Defence Science Journal, Vol. 66, No. 2, March 2016, pp.151-155, DOI : 10.14429/dsj.66.8423

Self-affine Fractal Modelling of Aircraft Echoes from Low-resolution Radars

Q.S. Li[#,*], J.H. Pei[!], and X.Y. Liu[#]

[#]*School of Physics and Electronic Information, Gannan Normal University, Ganzhou - 341 000, China*
[!]*Intelligent Information Institute, Shenzhen University, Shenzhen - 518 060, China*
[*]*E-mail: bjliqiusheng@163.com,*

ABSTRACT

For complex targets, the non rigid vibration of an aircraft as well as its attitude changes and the rotation of its rotating parts will induce complex nonlinear modulation on its echo from low-resolution radars. If one performs the fractal analysis of measures on an aircraft echo, it may offer a fine description of the dynamic characteristics which induce the echo structure. On basis of introducing self-affine fractal theory, the paper models real recorded aircraft echo data from a low-resolution radar using the self-affine fractal representation, and investigates the application of echo self-affine fractal characteristics in aircraft target classification. Results analysis shows that aircraft echoes from low-resolution radars can be modelled by using the self-affine fractal method, and the self-affine fractal features can be effectively applied to target classification and recognition.

Keywords: Low-resolution radar, self-affine fractal, echo modelling, target recognition

1. INTRODUCTION

Aircraft is a kind of non rigid targets with complex shapes. The non rigid vibration or attitude change of aircraft relative to the observation radar will induce complicated nonlinear modulations on the echo amplitude and its phase[1]. In addition, the jet engine modulation (JEM) induced by the rotation of the aircraft rotating parts, such as the rotor, empennage, propeller, turbine fan, etc., is also a typical nonlinear modulation, which embodies in the echo characteristics, such as amplitude, phase, frequency, and polarisation[1-3]. These kinds of nonlinear modulations reflect the complicated micro-motion modulation effects of various parts of aircraft and contain target attribute information, such as the geometric structure, material composition, etc.[4-5]. Different types of aircraft targets generally have different structure and rotating parts, and often have different nonrigid vibration and JEM characteristics. So if these nonlinear modulation signatures which reflect the physical characteristics of an aircraft target can be extracted effectively, then one may apply them to target classification and recognition directly[6-7].

So far, some scholars have proposed several theoretical models for aircraft echoes from low-resolution radars[8-12]. However, due to the complexity of the nonlinear modulation induced by the nonrigid vibration or attitude change, most models have paid more attention to the modelling of the JEM echo section, and simplified the modelling of the airframe echo section; so in some cases they are unsatisfactory in analyzing the nonlinear modulation characteristics of aircraft echoes from low-resolution radars. In recent years, some fractal geometry methods, such as mono-fractal, fuzzy fractal, multifractal, etc., have been introduced into the characteristic analysis of aircraft echoes from low-resolution radars[6,12-14]. However, it has not been reported that self-affine fractal theory has been applied to the modelling of real-recorded aircraft echo data from low-resolution radars so far. Therefore, the paper plans to take self-affine fractal theory as the tool to model aircraft echoes from low-resolution radars. On basis of introducing self-affine fractal theory, the text models aircraft echoes from low-resolution radars by using the self-affine fractal representation method[15], analyzes their self-affine fractal characteristics, and investigates the application of echo self-affine fractal signatures in aircraft target classification.

2. THE THEORETICAL BASIS

2.1 The Theoretical basis of Self-affine Fractal

Iterated function system[16] (IFS) theory is a powerful mathematical tool for the research of self-affine fractal, which is built on the basis of the compression mapping. Firstly, the definitions related to compression mapping are given as follows[17].

Definition 1: Let $(\boldsymbol{X}, d)$ be a metric space and ω be a mapping of $\boldsymbol{X} \rightarrow \boldsymbol{X}$. If there exists a positive constant, forming $d(\omega(x), \omega(y)) \le c \cdot d(x, y)$, $\forall x, y \in \boldsymbol{X}$ then ω is called a compression mapping, and c is its compression factor.

Definition 2: A hyperbolic IFS is composed of a complete metric space $(\boldsymbol{X}, d)$ and a set of compression mapping $\omega_i : \boldsymbol{X} \rightarrow \boldsymbol{X}$ with its compression factor c_i ($0 \le c_i < 1, i = 1, 2, \cdots, N$). The system can be expressed as $\{\boldsymbol{X}; \omega_i, i = 1, 2, \cdots, N\}$, and its compression factor $c = \max\{c_i, i = 1, 2, \cdots, N\}$.

If a time series is known, then one can construct an IFS and make its attractor approximate a specified sequence to model the series. Research on the attractor is usually carried out in the Hausdorff metric space. One can map the series to the

Received 09 April 2015, revised 17 February 2016, online published 22 March 2016

corresponding local through each of the self-affine transform of the IFS and piece these local fragments together, and then the error of the new series with the original series describes the degree of similarity between the attractor generated by the IFS and the original series. The smaller is the error, the higher is the degree of similarity between the two. For the error with the original series, the following collage theorem gives a metric method.

Collage theorem[18]: Let $(\boldsymbol{X}, d)$ be a complete metric space, and the elements of the space $H(\boldsymbol{X})$ consist of all non empty set of $\boldsymbol{X}$. If given $\boldsymbol{L} \in H(\boldsymbol{X})$ and $\varepsilon \geq 0$, one can select an IFS $\{\boldsymbol{X}; \omega_i, i = 1, 2, \cdots, N\}$ with its compression factor c ($0 \leq c < 1$), making

$$h\left(\boldsymbol{L}, \bigcup_{i=1}^{N} \omega_i(\boldsymbol{L})\right) \leq \varepsilon \tag{1}$$

then one can get $h(\boldsymbol{L}, \boldsymbol{A}) \leq \varepsilon / (1 - c)$, where $\boldsymbol{A}$ denotes the attractor of the IFS and $h(\cdot)$ is Hausdorff distance.

2.2 Representation of Self-affine Fractal

Authors described the theoretical basis of self-affine fractal for modelling a time series, below we further discuss the realisation of self-affine fractal[19]. Consider a linear affine mapping of point (t, x) in the $t - x$ plane, defined as

$$\omega\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} a & 0 \\ c & d \end{pmatrix}\begin{pmatrix} t \\ x \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix} \tag{2}$$

where a, c, d, e, and f are the mapping parameters, t is the time, and x is the amplitude of the signal. The mapping in Eqn (2) is called shear transformation; this is because it has different scaling in the horizontal and vertical direction. The parameter d is called the compression factor of the mapping, and $d \in (-1, +1)$. If the signal $x(t)$ defined in the interval $[t_0, t_f]$ is self-affine and the corresponding endpoint values are x_0 and x_f, then for the subinterval $[t_p, t_q]$, there exists a linear mapping ω_i which maps the signal in $[t_0, t_f]$ to the signal fragment in $[t_p, t_q]$. In particular, x_0 and x_f are mapped as $x_p = x(t_p)$ and $x_q = x(t_q)$, so

$$\begin{pmatrix} t_p \\ x_p \end{pmatrix} = \omega_i\begin{pmatrix} t_0 \\ x_0 \end{pmatrix}, \begin{pmatrix} t_q \\ x_q \end{pmatrix} = \omega_i\begin{pmatrix} t_f \\ x_f \end{pmatrix} \tag{3}$$

According to Eqn (3), a, c, e, and f can be calculated by the compression factor d and the corresponding boundary values, which can be expressed as

$$a = \frac{t_q - t_p}{t_f - t_0}, \tag{4}$$

$$e = \frac{t_f t_p - t_0 t_q}{t_f - t_0}, \tag{5}$$

$$c = \frac{x_q - x_p}{t_f - t_0} - d\,\frac{x_f - x_0}{t_f - t_0}, \tag{6}$$

$$f = d\,\frac{t_0 x_f - t_f x_0}{t_f - t_0} - \frac{t_0 x_q - t_f x_p}{t_f - t_0}. \tag{7}$$

Assume that $t_0 \leq t_p \leq t_q \leq t_f$, then the compression factor d can be obtained by the analytic technique[19]. Obviously, the affine mapping ω_i is completely determined by the compression factor d and the subinterval boundary condition. In practice, the signal is often a discrete time series. When ω_i compresses the whole series to a series subset according to the proportion, the number of sampling points will not be consistent before and after mapping. If assuming that the discrete series is $\{x_n\}, n = 1, 2, \cdots, N$ and $[p, q]$ is the subinterval of $[1, N]$, i.e., $1 \leq p \leq q \leq N$, where p, q and N are all positive integers, then ω_i is an approximate affine mapping, viz.

$$\omega_i\begin{pmatrix} n \\ x_n \end{pmatrix} = \begin{pmatrix} m \\ \omega_i(m) \end{pmatrix}, n = 1, 2, \cdots, N \tag{8}$$

with $p \leq m \leq q$. Here, the symbol $\omega_i(m)$ denotes the image of x_n in the m position. Attention is required, here m is not necessarily an integer. The optimal compression factor d for the mapping should make the mean square error (MSE) between the mapping value and the measured value smallest, and the error function[20] is

$$E_d = \sum_{k=p}^{q}\left[\tilde{\omega}(k) - x_k\right]^2 \tag{9}$$

Obviously, in the subinterval, the mapping points is N, while the measuring point has only $q - p + 1$. In Eqn (9), $\tilde{\omega}_i(k)$ denotes the average value of all $\omega(k)$, where $\mathrm{round}(m) = k$ ($\mathrm{round}(\cdot)$ represents the nearest integer). The method is used to overcome the inconsistency of the sampling points before and after mapping. The minimum variance estimator for the compression factor d is[19]

$$d = \frac{\sum_{n=1}^{N} B_n A_n}{\sum_{n=1}^{N} A_n^2}, \tag{10}$$

with

$$\begin{cases} A_n = x_n - \left[\xi_n x_1 + (1 - \xi_n) x_N\right] \\ B_n = x_{\mathrm{round}(a \cdot n + e)} - \left[\xi_n x_p + (1 - \xi_n) x_q\right] \end{cases}, \tag{11}$$

and

$$\xi_n = \frac{N - n}{N - 1}. \tag{12}$$

3. SELF-AFFINE FRACTAL MODELLING OF AIRCRAFT ECHOES

3.1 Self-affine Modelling of a Signal

Consider a discrete time series $\{x_n\}$, n = 1, 2, …, N, which can be divided into M overlapping subintervals with δ and φ as their sizes, there into, δ denotes the number of points in the subinterval, and φ denotes the overlapping points of two adjacent subintervals. Due to self-similarity, each subinterval is the replication of the whole series by shrinking proportionally and rotating corresponding to the compression factor. Segmenting data according to this method, we can get M contraction factors $d_i, i = 1, 2, \cdots, M$. The evolution process of the contraction factor can be modelled as a P-th order AR process[20], i.e.

$$d_j = \sum_{l=1}^{P} \alpha_l d_{j-l} + \varepsilon_j \tag{13}$$

where ε_j is the noise term and α_l are AR model coefficients. P can be selected by Akaike's Information Criterion (AIC)[21]. Then the $(M+1)$-th contraction factor can be estimated as

$$\hat{d}_{M+1} = \sum_{l=1}^{P} \alpha_l d_{M+1-l} \tag{14}$$

As can be known from Eqns (4) ~ (7), the mapping is completely determined by the two boundary values and the contraction factor. The estimator $\hat{d}_{M+1}$ of d_{M+1} can be obtained by Eqn (14). Because the data overlap, the right boundary can be estimated by the minimum mean square error between the estimation value and the actual value of the overlapping part of the $(M+1)$-th subinterval. And then it can be used to predict the values of $\{x_n\}, n > N$.

3.2 Echo Modelling Results based on Self-affine Fractal

The echo data used in the text are recorded from a surveillance radar, and they are from two different types of aircraft targets with the one fighter aircraft and the other civil aircraft. There into, the radar operates in the VHF band with its PRF 100 Hz and pulse width 25 μs, and the flight attitude of both types of aircraft targets contains two kinds: towards the radar station and off the radar station. In the working band of the experimental radar, the RCS values of the two kinds of aircraft targets fluctuate slowly. In the forward or backward-looking to the range of plus or minus 30 degrees, the RCS value of the fighter target is about 15 m^2, while the RCS value of the civil aircraft target is about 31.6 m^2. Figures 1 (a) and 1(b) show the self-affine fractal modelling results of a group of a normalised echo data from both types of aircraft targets when fly off the radar station. Among them, the echo data points N equals to 1024, the number of points in a subinterval δ equals to 32, and the number of overlapping data points φ equal to $\delta/2$. Figure 2 presents the relationship curves of the mean square modelling error E_x of these two groups of echo data with δ, and the number of overlapping data points φ are also $\delta/2$.

It can be seen from Fig. 1, compared to echoes from the fighter target, although echoes from the civil aircraft fluctuate more intensely, but the self-affine fractal model can effectively track the change trend of the two kinds of echo data, and therefore it can model them effectively. As can be seen from Fig. 2, the mean square modelling errors of echo data from the two types of aircraft targets increase with the increase of δ, but with the further increase of δ, both them grow more and more slowly and gradually stabilize. Comprehensively considering the factors such as the amount of calculation, 128 is selected as the number of points in a subinterval, i.e., $\delta = 128$, and φ is still taken as $\delta/2$.

Figure 3 shows the distributing circumstances of the 2-D signatures composed of the mean square modelling error E_x and the model order P of the AR process of the contraction factor d of echo data from both types of aircraft targets, with '★' and 'o' denoting the fighter aircraft and the civil aircraft, respectively. Among them, the group numbers of echo data from both types of aircraft are all 2560, with the echo group numbers for each flight attitude 1280. It can be seen from the figure, although there are some overlaps between the 2-D signatures of both types of aircraft targets, as a whole, the signatures belonging to

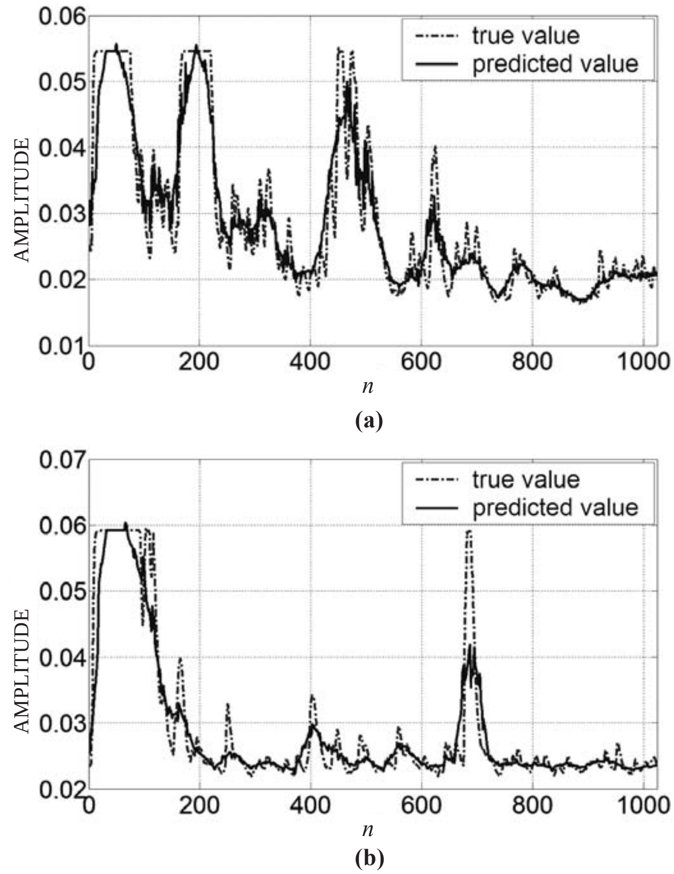

Figure 1. Self-affine fractal modelling of echo data from two types of aircraft targets (a) Civil aircraft and (b) Fighter aircraft.

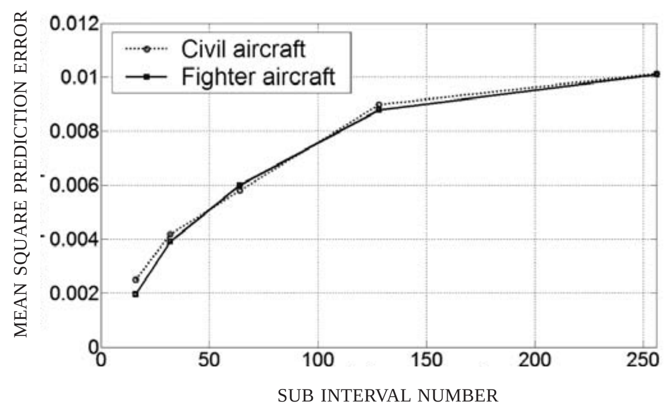

Figure 2. The relationship curves of the mean square modelling error E_x with the number of points in a subinterval δ.

different types of aircraft separate from each other distinctly. Therefore, if combining the two characteristic parameters together to identify different types of aircraft targets, it is hopeful to obtain a better performance.

4. AIRCRAFT TARGET CLASSIFICATION BASED ON SELF-AFFINE FRACTAL SIGNATURES

Section 3 shows that the self-affine fractal method can model an aircraft echo from low-resolution radar effectively. As pointed out in the introduction, echo data from different types

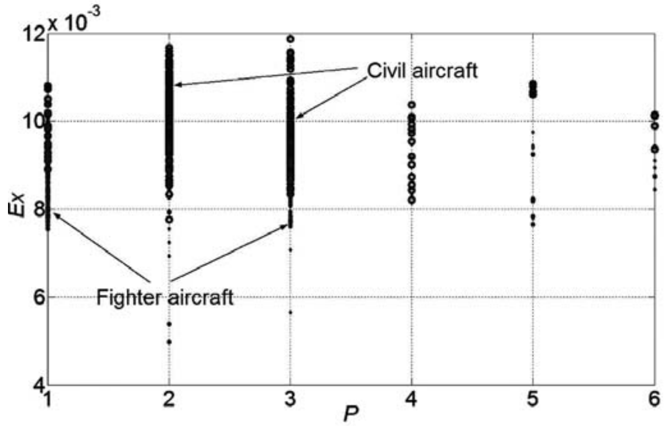

Figure 3. Distributing circumstances of 2-D signatures composed of E_x and P of echo data from both types of aircraft.

of aircraft targets often have different nonlinear characteristics, and thus they certainly will appear different self-affine fractal signatures. Therefore, it provides a probability for aircraft target classification and recognition with low-resolution radars. Based on the foregoing real recorded aircraft echo data, below the paper will investigate the application of self-affine fractal signatures in aircraft target classification with low-resolution radars.

Due to complexities of the actual target state and the environment, the target attitude, distance, background, etc. often change, which makes the raw target echo data cannot be directly used for feature analysis and extraction, and therefore one must do some data preprocessing to reduce the influence of these factors. Here the following two kinds of preprocessing will mainly be done: one is attitude partition, the other is energy normalisation. The specific method can be found in Literature 14.

Based on the difference between the distributing circumstances of the 2-D signatures of echo data from both types of aircraft targets shown by Fig. 3, here E_x and P are chosen as the characteristic parameters for target classification. Compared to other classifiers, support vector machine (SVM) has stronger generalisation abilities and a faster convergence rate[22], so in the experiment SVM using the Gaussian kernel function is taken as the classifier, and the kernel function parameters are selected rationally without going beyond the calculation burden.

Table 1 shows the classification results of the two types of aircraft targets, and as a contrast, the results using the raw echo data without performing any preprocessing are also presented. Among them, the group numbers of echo data from both types of aircraft targets are the same as those in Fig. 3, and for each type of aircraft targets, the signature data extracted from 512 groups of echo data are chosen as training samples (the group numbers for each of the two flight attitudes useful for classification are 256), with the rest signature data as testing samples. As can be seen from Tab. 1, the average correct classification rate (CCR) is more than 91 per cent, and the data preprocessing obtains a classification gain more than two percentage points. Therefore the classification effect is satisfactory. What should be pointed out is that the signature dimension reduction processing has been done in the classification experiment. If the whole signatures such as the AR model parameters of the contraction factor *d* are made full use of, the average CCR could still have an increase to a certain extent. Of course, this will lead to the increase of the feature dimension and the computation of the algorithm.

Table 1. Classification results

	Using raw echo data (%)	Using preprocessed echo data (%)
Fighter aircraft	86.86	91.77
Civil aircraft	90.89	90.48
Average CCR	88.77	91.11

5. CONCLUSIONS

Based on the complex nonlinear modulation characteristics induced by the nonrigid vibration and attitude change of aircraft targets along with the JEM effect, the paper models aircraft echoes from low-resolution radars from the viewpoint of self-affine fractal. On basis of introducing self-affine fractal theory, it models the real recorded aircraft echo data from low-resolution radars by using the self-affine fractal representation, and investigates the application of aircraft echo self-affine fractal signatures in target classification with low-resolution radars. The experimental results show that:

(i) It is an effective method to model aircraft echoes from low-resolution radar using a self-affine fractal model;
(ii) If one performs the self-affine fractal analysis of measures on an aircraft echo, it is hopeful to reveal its internal dynamics evolution mechanism;
(iii) Self-affine fractal characteristic parameters of aircraft echoes can be used as effective signatures for aircraft target classification with low-resolution radars.

REFERENCES

1. Huang, P.K.; Yin, H.C. & Xu, X.J. Radar target characteristics. Publishing House of Electronic Industry, Beijing, 2005.
2. Nalecz, M.; Andrianik, R.R. & Wojtkiewicz, A. Micro-Doppler analysis of signal received by FMCW radar. *In* Proceedings of International Radar Symposium, Germany, 2003, pp.231-235.
3. Ding, J.J. & Zhang, X.D. Automatic classification of aircraft based on modulation features. *J. Tsinghua University* (*Sci. Technol.*), 2003, **43**(7), 887-890. doi:10.3321/j.issn:1000-0054.2003.07.009.
4. Chen, V.C.; Li, F. Y.; & Ho, S.S. Micro-Doppler effect in radar: Phenomenon, model, and simulation study. *IEEE Trans. AES*, 2006, **42**(1), 2-21. doi: 10.1109/TAES.2006.1603402
5. Zhuang, Z.W.; Liu, Y.X. & Li, X. The achievements of target characteristic with micro-motion. *Acta Electron. Sinica*, 2007, **35**(3), 520-525. doi: 10.3321/j.issn:0372-2112.2007.03.028.
6. Li, Q.S. & Xie, W.X. Target classification with low-resolution surveillance radars based on multifractal features. *Progress Electromag. Res. B*, 2012, **45**, 291-

308.
doi:10.2528/PIERB12091509

7. Martin, J. & Mulgrew, B. Analysis of the effects of blade pitch on the radar return signal from rotating aircraft blades. *In* Proceedings of the IEEE 1990 International Radar Conference, 1990, pp. 569-572.
doi: 10.1109/RADAR.1990.201091
8. Bell, M.R. & Grubbs, R.A. JEM modelling and measurement for radar target identification. *IEEE Trans. AES*, 1993, **29**(1), 73-87.
doi: 10.1109/7.249114
9. Pizza, E. Radar signals analysis and modellisation in the presence of JEM application in the civilian ATC radars. *IEEE AES Magazine*, 1999, **14**(1), 35-40.
doi:10.1109/62.738353
10. Ding, J.J.; Zhang, X.D. & Lv, J.J. Modelling of the modulation signature for aircraft returns in the conventional radar. *Sys. Eng. Electron.*, 2003, **25**(11), 1407-1410.
doi:10.3321/j.issn:1001-506X.2003.11.028.
11. Chen, F.; Liu, H.W. & Du, L. Target classification with low-resolution radar based on dispersion situations of eigenvalue spectra. *Sci. China: Info. Sci.*, 2010, **53**, 1446-1460.
doi:10.1007/s11432-010-3099-5
12. Ni, J.; Zhang, S.Y. & Miao, H.F. Target classification of low-resolution radar based on fractional Brown feature. *Modern Radar*, 2011, **33**(6), 46-48.
doi:10.3969/j.issn.1004-7859.2011.06.011.
13. Li, Q.S. & Xie, W.X. Classification of aircraft targets with surveillance radars based on fuzzy fractal features. *Progress Electromag. Res. M*, 2013, **29**, 65-77.
doi:10.2528/PIERM12121601
14. Li, Q.S., Xie, W.X. Multifractal modelling of aircraft echoes from low-resolution radars based on structural functions. *Def. Sci. J.*, 2013, **63**(5), 515-520.
doi: 10.14429/dsj.63.3773
15. Guan, J; Liu, N.B. & Huang, Y. Fractal theory for radar target detection as well as its application. Publishing House of Electronics Industry, Beiijng, 2011.
16. Xie, H.P. & Xue, X.Q. The mathematical basis and method of fractal application. Science Press, Beijing, 1997.
17. Shi, B.; Wang, X.P. & Gai, M.J. The introduction and application of functional analysis. National Defence Industry Press, Beijing, 2009.
18. Packard, N.H.; Crutchfield J.P. & Farmer J.D. Geometry from a time series. *Phy. Rev. Lett.*, 1980, **45**(9), 712-716.
doi: 10.1103/PhysRevLett.45.712
19. Mazel, D.S. & Hayes, M.H. Using iterated function systems to modell discrete sequences. *IEEE Trans. Signal Process.*, 1992, **40**(7), 1724-1734.
doi: 10.1109/78.143444
20. Zhou, Y.F.; Yip P.C. & Leung H. On the efficient prediction of fractal signals. *IEEE Trans. Signal Process.*, 1997, **45**(7), 1865-1868.
doi: 10.1109/78.599962
21. Bozdogan, H. Model selection and Akaike's Information Criterion (AIC): The general theory and its analytical extensions. *Psychometrika*, 1987, **52**(3), 345-370.
doi: 10.1007/BF02294361
22. Duda, R.O.; Hart, P. E. & Stork, D. G. Pattern classification. Ed. 2nd. John Wiley and Sons, New York, 2001.

ACKNOWLEDGEMENTS

Authors would like to thanks Professor Huang Jianjun at Shenzhen University for offering the experiment data, the National Natural Science Foundation of China (Grant: 61561004, 61331021, 61271107), the Education Department of Jiangxi Province (Grant: GJJ14658) and the 2015 Bidding Project of Gannan Normal University (15zb04) for the support to this research work. They also wish to thank the anonymous reviewers for their help in improving this paper.

CONTRIBUTORS

Mr Q.S. Li, has received his BSc from Tongji University, Shanghai, in 1998, MSc from Beihang University, Beijing, in 2004, and PhD from Shenzhen University, Shenzhen, in 2013. Currently working in the School of Physics and Electronic Information, Gannan Normal University, Ganzhou, China. His fields of interests includes: Intelligent information processing, automatic target recognition and tracking.
In the current study, he has contributed in the self-affine fractal modelling of aircraft echoes and target classification experiments.

Prof. J.H. Pei, has received his BSc from from Beihang University, Beijing, in 1989, MSc and PhD from Xidian University, Xi'an, in 1994 and 1998, respectively. Currently working in the College of Information Engineering, Shenzhen University, Shenzhen, China. His fields of interests includes: Image and video processing and analysis, pattern recognition, fuzzy information processing, etc.
In the current study, he has contributed in the theory of self-affine fractal analysis.

Ms X.Y. Liu, has received her BSc and MSc from Nanchang University, Nanchang, in 2001 and 2005, respectively. Her fields of interests includes: Embedded systems, communications and information systems, etc.
In the current study, she has contributed in the data processing and graphics landscaping.