsilon_{xy}\end{Bmatrix} \quad (7)$$

where

$$Q_{11}=Q_{22}=\frac{E(z',T)}{\left(1-\nu^2(z',T)\right)}, Q_{12}=Q_{21}=\frac{\nu(z',T)E(z',T)}{\left(1-\nu^2(z',T)\right)},$$
$$Q_{44}=Q_{55}=Q_{66}=\frac{E(z',T)}{2\left(1+\nu(z',T)\right)}$$

2.3 Nonlinear Temperature Distribution

The temperature rise through the thickness direction can be expressed as

$$T(z')=T_m+\left(T_c-T_m\right)\frac{\int_{-h/2-d}^{z'}\frac{1}{K(z')}dz'}{\int_{-h/2-d}^{h/2-d}\frac{1}{K(z')}dz'} \quad (8)$$

2.4 Finite Element Analysis

A four-noded rectangular plate element is considered for modal analysis of the FGM plate in thermal environment.The rectangular plate element has one node at each corner and five degrees of freedom per node.The in-plane displacements are *u, v,* and *w* is the transverse displacement, and represents the rotations of *x* and *y* normals.

$$u=\sum_{i=1}^{4}N_iu_i, v=\sum_{i=1}^{4}N_iv_i, w=\sum_{i=1}^{4}N_iw_i, \theta_x=\sum_{i=1}^{4}N_i\theta_x^i, \theta_y=\sum_{i=1}^{4}N_i\theta_y^i \quad (9)$$

The element stiffness matrix are derived as

$$K_s^e=K_b^e+K_{sh}^e$$
$$K_b^e=\iint[B_0]^T[D_0][B_0]dxdy+\iint[B_1]^T[D_1][B_1]dxdy+\iint[B_2]^T[D_2][B_2]dxdy$$
$$K_{sh}^e=\iint[B_3]^T[D_3][B_3]dxdy+\iint[B_4]^T[D_4][B_4]dxdy \quad (10)$$

where the submatrices are

$$D_0=\frac{E(z',T)}{\left(1-\nu^2(z',T)\right)}\begin{bmatrix}1&\nu(z',T)&0\\ \nu(z',T)&1&0\\ 0&0&\frac{1-\nu(z',T)}{2}\end{bmatrix}dz'$$

$$D_1=\frac{E(z',T)z'^2}{\left(1-\nu^2(z',T)\right)}\begin{bmatrix}1&\nu(z',T)&0\\ \nu(z',T)&1&0\\ 0&0&\frac{1-\nu(z',T)}{2}\end{bmatrix}dz' \quad D_3=\frac{E(z',T)}{2\left(1-\nu(z',T)\right)}\begin{bmatrix}1&0\\ 0&1\\ 0&0\end{bmatrix}dz'$$

The element mass matrix is derived considering translational and rotational kinetic energies and is given as

$$M^e=\int_v[N]^T\rho[N]dv+I\iint[N]^T[N]dxdy \quad (11)$$

The element geometric stiffness matrix is derived considering work done by the in-plane load and is expressed as

$$\left[K_g^e\right]=N_x\iint\frac{\partial N^T}{\partial x}\frac{\partial N}{\partial x}dxdy+N_y\iint\frac{\partial N^T}{\partial y}\frac{\partial N}{\partial y}dxdy+2N_{xy}\iint\frac{\partial N^T}{\partial x}\frac{\partial N}{\partial y}dxdy \quad (12)$$

The element thermal stiffness matrix is derived considering the work done by thermal load and is given as

$$\left[K_T^e\right]=\iint N_{tx}\left(\frac{\partial N}{\partial x}\right)^T\left(\frac{\partial N}{\partial x}\right)dxdy+\iint N_{ty}\left(\frac{\partial N}{\partial y}\right)^T\left(\frac{\partial N}{\partial y}\right)dxdy \quad (13)$$

where $N_{tx}=N_{ty}=\int_{-h/2}^{h/2}E(z)\psi(z)T(z)dz$ thermal load.

The Pasternak foundation element stiffness matrix K_F^e is derived from the potential energy U_F^e of the foundation

$$k_w^e=K_w\int_a[N]^T[N]dxdy \quad (14)$$

$$k_s^e=K_s\int_a\left(\left(\frac{\partial N}{\partial x}\right)^T\left(\frac{\partial N}{\partial x}\right)+\left(\frac{\partial N}{\partial y}\right)^T\left(\frac{\partial N}{\partial y}\right)\right)dxdy \quad (15)$$

$$K_F^e=k_w^e+k_s^e \quad (16)$$

where k_w^e and k_s^e are the Winkler's foundation and shear layer element stiffness matrix, respectively.

3. GOVERNING EQUATION OF MOTION

The equation of motion of the plate on elastic foundation in global matrix form can be expressed as

$$[M]\{\ddot{q}\}+[K_{ef}]\{q\}-P(t)[K_g]\{q\}=0 \tag{17}$$

where M, K_{ef}, K_g are global mass, effective elastic stiffness, and geometric stiffness matrices, respectively and $[q]$ is the global displacement vector.

The Eqn (18) represents a system of differential equations with periodic coefficients of Mathieu-Hill type.The regions of instability can be developed from Floquet's theory, which solve for the existence of periodic solutions of periods T and $2T$.

For practical importance the condition for existence (Ω=2ω) of these boundary solutions with period $2T$ can be achieved in theform of the trigonometric series:

$$q(t)=\sum_{b=1,3,..}^{\infty}\left[\{c_b\}\sin\frac{b\Omega t}{2}+\{d_b\}\cos\frac{b\Omega t}{2}\right] \tag{18}$$

Equating the coefficients of the sine and cosine terms leads to a series of algebraic equations for the vectors $\{c_b\}$ and $\{d_b\}$in the determination of instability regions. A sufficiently close approximation of the infinite eigen value problem is obtained by taking b=1 in the expansion in Eqn (19).The instability boundaries are obtained, putting the determinant of the coefficient matrices of the first-order equal to zero.

Hence, the condition for existence of these boundary solutions with period $2T$ is given by

$$\left|[K_{ef}]-(\alpha\pm\beta/2)P_{cr}[K_g]-\frac{\Omega^2}{4}[M]\right|=0 \tag{19}$$

4. RESULTS AND DISCUSSIONS

4.1 Validation

In this section, the validation of the present method is established using available results in the literature for fully simply-supported FGM plates. For a square FGM (Al/Al_2O_3) plate, the natural frequency parameter (ϖ) values from the present work are compared with those of Baferani[1], *et al.*, and are listed in Table 1. It can be concluded that a good agreement exits between the results.

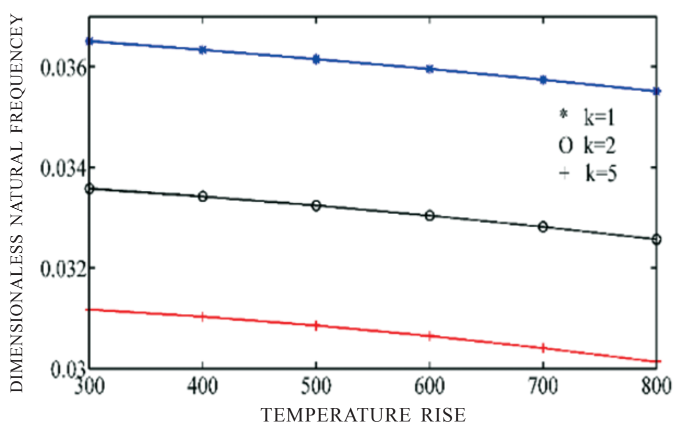
4.2 Free Vibration and Buckling Analyses

The FGM ($SUS304/Al_2O_3$) plate composed of steel (as metal) and alumina (as ceramic) on elastic foundation is considered. The side and thickness of square FGM plate are L = 1 and h = 0.05 m, and the Winkler and shear layer constants are k_w= 50 and k_s = 50, respectively. Figure 2 illustrate the effect of temperature rise on the first two dimensionless natural frequencies of simply-supported FGM plate on elastic foundation for nonlinear temperature environment. It was observed that the first-and second-mode dimensionless natural frequencies have decreasing tendency with increase in temperature. A distinct decrease was observed for increase in index value k = 1, 2, and 5.

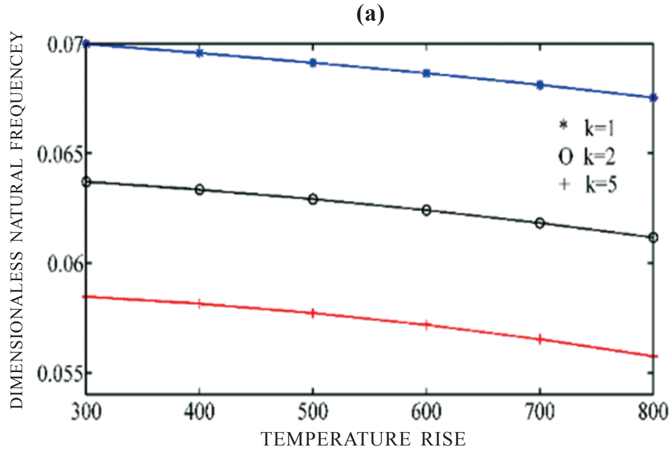
Table 1. The natural frequency parameter of FG square plate versus the shear and Winkler parameters, power law index and thickness

k_w	k_s	h/L	K=0		k=5	
			Ref. [1]	Present	Ref. [1]	Present
0	0	0.05	0.0291	0.0286	0.0197	0.0200
		0.1	0.1134	0.1114	0.0767	0.0771
		0.15	0.2454	0.2407	0.1648	0.1651
		0.2	0.4154	0.4070	0.2765	0.2733
100	0	0.05	0.0298	0.0295	0.0210	0.0214
		0.1	0.1162	0.1153	0.0821	0.0828
		0.15	0.2519	0.2501	0.1775	0.1778
		0.2	0.4273	0.4250	0.2999	0.2989
0	100	0.05	0.0406	0.0403	0.0381	0.0382
		0.1	0.1599	0.1591	0.1515	0.1515
		0.15	0.3515	0.3512	0.3362	0.3362
		0.2	0.6080	0.6105	0.5879	0.5888
100	100	0.05	0.0411	0.0410	0.0388	0.0390
		0.1	0.1619	0.1619	0.1543	0.1545
		0.15	0.3560	0.3577	0.3427	0.3430
		0.2	0.6162	0.6224	0.5993	0.6001

Length ratio for simply supported boundary conditions. $\varpi=\omega h\sqrt{\rho_m/E_m}$



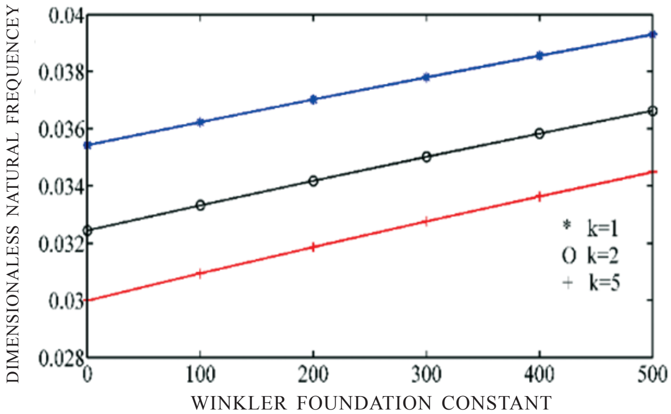
(a)



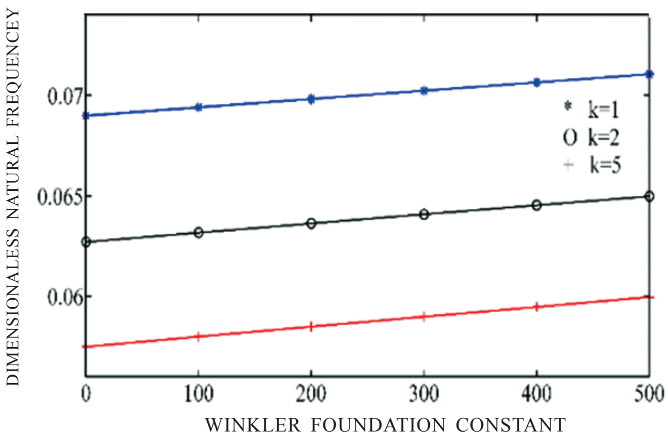
(b)

Figure 2. Dimensionless natural frequency vs temperature rise with nonlinear temperature field for various index values (k=1, 2, and 5, k_w=50, k_s=50): (a) First mode and (b) Second mode.

Figures 3(a) and 3(b) show the effect of Winkler foundation constant on natural frequency of FGM plate for first and second-mode, respectively. The Winkler foundation constant various from 0 to 500. It can be anticipated that the first two mode natural frequencies increase as the Winkler constant increases for power law index values k = 1, 2, and 5. The increase of index value reduces the frequency parameter.



(a)



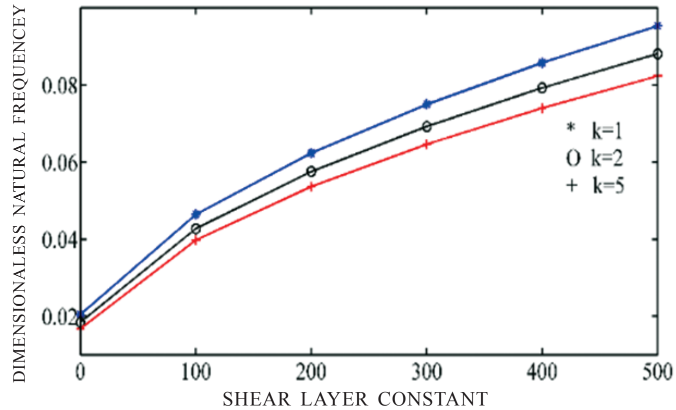
(b)

Figure 3. Effect of Winkler constant on first two frequency parameters (k_s=50, ΔT =200 K): (a) First mode and (b) Second mode.

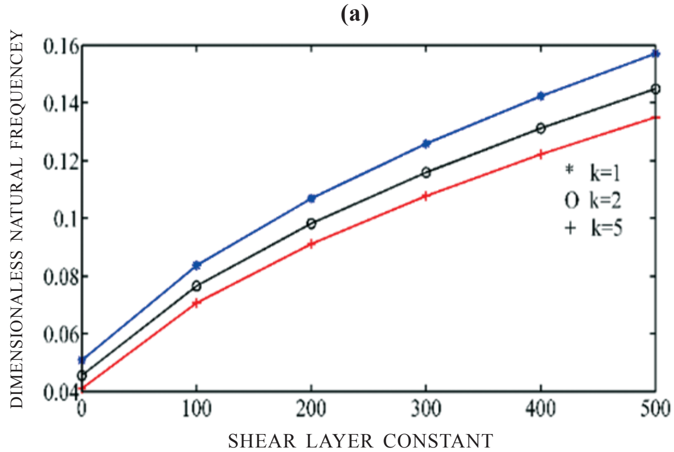
Figures 4(a) and 4(b) describe the effect of shear layer constant on the first two-mode natural frequency for different values of power law index k = 1, 2, and 5, respectively. Figure 5 shows that the natural frequency of FGM plate increases with increase in the value of shear layer constant. This tendency is observed because effective stiffness increases as the shear layer constant increases and consequently, the stiffer effective stiffness increase the natural frequencies.

4.3 Parametric Instability

The first-and second-mode principal instability regions of FGM plate resting on Pasternak foundation (k_w=50, k_s=50) for different index values k = 1, 2, and 5 are shown in Fig. 5. It is observed that the first-and second-mode instability regions shift towards the dynamic load axis with increase in index values k = 1, 2, and 5. So the increase in index value increases the instability of FGM plate on elastic foundation. This happens due to the reduced natural frequencies with increased power



(a)



(b)

Figure 4. Effect of shear layer constant on first two frequency parameters (k_w=50, ΔT =200 K): (a) First mode and (b) Second mode.

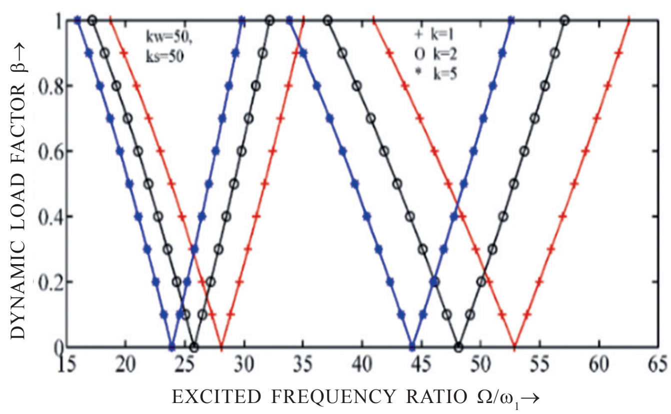


Figure 5. Regions of instability for first two-mode of FGM plates on Pasternak foundation (k_w=50, k_s=50).

law index. Increase in power law increases the metal content, and hence, effective stiffness of the FGM plate reduces.

Figure 6 illustrates the influence of the temperature rise on the first two-mode instability regions of FGM plate on Pasternak foundation (k_w=50, k_s=50). The temperature differences considered are 0 K, 300 K and 600 K above the ambient temperature. It was noticed that with increase in temperature the instability regions relocate towards the lower excitation frequencies. This means that increase in temperature enhances the dynamic instability of the FGM plate. Due to

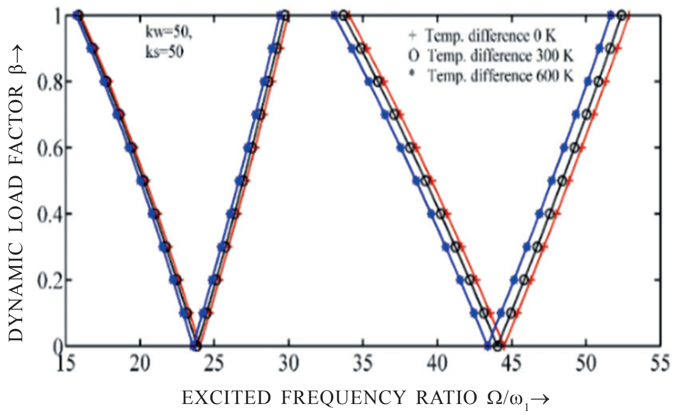
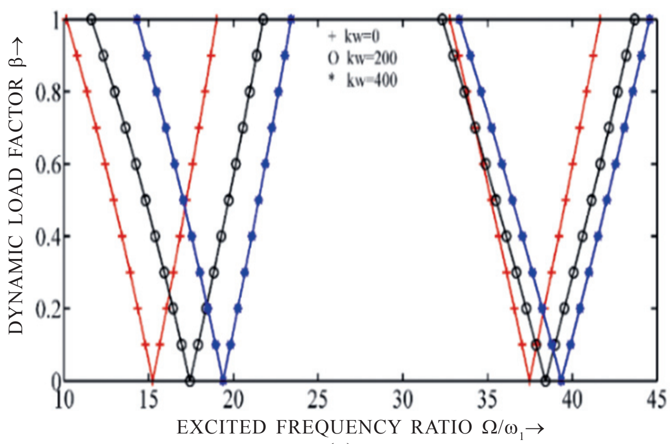


Figure 6. Dynamic instability regions of FGM plate resting on Pasternak foundation (k_w=50, k_s=50), (k=1).

the increased temperature, the effective Young's modulus decreases, and there is a reduction in natural frequencies. This leads to occurrence of instability at lower frequencies of excitation. The effect of temperature rise on second-mode instability regions is found to be more prominent than on the first-mode instability regions.

Figures 7(a) and 7(b) show the first two-mode instability regions of FGM plate supported on Winkler foundation (k_w=0, 200, and 400) and shear layer constant (k_s=50) for index values



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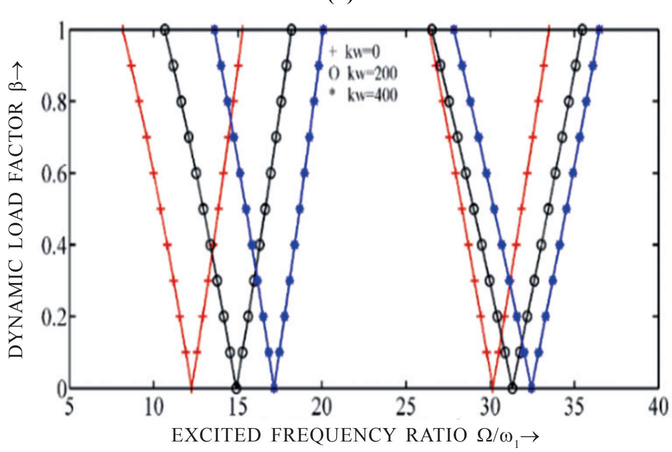
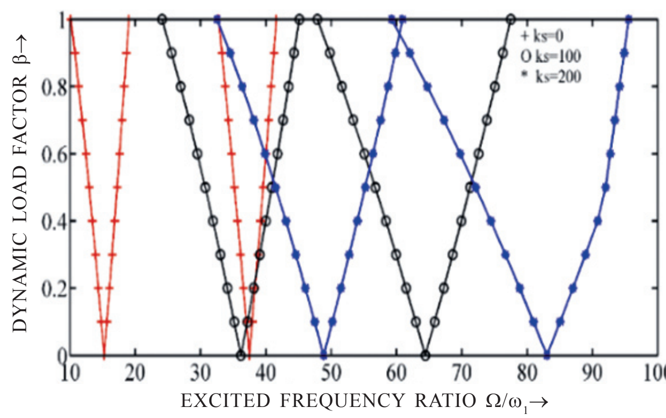


(b)

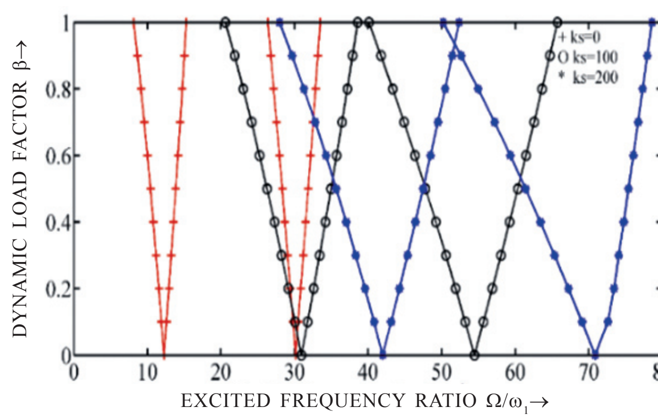
Figure 7. Effect of Winkler foundation constant on first-and second-mode instability of FGM plate for index values (a) *k*=1 and (b) *k*=5.

k=1, and *k*=5, respectively. Increase in Winkler foundation constant moves the instability regions away from the dynamic load factor axis.This happens due to the fact that the natural frequencies increase with increase in foundation stiffness. So, the increase in Winkler foundation constant increases the stability of the plate. It was observed that effect on the first-mode instability regions is more prominent than on the second mode instability regions.

The first two mode instability regions of FGM plate with k = 1 and k = 2 resting on Winkler foundation (k_w=50) and shear layer constant (k_s= 0, 100 and 200) are shown in Figs. 8 (a) and 8(b), respectively. Here the instability regions are located farther from the dynamic load factor axis with increase in shear layer constant. It was observed that increase of shear layer constant has got more influence on the stability of plate that than of Winkler elastic foundation constant.



(a)



(b)

Figure 8. Effect of Shear layer constant on first-and second-mode instability of FGM plate with index values (a) *k*=1 and (b) *k*=5.

5. CONCLUSIONS

The study of free vibration and dynamic stability of a FGM plate resting on elastic foundation under high temperature thermal environment has been studied introducing third-order shear deformation theory. The first two natural frequencies of FGM plate resting on elastic foundation decrease with increase in temperature and power law index values and it is due to reduction of effective stiffness. The frequencies of first two modes increase with increase of Winkler foundation constant and shear layer constant.

Increase of environment temperature and power law index has a destabilising effect on the dynamic stability of the FGM plate. On the contrary, increase in Winkler foundation constant and shear layer constant enhances the stability of the plate. The shear layer constant has a dominant influence on the dynamic stability of the FGM plate compared to the Winkler foundation constant.

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