

## Path Planning Algorithm based on Arnold Cat Map for Surveillance UAVs

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### ABSTRACT

During their task accomplishment, autonomous unmanned aerial vehicles are facing more and more threats coming from both ground and air. In such adversarial environments, with no a priori information about the threats, a flying robot in charge with surveilling a specified 3D sector must perform its tasks by evolving on misleading and unpredictable trajectories to cope with enemy entities. In our view, the chaotic dynamics can be the cornerstone in designing unpredictable paths for such missions, even though this solution was not exploited until now by researchers in the 3D context. This paper addresses the flight path-planning issue for surveilling a given volume in adversarial conditions by proposing a proficient approach that uses the chaotic behaviour exhibited by the 3D Arnold's cat map. By knowing the exact location of the volume under surveillance before take-off, the flying robot will generate the successive chaotic waypoints only with onboard resources, in an efficient manner. The method is validated by simulation in a realistic scenario using a detailed Simulink model for the X-4 Flyer quadcopter.

**Keywords:** Unmanned aerial vehicle, adversarial environment, chaotic path, 3D Arnold's cat map, volume surveillance

### 1. INTRODUCTION

Unmanned aerial vehicles (UAVs) represent a rapidly emerging technology that has attracted significant research interest in the last decades due to its high demand in both military and civil applications. With structural forms derived from airplanes, helicopters or dirigibles, these flying robots exhibit artificial intelligence capabilities derived from their sensing, computing and actuating attributes. The UAVs are currently being utilized in a wide spectrum of tasks ranging from reconnaissance<sup>1</sup> or surveillance<sup>2</sup> to target search and destroy missions<sup>3</sup>.

Under normal circumstances a patrol UAV tasked with a surveillance mission of a specified three-dimensional zone, pursues an optimized path that covers each section of the patrolled volume at regular time intervals. In adversarial conditions this task-oriented approach has to be enriched using unpredictable trajectories to cope with enemy entities.

To solve this problem we started from the lessons learned in the field of Unmanned Ground Vehicles (UGVs), where coping with enemies was addressed following two different lines of thinking: game theory and chaotisation of the UGV's path:

- The game theoretic perspective uses mathematical models to characterize patrolling tasks (models for the environment, for the UGV itself and for the enemies) and try to solve the problem by reducing it to classic pursuit-evasion games. Significant game theoretic approaches have been designed for various patrolling settings like single<sup>4</sup> or multi-robot patrolling<sup>5</sup>, with complete or partial

information about adversaries<sup>6</sup>. In spite of their reliable theoretical foundation, due to simplifying hypotheses and computational intricacy, the game theory methods are inappropriate in real-life applications where UGVs are facing dynamically environments and unidentified opponents.

- The approaches based on chaotisation of UGV's trajectories are covering three basic patrol missions – area surveillance<sup>7,8</sup>, points of interest surveillance<sup>9,10</sup> and boundary surveillance<sup>11</sup>, and rely on the proven unpredictability of the chaotic paths. In these methods the normal patrolling trajectory is changed into a misleading one using the chaotic dynamics revealed by well-known chaotic systems (e.g. Henon, Lorenz, Chua, etc.).

While in the case of UGVs a series of relevant scientific research involving chaotic dynamics has been reported, in the case of flying robots no solutions have been proposed so far. Trying to cover this research gap, present paper proposes an efficient approach for a patrol UAV to surveil a given 3D sector. For this, the flight path is described by a sequence of waypoints generated based on an adapted version of the 3D Arnold's chaotic cat map. The unpredictability of such trajectories is ensured by an intrinsic characteristic of any chaotic system: sensibility to initial conditions, also known as the butterfly effect<sup>12</sup>. Our method, due to its low computational complexity and memory requirements, can be efficiently performed using only UAV's onboard resources. Thus, before take-off, the UAV is programmed with the precise spatial location of the 3D zone under investigation and then the waypoints are computed one-

by-one during the flight. To validate our methodology, we present an illustrative simulation case study involving a highly maneuverable UAV (a quadcopter) that evolves in a specified volume of 640 cubic meters.

## 2. PROBLEM STATEMENT AND PRELIMINARY ANALYSIS

To patrol a three-dimensional zone in adversarial conditions, an autonomous UAV should accomplish its task using a path that is unpredictable for enemies. This problem can be formalised as follows:

**Problem Formulation:** Consider a given rectangular parallelepipedic 3D zone to be covered by an autonomous UAV exhibiting an unpredictable character for any enemy entity. Our task is to find a UAV path, described by uniformly distributed waypoints inside the zone and computed onboard with limited computational and energy resources.

In principle, the process of imparting unpredictability to UAVs paths may be attacked from two different perspectives: using random sequences or using chaotic sequences of intermediary flight points. While chaotic sequences are generated in a deterministic manner by a chaotic dynamical system<sup>12</sup>, the random sequences are fundamentally non-deterministic therefore irreproducible even if the initial states are exactly identical<sup>13</sup>.

In the case of our problem, there are two reasons that make the chaotic alternative more attractive: a) in the case of a chaotic waypoints generator, the ally entities, having full knowledge about the way intermediary points are obtained (the chaotic sequence generator is deterministic), can predict the future movements of the UAV and make appropriate decisions; and b) a truly random sequence cannot be implemented in practice, its pseudorandom variant having in fact a deterministic character.

We identified a set of three essential features that must be met by a chaotic system to efficiently solve the above-mentioned problem:

- (i) To provide a pseudo-random uniform distribution of waypoints in a predetermined volume  $V$  of the 3D space
- (ii) The volume  $V$  to be a trapping region<sup>14</sup> for the chaotic system and
- (iii) To allow a real-world implementation using only the low on-board computational and energy resources.

Analyzing various chaotic maps presented in scientific literature, we found an ideal candidate - a variant of the 3D Arnold's cat map.

## 3. THE 3D CHAOTIC CAT MAP

The Thom diffeomorphism of the 2-torus<sup>15</sup>, better known as the classic Arnold cat map<sup>16</sup> is a hyperbolic toral automorphism described by the following formula:

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \end{bmatrix} \text{ mod } 1 \quad (1)$$

where the state variables  $x_i$  and  $y_i$  are restricted to the interval  $[0,1)$  using the *mod*1 operation which extracts the fractional part.

The Arnold cat map gained its popularity due to numerous applications in the field of image encryption, steganography

and watermarking. In the last fifteen years, the cat map was generalized in the three-dimensional space<sup>17</sup> offering new application opportunities. To develop a chaotic path for UAVs accomplishing surveillance missions in a given bounded volume of space, we adopted the 3D cat map variant described by Chen<sup>17</sup>, *et al.* in the form:

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{bmatrix} = \begin{bmatrix} 1 + a_x a_z b_y & a_z \\ b_z + a_x b_y + a_x a_z b_y b_z & 1 + a_z b_z \\ a_x b_x b_y + b_y & b_x \end{bmatrix} \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} \text{ mod } 1 \quad (2)$$

where all the parameters  $(a_x, a_y, a_z, b_x, b_y, b_z)$  are positive integers.

By particularising all these six parameters to be equal to one, we obtain the 3D cat map version used in our research:

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & 5 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} \text{ mod } 1 \quad (3)$$

that is an invertible volume-preserving map (the determinant of  $A$  matrix is equal to 1), which also exhibits a typical chaotic behavior: the three eigenvalues of  $A$  are  $\sigma_1 = 7.1842 > 1$ ,  $\sigma_2 = 0.2430 < 1$  and  $\sigma_3 = 0.5728 < 1$ , while the maximal Lyapunov exponent is  $\lambda_1 = \log \sigma_1 = 0.8563 > 0$ . Because the origin  $O(0,0,0)$  is a fixed point of transformation (3), we should take any other starting point of the bounded volume  $[0,1) \times [0,1) \times [0,1)$  to obtain chaotic trajectories (Fig.1).

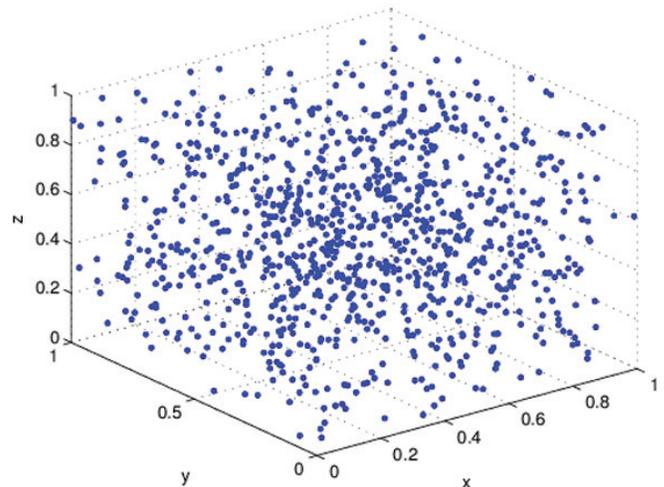


Figure 1. 3D Arnold cat map.

The 3D cat map extends the uniform pseudorandom distribution of the state variables from 2D space to 3D space<sup>18</sup> and represents the kernel of our methodology to solve the effective coverage of a 3D zone when an unpredictable trajectory is required.

#### 4. CHAOTIC PATH PLANNING METHOD

The approach we propose herein is divided in four steps and covers the entire path planning mechanism from the way the coordinate system is selected to the onboard waypoints generating as follows:

**Step 1:** Specify the local North-East-Down (NED) coordinate system in which the UAV will evolve; in the practice of aircraft or rotorcraft navigation, several coordinate systems are intensively utilised<sup>19</sup>: the geodetic, the local NED, the vehicle-carried NED, the earth-centered earth-fixed (ECEF) and the body coordinate systems. Using specific transformation formulas<sup>20</sup> all of these coordinate frames can be converted into one another, so a particular choice does not affect the generality of our approach. Due to the ease of coordinates' manipulation inside the methodology and for the clarity of our explanation we chose the local NED coordinate system to be the chosen frame. This Cartesian coordinate system is considered to be intuitive and adequate for flight navigation scenarios extended to no more than some tens of kilometers, covering most situations in which UAVs are involved. To specify a local NED frame<sup>20</sup> we need to establish the origin (a chosen fixed point on Earth's surface) and to consider the x and y axes pointing towards the geodetic North and geodetic East respectively, while the z-axis is pointing downward as presented in Fig. 2.

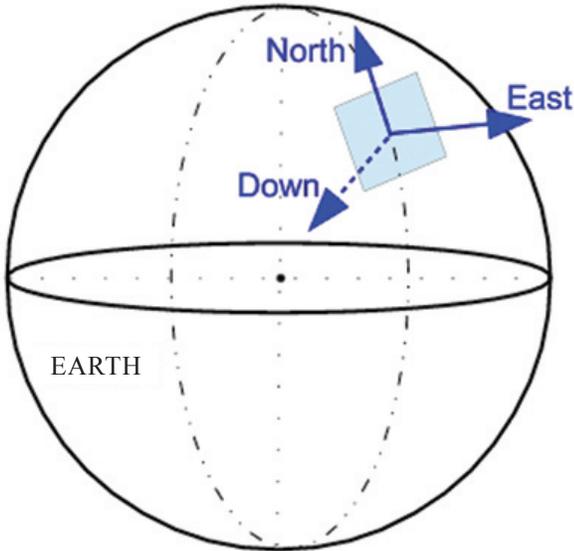


Figure 2. Local NED coordinate system.

**Step 2:** Choose the 3D zone that has to be covered by the UAV; we will select a rectangular parallelepiped with the bases parallel to the ground (xoy plane in local NED coordinate system) that will permit the accomplishment of UAV's surveillance task. This particular parallelepipedic shape can be considered as the image of the unit cube through a one-to-one affine transform as presented in Fig. 3.

The  $\Lambda$  transformation matrix using homogenous coordinates results by combining four elementary affine transformations: a translation with the transformation matrix  $\Lambda_{T1}$  that will move the center of the cube in the origin of the coordinate system; a scaling with the matrix  $\Lambda_S$  that will adapt the lengths of the edges; a rotation around z-axis described

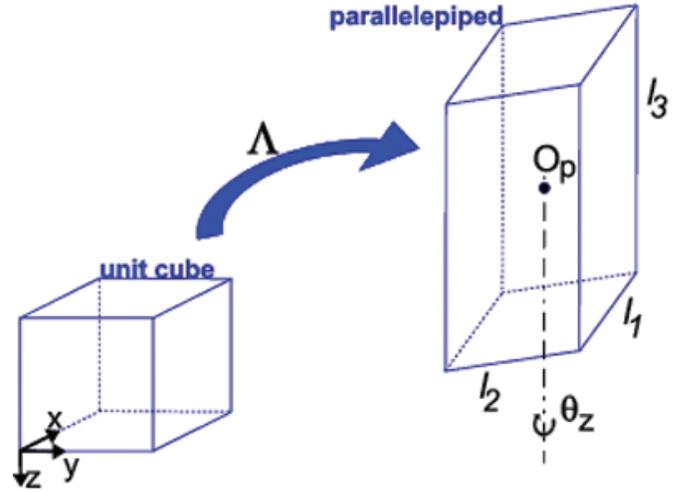


Figure 3 Affine transform  $\Lambda$ .

by the matrix  $\Lambda_R$ ; and finally a translation with the transform matrix  $\Lambda_{T2}$  that will move the parallelepiped in the desired place, as follows:

$$\Lambda = \Lambda_{T2} \Lambda_R \Lambda_S \Lambda_{T1} = \begin{bmatrix} 1 & 0 & 0 & O_{px} \\ 0 & 1 & 0 & O_{py} \\ 0 & 0 & 1 & O_{pz} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_z & -\sin \theta_z & 0 & 0 \\ \sin \theta_z & \cos \theta_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_1 & 0 & 0 & 0 \\ 0 & l_2 & 0 & 0 \\ 0 & 0 & l_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & -0.5 \\ 0 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

where  $\theta_z$  is the rotation angle with respect to z-axis,  $l_1$ ,  $l_2$ ,  $l_3$  are the length of the parallelepiped edges and  $O_{px}$ ,  $O_{py}$  and  $O_{pz}$  are the coordinates of the parallelepiped's center.

Computing the matrices multiplication in (4), the concatenated affine transformation  $\Lambda$  can be expressed in the form:

$$\Lambda = \begin{bmatrix} l_1 \cos \theta_z & -l_2 \sin \theta_z & 0 & -0.5l_1 \cos \theta_z + 0.5l_2 \sin \theta_z + O_{px} \\ l_1 \sin \theta_z & l_2 \cos \theta_z & 0 & -0.5l_1 \sin \theta_z - 0.5l_2 \cos \theta_z + O_{py} \\ 0 & 0 & l_3 & -0.5l_3 + O_{pz} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

that will map each point inside the unit cube into the desired parallelepiped where the UAV will evolve.

**Step 3:** Generate the chaotic sequence of waypoints in the desired parallelepipedic volume; the succession of points  $p_i = [x_i \ y_i \ z_i]^T$  generated inside the unit cube using the 3D cat map (3) are mapped using the resultant affine transformation  $\Lambda$  to obtain the chaotic sequence of waypoints  $\tilde{p}_i = [\tilde{x}_i \ \tilde{y}_i \ \tilde{z}_i]^T$  within the desired parallelepiped. This operation is expressed in homogeneous coordinates by the equation:

$$\tilde{p}_i = \Lambda \times p_i \quad (6)$$

that can be rewritten in terms of its components, in the following form:

$$\begin{cases} \tilde{x}_i = l_1 \cos \theta_z x_i - l_2 \sin \theta_z y_i - 0.5l_1 \cos \theta_z + 0.5l_2 \sin \theta_z + O_{px} \\ \tilde{y}_i = l_1 \sin \theta_z x_i + l_2 \cos \theta_z y_i - 0.5l_1 \sin \theta_z - 0.5l_2 \cos \theta_z + O_{py} \\ \tilde{z}_i = l_3 z_i - 0.5l_3 + O_{pz} \end{cases} \quad (7)$$

Due to the *mod1* operation that is applied inside the 3D cat map, the Eqn (7) cannot be used as variables' changing relations, so this step needs to be implemented in two stages: first the points  $p_i$  are generated inside the unit cube using Eqn (3) and after that each of these points are mapped inside the desired rectangular parallelepiped using Eqn (7).

**Step 4:** Following the path; having the sequence of waypoints  $\tilde{p}_i$ , the UAV will start moving from one waypoint to the next one accomplishing the shortest possible trajectory. This trajectory between two successive waypoints even it is desirable to be a line segment is affected by flight dynamic constraints of the UAV<sup>21</sup>.

### 5. UAV FLIGHT SIMULATION

Unmanned aerial vehicles can be categorized into three different subsets depending on their lift-creating elements: fixed wing aircrafts (airplanes), rotor wing aircrafts (helicopters) and lighter than air vehicles (airships or dirigibles). Our methodology is aimed for all of these UAV types, but the effectiveness of the approach is superior for highly maneuverable aircrafts (they can easily follow the sudden direction changes required by the sequence of waypoints). For this reason, our approach is exemplified on a quadcopter (quadrotor helicopter) which is a rotorcraft with two pairs of counter-rotating vertical propellers that exhibits agile manoeuvrability correlated with vertical take-off and landing (VTOL) capability and hovering ability.

The flight through the sequence of waypoints (Fig. 4) is simulated using the *mdl\_quadcopter* Simulink model for the X-4 Flyer<sup>22</sup> included in the Matlab Robotics Toolbox<sup>23</sup>, while the waypoints are generated by a new Matlab block that we developed based on our methodology.

The complex control system is based on a decoupled aerodynamic model which separates the longitudinal from

azimuthal modes and employs roll-pitch-yaw angles to represent the orientation and orientation rate of the quadrotor. A set of linear SISO control loops is used to control the attitude or to compute the required thrust and torques to move the UAV to the desired waypoint. The pitch and roll angles are controlled using a cascade structure with an inner attitude control loop and an outer position loop. Two other independent control loops are used to control the yaw and the altitude. All these loops are based on proportional-derivative controllers.

The flying path between two waypoints cannot be accomplished on the straight line segment joining the pair of points due to inherent kinematic and dynamic constraints described by the flight envelope and the actuators' limitations. The flight envelope is represented by the pitch and roll angles and the actuators' limitations are the maximal thrusts that can be generated by the rotors<sup>24</sup>. By considering the mentioned constraints, the real trajectory will be smoother but its global unpredictability will not be affected.

To examine the uniform pseudorandom distribution of the waypoints inside the surveilled volume, we split the unit cube in 125 cube-shaped cells (each dimension was split in 5) and we generated one million successive values using Eqn (5) starting with the initial point having the following coordinates:  $x_0=0.0831$ ,  $y_0=0.6811$ ;  $z_0=0.7289$ . For an ideal uniform distribution, all 125 bins have equal number of points: 8000. In our case we obtained a minimal value of 7792 and a maximal value of 8207 which correspond to a maximum absolute deviation of around 2.6 per cent that confirms the uniform pseudorandom distribution. In Fig. 5 we represented the number of values in every cell.

To estimate the effectiveness of the volume coverage accomplished by the proposed approach we used the coverage rate described by the formula:

$$C = \frac{1}{M} \cdot \sum_{i=1}^M l(i) \quad (8)$$

where  $M$  represents the total number of cubic cells in which the volume was split, while  $l(i)$  is a binary variable reflecting the status (visited or not visited) of the cell:

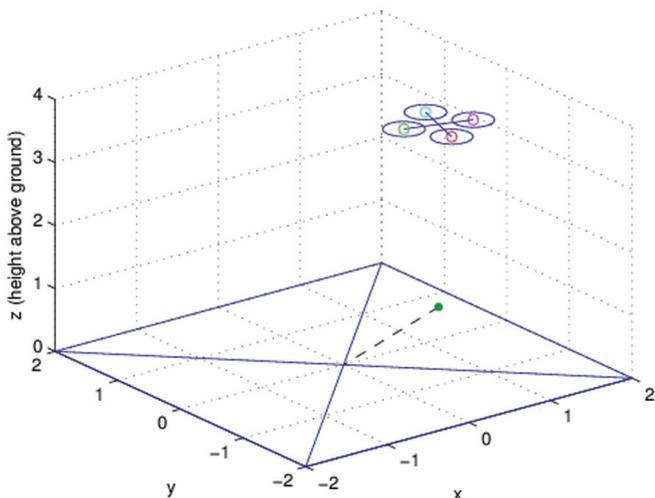


Figure 4. A snapshot of the quadcopter performing the surveillance task.

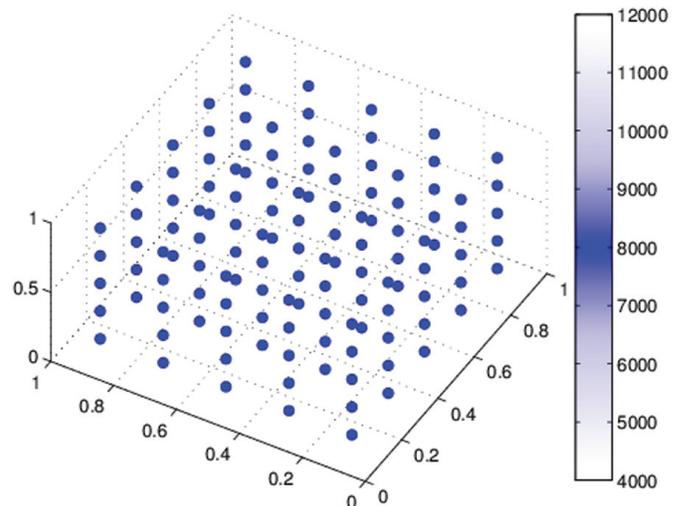


Figure 5. Evaluation of the uniform pseudorandom distribution.

$$l(i) = \begin{cases} 1 & \text{if the cell } i \text{ was visited} \\ 0 & \text{if the cell } i \text{ was not visited} \end{cases} \quad (9)$$

If we choose to divide each dimension of the entire volume to be covered in  $n$  parts, we will obtain a total of  $M=n^3$  cubic cells. Figure 6 depicts the coverage rate progression in three particular cases:  $n=5$  ( $M=125$  cells),  $n=8$  ( $M=512$  cells) and  $n=10$  ( $M=1000$  cells).

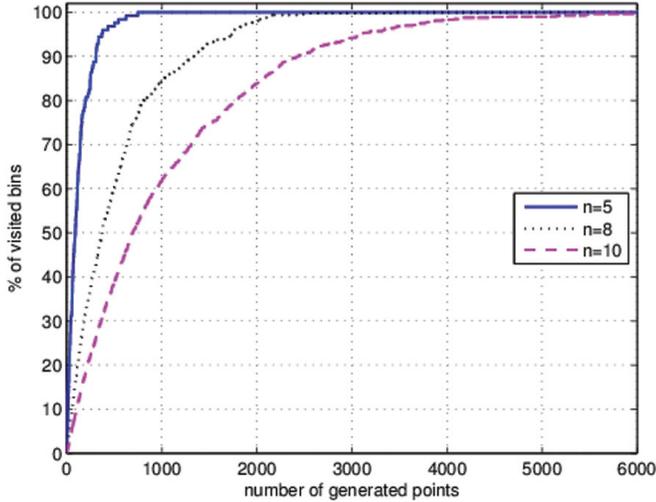


Figure 6. Volume coverage rate.

In the following paragraphs an illustrative simulation case study is presented. The 3D zone that will be covered by the quadcopter in its chaotic flight is represented in the local NED coordinates by a rectangular cuboid having the center  $O_p = [0 \ 0 \ 6m]^T$ , the lengths of the edges  $l_1 = l_2 = 8m$  and  $l_3 = 10m$  and the rotation angle about  $z$  axis  $\theta_z = \pi/4$ . In this case, the waypoints are obtained applying the affine transformation (7) in the following particular form:

$$\begin{cases} \tilde{x}_i = 4 \cdot \sqrt{2} \cdot (x_i - y_i) \\ \tilde{y}_i = 4 \cdot \sqrt{2} \cdot (x_i + y_i - 1) \\ \tilde{z}_i = 10 \cdot z_i + 1 \end{cases} \quad (10)$$

In Fig. 7 we illustrated the quadrotor's path pursued during 8 minutes of flight time. The UAV's flight simulation

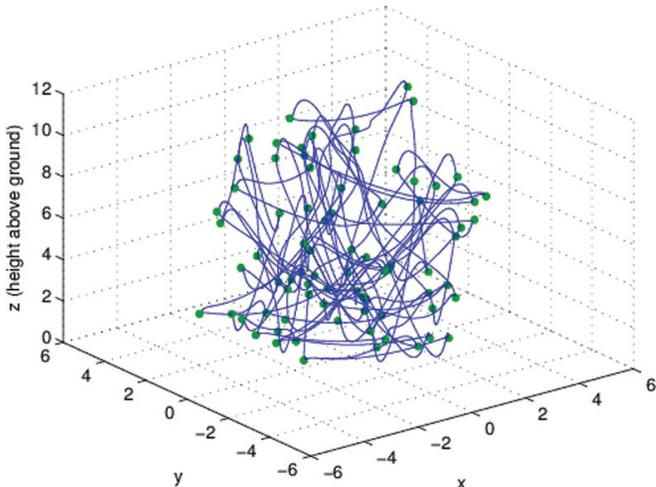


Figure 7. UAV's trajectory and related waypoints.

starts from the initial point  $p_0 = [0.2 \ 0.2 \ 0.05]^T$  of the unit cube or equivalently at the  $\tilde{p}_0 = [0 \ 3.38 \ 1.5]^T$  inside the surveilled parallelepiped and navigates through 79 consecutive waypoints. Derived from the 'sensitivity to initial conditions' feature of the 3D cat map chaotic system, the unpredictability of the UAV's path is ensured by the waypoints' generating method.

The path is generally situated inside the given rectangular cuboid as presented in the xoy flight's projection (Fig. 8), where the dashed line represents the projection of the chosen boundary parallelepiped on the ground plane.

The parallelepipedic boundary is rarely crossed, this situation occurring only for short intervals due to the above-mentioned flight constrains of the quadcopter.

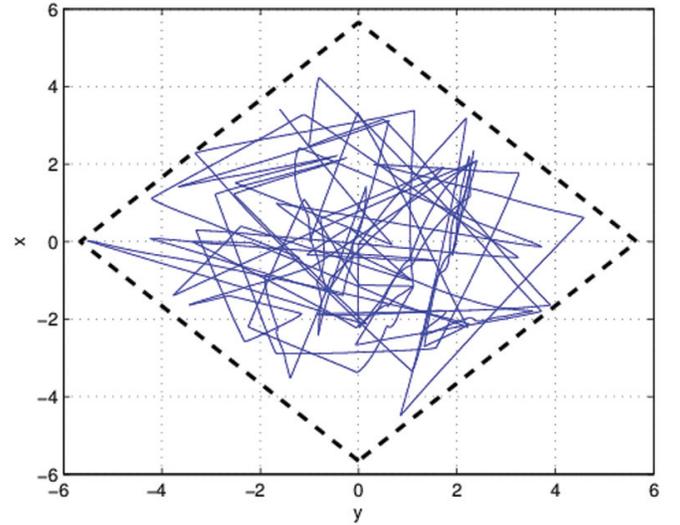


Figure 8. Flight path projection on the ground plane.

## 6. CONCLUSIONS

Autonomous unmanned aerial vehicles are considered to be adequate solutions for diverse types of surveillance or reconnaissance missions in harsh or hazardous environments. In adversarial conditions, the unpredictability of the UAV's path represents a critical and challenging issue. In this context, the present paper introduces a novel method for generating flight waypoints sequences based on the chaotic dynamics exhibited by a carefully constructed variant of the 3D Arnold's cat map. Due to its low computational cost and memory requirements, the method is suitable to be carried out using only UAV's onboard resources. Therefore, the precise spatial location of the 3D sector to be surveilled is loaded in the UAV's memory before its take-off, the waypoints being computed one-by-one during the flight.

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