Discrete Electronic Warfare Signal Processing using Compressed Sensing Based on Random Modulator Pre-integrator

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ABSTRACT

Electronic warfare receiver works in the wide electromagnetic spectrum in dense radar signal environment. Current trends in radar systems are ultra wideband and low probability of intercept radar technology. Detection of signals from various radar stations is a concern. Performance and probability of intercept are mainly dependent on high speed ADC technology. The sampling and reconstruction functions have to be optimized to capture incoming signals at the receiver to extract characteristics of the radar signal. The compressive sampling of the input signal with orthonormal base vectors, projecting the basis in the union of subspaces and recovery through convex optimisation techniques is the current traditional approach. Modern trends in signal processing suggest the random modulator pre-integrator (RMPI), which sample the input signal at information rate non-adaptively and recovery by the processing of discrete and finite vectors. Analysis of RMPI theory, application to EW receiver, simulation and recovery of EW receiver signals are discussed.

Keywords: Compressed sampling, digital receiver, low probability of intercept, random modulator pre-integrator, ultra wideband

1. INTRODUCTION

The electronic warfare (EW) systems perform the control and coordination of the electromagnetic spectrum. Recent development of unmanned air vehicles and net centric warfare concept increases airborne threat density and also enables the flow of information at longer distances. Presently ultra wideband and low probability of intercept radar technology is a concern in EW field. The EW receiver systems operating frequency range 0.5 GHz to 40 GHz and are designed to capture the electromagnetic scenario in real time with high probability of intercept. The wideband EW systems have a better probability of intercept, moderate sensitivity and suffer availability of Nyquist ADCs or fail in the processing of aliased spectra of signals. Current EW receiver systems work on digital receiver technology enabling digital processing by traditional signal processing software algorithms. Digital receiver is designed using wideband and narrow band architectures together to optimise the probability of intercept and sensitivity of receiving signals. ADC technology lies at the heart of this revolution. The electromagnetic signals are linear, but when transmitted into space and received at EW receiver the signal is corrupted by noise and makes the received signal as nonlinear. Current radar parameter measurement and processing in EW receivers need to be improved to obtain higher resolutions for better radar signature identification. Compressed Sensing theory[2-4] using random modulator and pre-integrator (RMPI) states that a receiver signal can be sampled randomly without loss at a rate close to its information content and recover with stable recovery mechanism in the presence of noise and corruption.

The RMPI provides with increased bandwidth and resolution without utilising superior ADCs below Nyquist rate. The RMPI is linear and time invariant within a sample period[5,6]. The paper is organised as RMPI theory, application to EW receiver, Simulation and recovery of radar signals.

2. THEORY OF RANDOM MODULATOR PRE-INTEGRATOR

The input signal $x(t)$ is randomised by the random sequence which spreads the spectrum over the entire bandwidth. The RMPI encodes compressed samples by modulating the input signal $x(t)$ with a *PN* sequence $p(t)$ within ±1s. Its chipping rate must be faster than the Nyquist rate of the input signal. The purpose of modulation is to provide randomness necessary for successful CS recovery. The modulation is followed by an Integrator with impulse response $h(t)$. Finally, the signal is sampled at rate M using a traditional low rate ADC. The model used for RMPI theory is as shown in Fig. 1. The signal $x(t)$ is defined on the interval $[0,T]$ decomposes according to some set of N orthonormal basis functions $\{pn(t)\}$ as given in the following inner product.

$$x(t) = \sum_{n=1}^{N} x_n, p_n(t) \qquad (1)$$
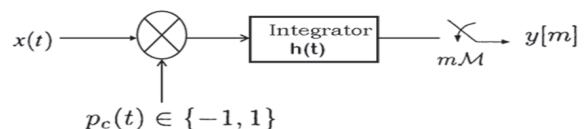
Figure 1. The model used for RMPI analysis[1].

where $p_n(t)$ satisfy $<p_n(t), pm(t)> = 1$ for $n = m$ and 0 otherwise. The number of basis functions that represent all the waveforms $\{x(t)\}$ may be infinite, $x_n = n \Delta T$, and $n = 0,1,2….N\text{-}1$; i.e. N may equal ∞. Using the set of basic functions, the function $x(t)$ maps to a set of N real numbers in a vector $\{xi\}$; these real-valued scalar coefficients assemble into an N-dimensional real-valued vector.

$$X = [X_1, X_2, X_3, ………..X_N] \tag{2}$$

The discrete signal $x(n)$ and $p(n)$ are discrete samples of $x(t)$ and $p(t)$. Random chip sequence $pc(t)$ is periodic chipping sequence[13]. Discrete sequence $p(n)=\{p0, p1, p2, p3 - - - - -;pN\}$, $p(n) \in \{+1;-1\}$ to be the pseudo-random bit sequence (PRBS). Discrete samples of $x(t)$ are given by

$$x(n) = w(t - nT) \int_{n-1}^{n} x(t) dt \tag{3}$$

where $\int_{n-1}^{n} x(t) dt$ is the average value of $x(t)$ on the nth interval giving a discrete value of $x(n)$. w $(t\text{-}nT)$ is a rectangular window function having magnitude one in interval $[-\Delta T/2, \Delta T/2]$ and 0 elsewhere. The frequency domain representation using Discrete Fourier Transform of input signal $x(n)$ is given by

$$c(n) = \sum_{n=0}^{N-1} q(n) h(mH - 1) \tag{4}$$

The frequency domain representation using Discrete Fourier Transform of signal $p(n)$ is given by

$$P(K) = \sum_{n=0}^{N-1} p(n) e^{-j\frac{2\pi kn}{N}}, k = 0,1,….. N-1 \tag{5}$$

Discrete signal model after the sampler is given by

$q(n)= p(n). x(n)$ (Time domain product) $\tag{6}$

The time domain product will have samples at the time instants of input signal at the rate of $p(n)$ and will have the bandwidth of the sum of the input signal and $p(n)$ pulse[4]. Hence, after the sampler the pulse output is spread and spreading is proportional to bandwidth $p(n)$ sequence.

$Q(K)= P(K) ©X(K)$ (Circular convolution) $\tag{7}$

$$Q(K) = \sum_{l=0}^{N-1} X(1) | P(K-1) |_N \text{ (Periodic convolution)} \tag{8}$$

The discrete frequency domain samples are compressed. The input signal is spreading in the time domain and compressing in the frequency domain. The input signal is sampled with base vectors; sparse coefficients are extracted and input signal recovery is called compressed sensing. Since $p(n)$ is truncated the sequence $q(n)$ also must be converged. The reason of convergence for discrete sequence $q(n)$ is important for stability and analysis using transform theory. In CS theory, inner product can be implemented by using discrete components and transform coding. The current transform coding theory suggests that time, frequency localisation and high resolution scaling of frequency domain to study the new signal properties. DFT coefficients are periodic and non-coherent. Optimum base coefficients with high resolution frequency domain scaling can be obtained using curve lets transform. The literature survey suggests that the use of directional transforms like discrete curve lets transform and shear-lets transform to study the physical signal new properties in CS receiver applications. Integrate the modulator output over discrete periodic time sequence with

period N. The number of output samples is 'm' and sampling rate is M and the integrator impulse response is h (mM). The output of the integrator is convolution of input signal amplitude samples $q(n)$ and impulse response of integrator $h(mM)$.

$$c(n) = q(n) * h(mM) \tag{9}$$

$$c(n) = \sum_{n=0}^{N-1} q(n) h(mM - 1) \tag{10}$$

$$C(K) = Q(K) . H(K) \tag{11}$$

The time domain sequence $c(n)$ can be obtained through IDFT of $C(K)$.

$$c(n) = \frac{1}{N} \sum_{k=0}^{N-1} C(K) e^{\frac{j2\pi kn}{N}}, k = 0,1,….. N-1 \tag{12}$$

Digitise the integrator output by sampling with a low-rate ADC, at the end of each time interval.

$$y(m) = \sum_{n=0}^{m} c(n) \delta(n - mT) = c(mT) \tag{13}$$

The $y(m)$ samples are m sparse and $m<< N$. Sensing is carried out by computing the linear projection $y = px$. Discrete signal model after the sampler is given by $y(m) = p(n).x(n)$. Discrete signal can be recovered by convex optimisation theory through L1-norm minimisation using Linear programming (LP) methods. The LPs are solved using a generic path-following primal-dual method. Signal recovery via $\ell 1$ minimisation[12]

Recovery: Given $Y(m) = p(n) . X(n)$

Optimisation: $X(n)= \arg \min \| x \| 1$

$Y = px$

The concept of RMPI is a parallel architecture and has several of multiplier - integrator pairs running in parallel using distinct sign sequences for wideband applications.

3.   RMPI APPLICATIONS TO EW

The radar pulse consists of a pulse envelope, which is nearly trapezoidal except smoothed at the corners, which is modulated by a high-frequency carrier signal. The relevant features of a radar pulse is frequency, pulse width, time of arrival, angle of arrival, radar signal polarisation, modulation method (time, frequency, Phase). The radar signal is sparse in time, since pulse width is finite duration and contains bandwidth of 10 MHz. The radar signal is sparse in frequency, Since PRF is around 10 kHz, but the gaps between the pulses are greater than the length of the pulses. Modern signal processing theory suggests that exploring orthogonality, coherence, random projections and study of received signal in the union of subspaces to improve radar signal detection and radar parameter resolutions. The compressive sampling is a generalisation of conventional point-sampling in which samples are inner products between an unknown signal vector and a set of user defined base vectors. RMPI compresses the information, while sampling, avoiding processing a huge redundant data and lowering the sampling rate[14]. Ultra wideband radars are impulse radar, noise radar use Gaussian pulse and its derivatives. These systems use electromagnetic (EM) pulses for transmission with and without high frequency modulation. The radar emits an EM signal at specific time instants with the shape of a pulse and pause a given time and then a sample of the antenna voltage are taken for the echo[15]. The receiver uses correlation techniques

with rake receiver, extremely high sampling rates, and timing requirements are extremely tight, often requires sophisticated devices exceeds the state-of-the-art technology. Compressed sensing offers better performance, including multipath signal handling, high sampling rates compared to all other techniques used in UWB systems

The basic pulse used for UWB is a Gaussian monocycle in which the width determines the center frequency and the bandwidth[16]. These systems use very low power spectral density with carrier frequency 3.1 GHz to 10.6 GHz, 14 bands each of 500 MHz or having a fractional bandwidth of larger than 20 per cent and PRF could be from 1 MHz to 50 MHz. The PRI is often modulated to carry information or coding with radiated power cannot exceed - 41.3 dBm/MHz .The pulses are of 0.2 ns and 0.5 ns width and used as the elementary pulse shaping to carry the information according to the Federal Communication Commission (FCC).

Low probability of intercept (LPI): The radar uses LPI waveforms which are intentionally designed to make the detection process nearly impossible. The LPI radar uses various intra-pulse modulations such as Chirp, Barker, poly-phase and poly-time codes. The pulse descriptor words received by EW receiver contain all the information about modulation. Initially preprocessing logic for all other radar will be processed and extracted, and then post processing logic extracts LPI parameters. A combination of pre and post processing extracts all the information of LPI pulse. Low probability of intercept (LPI) radar technology analysis shows the effect of scenario-dependent parameters and detection-threshold factors in jamming and anti-jamming environments. The composite signal received by EW system follows A Poisson distribution. The poisson distributed signal can be recovered by compressed sensing with random modulator principle. The recovery of ultra wideband pulse, LFM modulated pulse, Barker code pulse and random pulse spikes gives confidence to use in EW receiver technology.

4. SIMULATION AND RECOVERY OF UWB AND LPI SIGNALS

The PN sequence is simulated in MATLAB\R2013b and tried to multiply with input signal $x(t)$. The input signal $x(t)$ and PN sequence has to be maintained same dimension. The physical signal will not match with PN dimension. Hence the random noise sequence in standard MATLAB\R2013b Library and PN sequence both are used for simulation. The input signal samples are modulated with random sequence $p(t)$. The output of the modulator was integrated and sampled at 100 MHz rate using low frequency ADC and output samples are formed into an orthonormal vector matrix and the signal is recovered using L1-norm convex optimisation algorithm. The RMPI concept is simulated with an input signal $x(t)$ of 4 GHz frequency of UWB pulse. The Fig. 2(a), 2(b), 2(c) describes the sequence of simulation logics and recovery.

Random Pulse Signal Simulation: Pulse signal of input signal $x(t)$ is sampled 1000 MHz rate and modulated with random sequence $p(t)$. The Fig. 3(a), 3(b) describes the sequence of random modulator with pulse spikes simulation logic and recovery. Simulation and recovery of LFM signal: The LFM
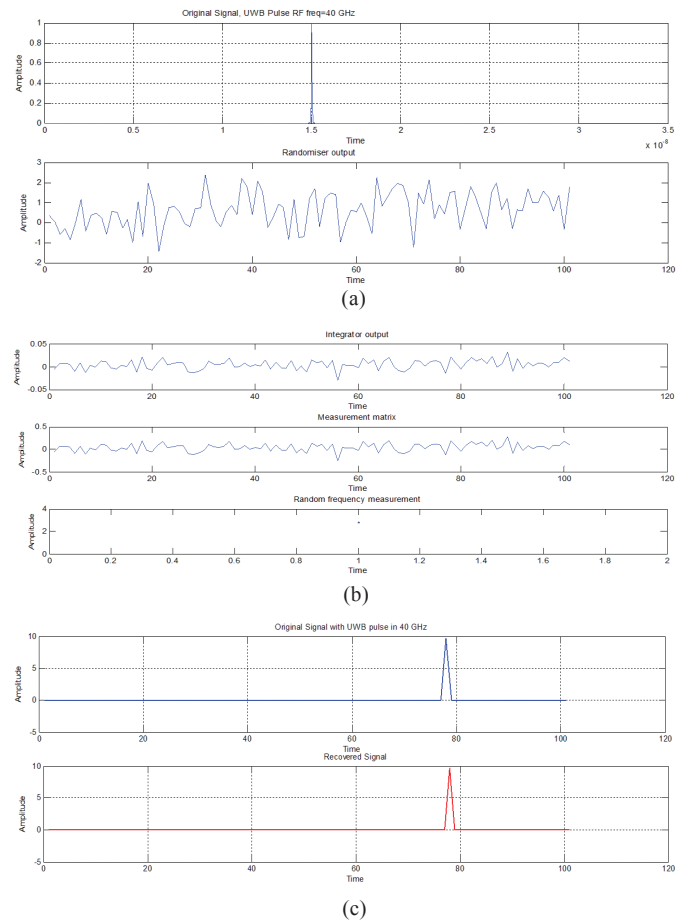

Figure 2. (a) Original signal and randomizer, (b) Integrator output, measurement matrix and random frequency, and (c) Original pulse and recovered pulse
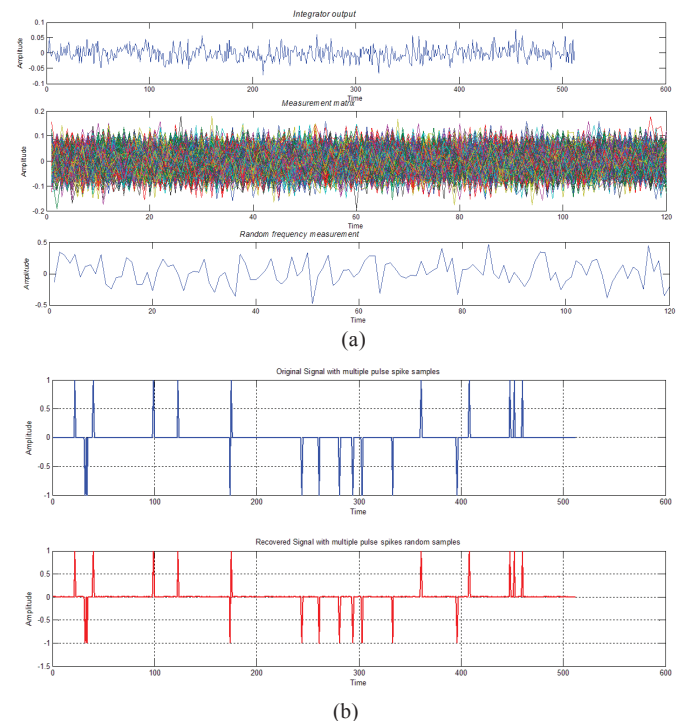

Figure 3. (a) Integrator output, measurement matrix and random frequency and (b) Original pulse and Recovered pulse spikes.

signal is simulated and used as input signal $x(t)$. The signal is sampled at 1250 MHz, modulated with random sequence $p(t)$, integrated, sampled at 100 MHz rate and recovered the signal. The signal is recovered using L1-norm convex optimisation algorithm. The Fig. 4 (a), 4(b), 4(c) describes the sequence of LFM signal simulation logic and recovery.

Simulation of Barker code signal: The Barker code modulated radar signal is simulated and used as input signal $x(t)$. The signal is sampled with 1250 MHz modulated with random sequence $p(t)$, integrated and sampled at 100 MHz rate and recovered using L1-norm convex optimisation algorithm. The following describes the sequence of Barker code simulation logic and recovery in Fig. 5(a), 5(b), 5(c).
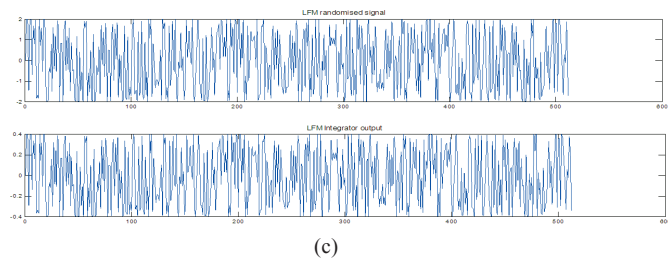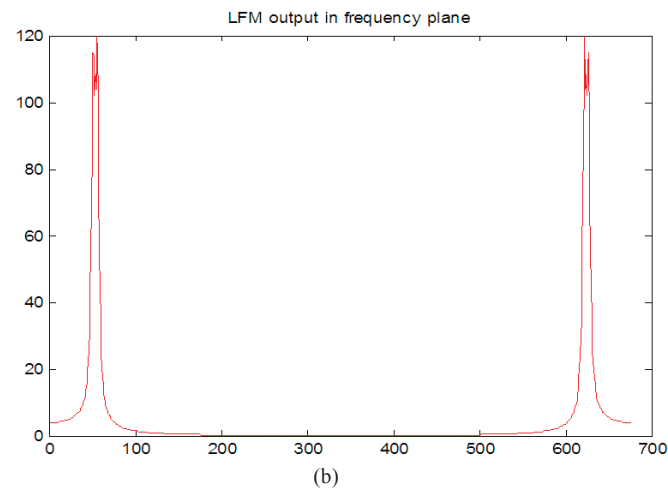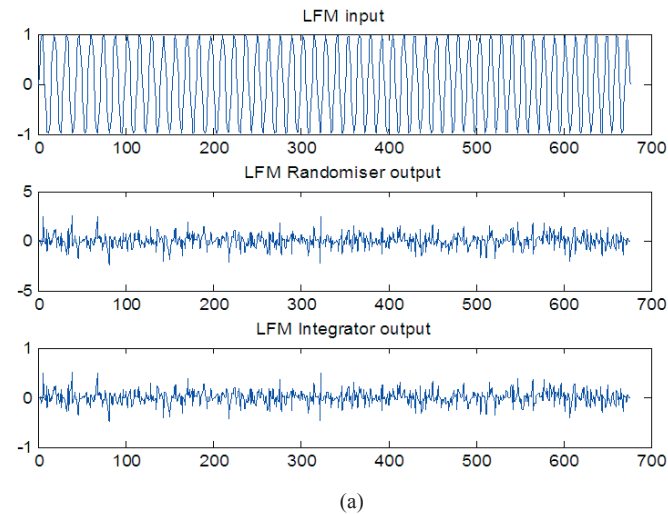

(a)

(b)

(c)

Figure 4.   (a) LFM Pulse, randomiser and integrator, (b) LFM in frequency plane, and (c) Original and recovered signal with LFM pulse.
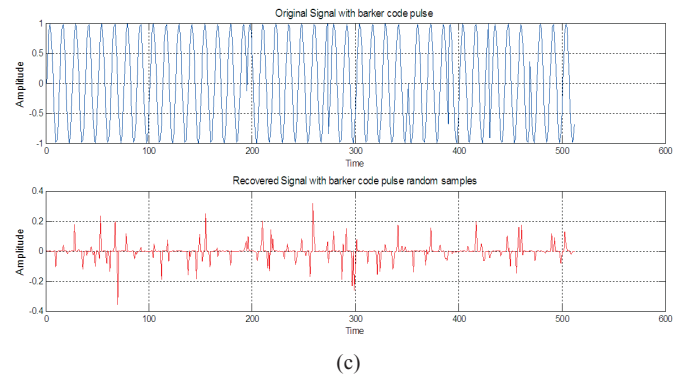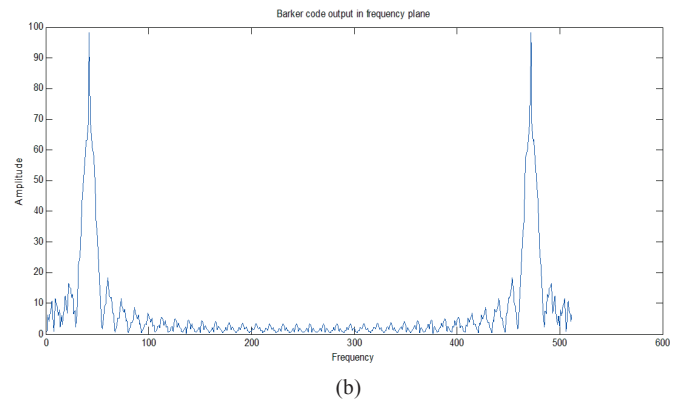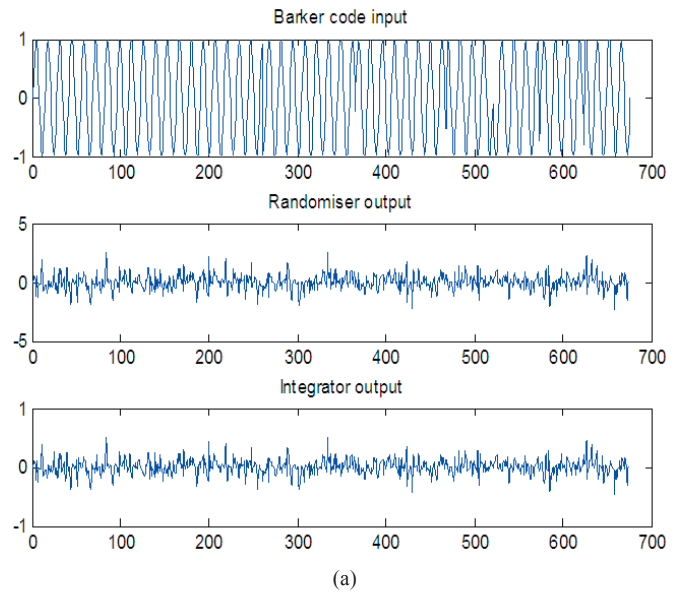

(a)

(b)

(c)

Figure 5.   (a) Barker coded pulse, randomiser and Integrator output, (b) Frequency plane output, and (c) Original and recovered signal with Barker pulse.

5.   CONCLUSION

Simulation of discrete EW signal processing using compressed sensing based on RMPI technique. The RMPI discrete processing theory is analyzed by the study of the same for EW signal processing. The ultra-wideband signal, LPI pulse signal, random pulse spikes, LFM modulated pulse and Barker code pulse are simulated, recovered the signal using L1 norm minimisation and RMPI concept in compressive sensing procedure. The simulation proves the applicability of RMPI for the electronic warfare application.

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