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Enhanced Singular Value Decomposition based Fusion for Super Resolution Image Reconstruction

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ABSTRACT

The singular value decomposition (SVD) plays a very important role in the field of image processing for applications such as feature extraction, image compression, etc. The main objective is to enhance the resolution of the image based on Singular Value Decomposition. The original image and the subsequent sub-pixel shifted image, subjected to image registration is transferred to SVD domain. An enhanced method of choosing the singular values from the SVD domain images to reconstruct a high resolution image using fusion techniques is proposesed. This technique is called as enhanced SVD based fusion. Significant improvement in the performance is observed by applying enhanced SVD method preceding the various interpolation methods which are incorporated. The technique has high advantage and computationally fast which is most needed for satellite imaging, high definition television broadcasting, medical imaging diagnosis, military surveillance, remote sensing etc.

Keywords: Singular value decomposition, image fusion, super resolution, image registration, interpolation

1. INTRODUCTION

In most electronic imaging applications, images with high resolution (HR) are desired and often required. HR means that pixel density within an image is high and therefore an HR image can offer more details that may be critical in various applications. For example, HR medical images are very helpful for a doctor to make a correct diagnosis. It may be easy to distinguish an object from similar ones using HR satellite images and the performance of pattern recognition in computer vision can be improved if an HR image is provided. Since the 1970's, charge coupled device (CCD) and CMOS sensors have been widely used to capture digital images. Although these sensors are suitable for most imaging applications, the current resolution level and consumer price will not satisfy the future demand. For example, people want an inexpensive HR digital camera or camcorder or see the price gradually reduce, and scientists often need a very HR level close to that of an analog 35 mm film that has no visible artifacts when an image is magnified. Thus, finding a way to increase the current resolution level is needed.

Super resolution (SR) image may be defined as a constructed HR image by combining group of lower resolution (LR) images that are sub pixel altered from one another. The super resolution image reconstruction can be classified based on the number of images used. If a single image is used it is called single-image super resolution. If more than one image is used it is called super resolution from multiple-images. The single-image super resolution method requires huge quantity of training data for creating effective learning models. Super resolution from multiple-images has a problem in processing matrices of large dimension[1-5]. To incorporate information from original images and sub-pixel shifted image, subjected to image registration and make them into a unified image, fusion is used. Since image fusion is capable of retaining the useful features it can be used in super resolution image reconstruction. Wavelet transform is used for fusion and the images are registered using affine transform[6]. The image is decomposed using the wavelet transform and the wavelet co-efficient is further used for fusion to increase the resolution of the image. Super resolution reconstruction is done using framelet transform[7]. The framelet transform is improved form of wavelet transform by overcoming the problems such as symmetry and perfect reconstruction using tight frames. SVD and mutual information based fusion is used for detection of boundaries in a video sequence[8]. Here the SVD is done on the histogram of the frames and the mutual information between the frames is used for evaluating it. SVD-based fusion is used for face recognition problem[9]. Here the face images are decomposed into three slices. The slicing is done based on the energy distribution. SVD-based fusion for super resolution image reconstruction is used[10]. In this method the maximum of the Eigen value is found out and the corresponding matrix of singular values are used for reconstruction of high resolution image.

As the singular values determine the intensity information for the reconstructed image choosing of the singular value need to be done precisely. To achieve this, authors has enhanced the method of choosing the singular values[10]. The

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proposed method of choosing the singular values has improved the quality measures of the image under various parameters like mean square error, peak signal to noise ratio, index sharpness.

2. SINGULAR VALUE DECOMPOSITION

Consider a matrix A of size m x n which has entries from a field K. By applying singular value decomposition (SVD) on the matrix A, it gets decomposed into three matrices namely U,S,V. The decomposition[10,11] is given by Eqn (1).

$$A = USV^T \tag{1}$$

where $U^TU = 1$, U is a m x m unitary orthogonal matrix over K, U^T is the conjugate transpose of U. S is a m x n diagonal matrix with non-negative real numbers on the diagonal which are the square roots of the non-zero eigen values of both A^TA and AA^T.

$V^TV = 1$, V which is n x n unitary orthogonal matrix over K, V^T is the conjugate transpose of V.

The columns of U are called the left singular vectors which are the Eigen vectors of A^TA. The columns of V are called the right singular vectors which are the Eigen vectors of AA^T. The diagonal elements of diagonal matrix $S = diag\,(\alpha_1, \alpha_2, \alpha_n)$ are called the singular values of A. The singular values give the intensity information of image. Higher the singular value greater is the information regarding the image.

3. SVD ALGORITHM FOR SUPER RESOLUTION RECONSTRUCTION

The proposed algorithm consists of ; (i) image registration, (ii) SVD based image fusion, and (iii) interpolation. Figure 1 explains the flow of the process, where A_o and A are two frames taken from a video sequence.

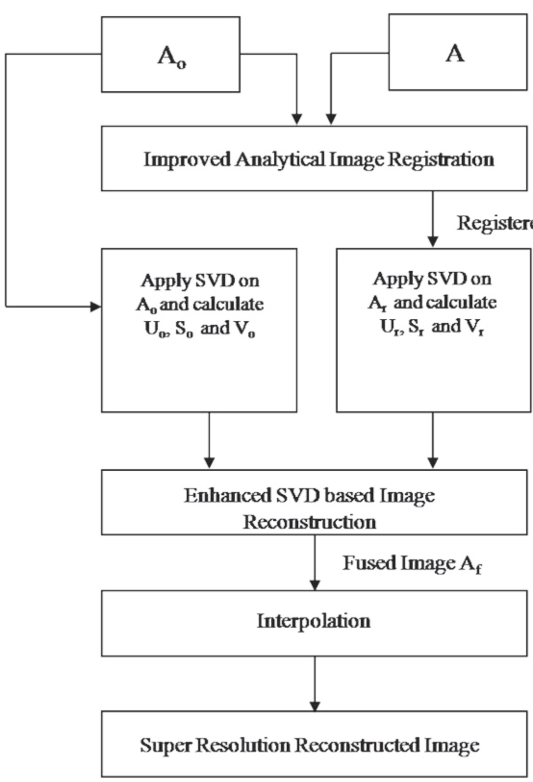


Figure 1. SVD based super resolution image reconstruction flowchart.

3.1 Analytical Method of Image Registration

There are numerous algorithms for image registration. Among these the gradient method[12] suits more for practical applications. But it works best in case the shift is small for LR images. To overcome this we have integrated the iterative nearest neighbor technique proposed[13] with the gradient image registration algorithm proposed[12]. Here we consider the first frame $r_i(x, y)$ as the reference image. Considering this as reference the shifts in the remaining frames are calculated. Assume X denotes the total frames contained in a video, p_i and v_i denotes the shifts in horizontal direction and vertical direction respectively for the i^{th} frame. The i^{th} frame can be expressed as

$$r_i(x, y) = r_i(x + \mathrm{p}_i, y + v_i) \tag{2}$$

where $i \in \{2, 3, 4..., X\}$. By[14] approximating Eqn (2) we get

$$r_i \approx r_1(x, y) + p_i \frac{\partial r_1(x, y)}{\partial x} + v_i \frac{\partial r_1(x, y)}{\partial y} \tag{3}$$

For solving the parameters p_i and v_i we apply the method of least square. For least square solution[15] the error Eqn (4)

$$E_i(p_i, v_i) \approx \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} [r_i(m,n) - r_1(m,n) - p_i \frac{\partial r_1(x, y)}{\partial x} + v_i \frac{\partial r_1(x, y)}{\partial y}]^2 \tag{4}$$

should be minimized, where

m- and n- discrete variables for x and y directions respectively

M- and N - total number of pixels in x and y directions respectively

p_i and v_i - are the translational shifts in x direction and y direction between i^{th} frame and original image.

Way of minising is to differentiate Eqn (4) with respect to p_i and v_i, making the derivative equal to zero. Simultaneously solving the deriving equation, it can be represented as Eqn (5).

$$\begin{pmatrix} \sum_{m=1}^{M} \sum_{n=1}^{N} (\frac{\partial r_1(m,n)}{\partial m})^2 & \sum_{m=1}^{M} \sum_{n=1}^{N} (\frac{\partial r_1(m,n)}{\partial m} \frac{\partial r_1(m,n)}{\partial n}) \\ \sum_{m=1}^{M} \sum_{n=1}^{N} (\frac{\partial r_1(m,n)}{\partial m} \frac{\partial r_1(m,n)}{\partial n}) & \sum_{m=1}^{M} \sum_{n=1}^{N} (\frac{\partial r_1(m,n)}{\partial n})^2 \end{pmatrix} \begin{pmatrix} p_i \\ v_i \end{pmatrix} = \begin{pmatrix} \sum_{m=1}^{M} \sum_{n=1}^{N} (r_i(m,n) - r_1(m,n) \frac{\partial r_1(m,n)}{\partial m} \\ \sum_{m=1}^{M} \sum_{n=1}^{N} (r_i(m,n) - r_1(m,n) \frac{\partial r_1(m,n)}{\partial n} \end{pmatrix} \tag{5}$$

can be written in matrix form as $M \bullet S = V$

where $$V = \begin{pmatrix} \sum_{m=1}^{M} \sum_{n=1}^{N} (r_i(m,n) - r_1(m,n) \frac{\partial r_1(m,n)}{\partial m} \\ \sum_{m=1}^{M} \sum_{n=1}^{N} (r_i(m,n) - r_1(m,n) \frac{\partial r_1(m,n)}{\partial n} \end{pmatrix}$$

The registration parameter can be calculated as $S = M^{-1}V$.

The frame $r_i(m,n)$ is shifted by using these estimated shifts so that it closely matches with $r_1(m,n)$. The procedure is repeated till the registration estimates becomes as small as possible.

3.2 Enhanced Singular Value Decomposition Fusion

The original frame (A_o) and any subsequent frame (A) which is sub-pixel shifted are considered as inputs. The sub-pixel shifted frame is subjected to images registration which gives a registered image (A_r). The original image (A_o) and the registered image (A_r) are subjected to singular value decomposition. Performing SVD on the original image (A_o) decomposes the original image (A_o) as U_o, S_o and V_o. Similarly performing SVD on registered image decomposes registered image (A_r) as U_r, S_r and V_r. The main objective is during the process of fusion, there should not be loss of any important information. To achieve this singular value (α) should be carefully chosen, by comparing them between original and the registered images. The proposed way of choosing the singular values (α) is compare first singular value of original image A_o (α_{o1}) with the first singular value of registered image A_r (α_{r1}). If value of α_{o1} is greater than α_{r1} choose α_{o1} as the first singular value for the new matrix S_{new}. Incase α_{r1} is greater than α_{o1} choose α_{r1} as the first singular value for the new matrix S_{new}. Based on the above statements S_{new} is given by Eqn (6). Figure 2 shows the how element wise comparison is done for singular values contained in S_o and S_r.

Likewise element wise comparison is done for all the singular values contained in S_o and S_r and the highest among the two singular values is chosen for S_{new}.

$$S_{new} = \begin{Bmatrix} \alpha_{oi} \text{ if } \alpha_{oi} > \alpha_{ri} \ , \ i = 1,2,3,\ldots \\ \alpha_{ri} \text{ if } \alpha_{oi} < \alpha_{ri} \ , \ i = 1,2,3,\ldots \end{Bmatrix} \quad (6)$$

The S_{new} is combined with left singular vector U_o and complex conjugate of right singular vector V_o to form the fused image A_f which is given by Eqn (7).

$$A_f = U_o S_{new} V_o^T \quad (7)$$

This is done prior to the interpolation step because interpolation may cause some loss in the information.

4. RESULTS AND DISCUSSIONS

In this section we present simulated results prior to interpolation and after the process of interpolation. The results are tabulated using five different types of interpolations namely nearest neighbor interpolation, bilinear interpolation, bicubic interpolation, box kernel interpolation, Lanczos-3 kernel interpolation. All these results are compared with reported paper on SVD based fusion method[10].

The performance measure for the reconstructed image is done based on three quality metrics, (i) Peak signal to noise ratio (PSNR) (ii) Mean square error (MSE) (iii) Sharpness index measure

(i) Peak Signal to Noise Ratio

$$PSNR = 10\log_{10}\left(\frac{\max^2}{MSE}\right) \quad (8)$$

where max is maximum pixel value of reconstructed image.

(ii) Mean Square Error

$$MSE = \frac{1}{pq}\sum_{i=1}^{p}\sum_{j=1}^{q}(X_{ij} - Y_{ij})^2 \quad (9)$$

where p denotes number of rows, q denotes no of columns, X_{ij} denotes pixel density value of original image and Y_{ij} denotes pixel density value of the reconstructed image.

(iii) Sharpness Index Measure

To calculate the sharpness of the image, a sharpness index $M$is calculated[16].

$$M = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\left\|L(x,y)\right\| \quad (10)$$

where N x N is the size of the image, $L(x,y)$ is the output of the Laplacian filter of pixel (x,y). Higher the value of M greater is the quality of the image.

Table 1 shows the performance measures prior to the interpolation step for original image, reconstructed image based on proposed method. Table 2 shows the PSNR values for original image and super resolution image after the

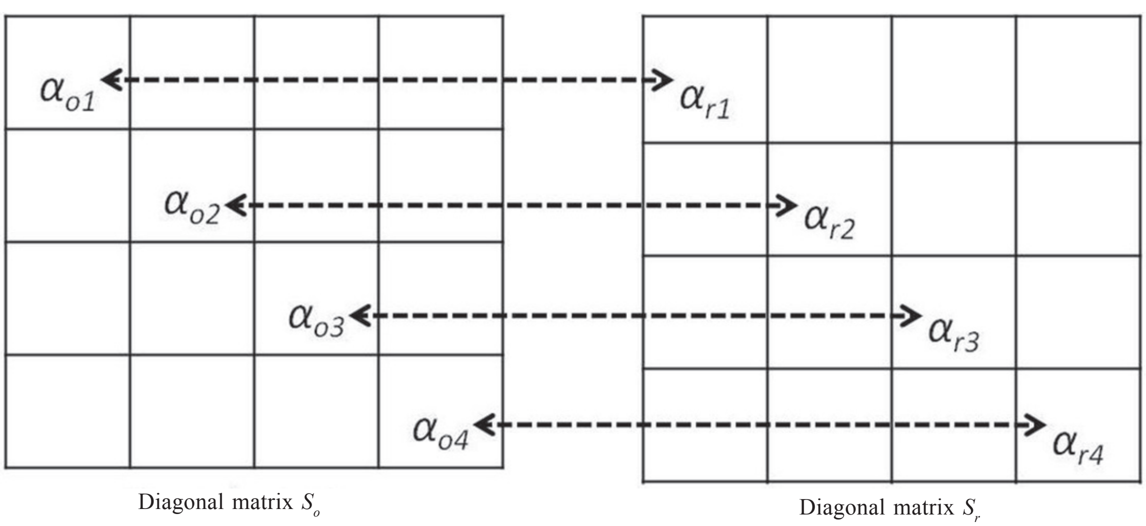


Figure 2. Element wise comparison if singular values in diagonal matrices S_o and S_r.

Table 1. Performance measure prior to interpolation

Quality metrics	Original image	Reconstructed image
PSNR	24.12	54.68
Sharpness index measure	15.6753	15.9786

Table 2. PSNR measure after interpolation based on method[10] and proposed method

Test image / Method	Nearest Neighbor interpolation	Bilinear interpolation	Bicubic interpolation	Box kernel interpolation	Lanczos-3 interpolation
Original image	22.5178	22.4997	22.5137	22.5178	22.5214
SR image / earlier method[10]	52.892	52.6378	52.2447	52.8920	51.8912
SR image / proposed method	53.0783	52.8339	52.4252	53.0783	52.0476

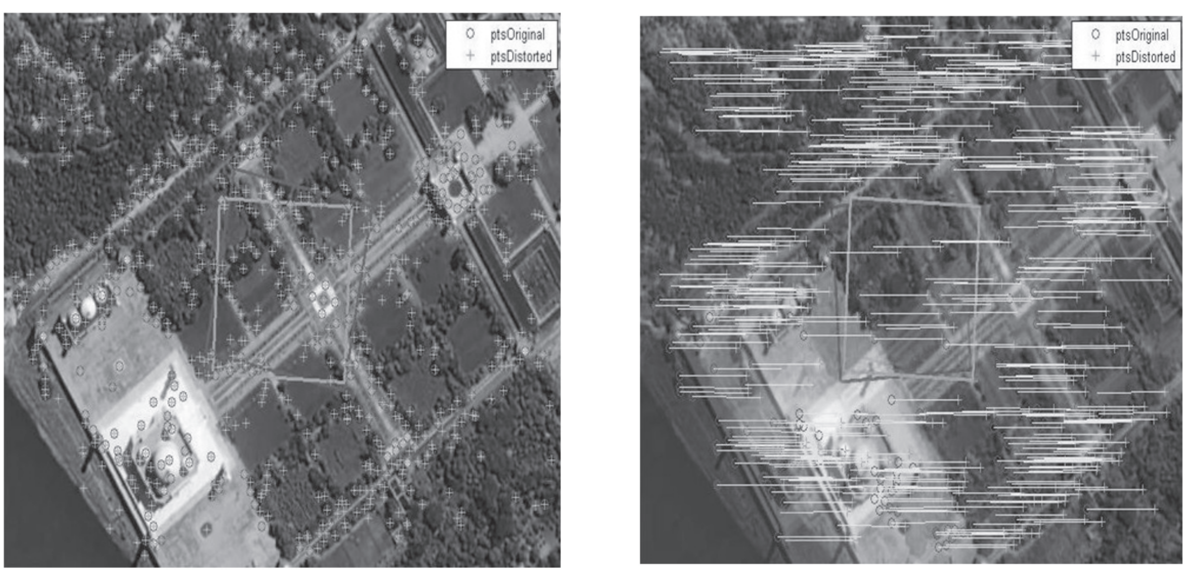
Table 3. Sharpness Index measure after interpolation based on method[10] and proposed method

Test image / Method	Nearest Neighbor interpolation	Bilinear interpolation	Bicubic interpolation	Box kernel interpolation	Lanczos-3 interpolation
Original image	13.7880	11.9834	13.4349	13.7888	14.0960
SR image / earlier method[10]	14.0639	12.1843	13.6891	14.0639	14.3733
SR image / proposed method	14.0640	12.1849	13.6894	14.0640	14.3737

interpolation process based on method[10] and proposed method. Table 3 shows the Sharpness Index Measure values for original image and super resolution image after the interpolation process based on method[10] and proposed method.

The experimental results tabulated in table shows that reconstructed image by proposed method has lower MSE higher PSNR and high sharpness index values compared to original image and earlier reported method[10]. These three results evidently prove that enhanced singular value decomposition based fusion has greatly improved the image quality. Figure 3 is an illustration of the image registration process showing the pixels which are sub pixel shifted video sequence. Figure 4 shows the simulated output prior to interpolation. Figure 5 shows the output based on proposed enhanced singular value decomposition fusion using different interpolation techniques.

The singular values give the intensity information of image. In the proposed method the singular values of original and registered images are compared. The maximum value among these singular values is chosen for the SR image. Higher the singular value greater is the information regarding the image. This makes the proposed method better compared to the existing method.

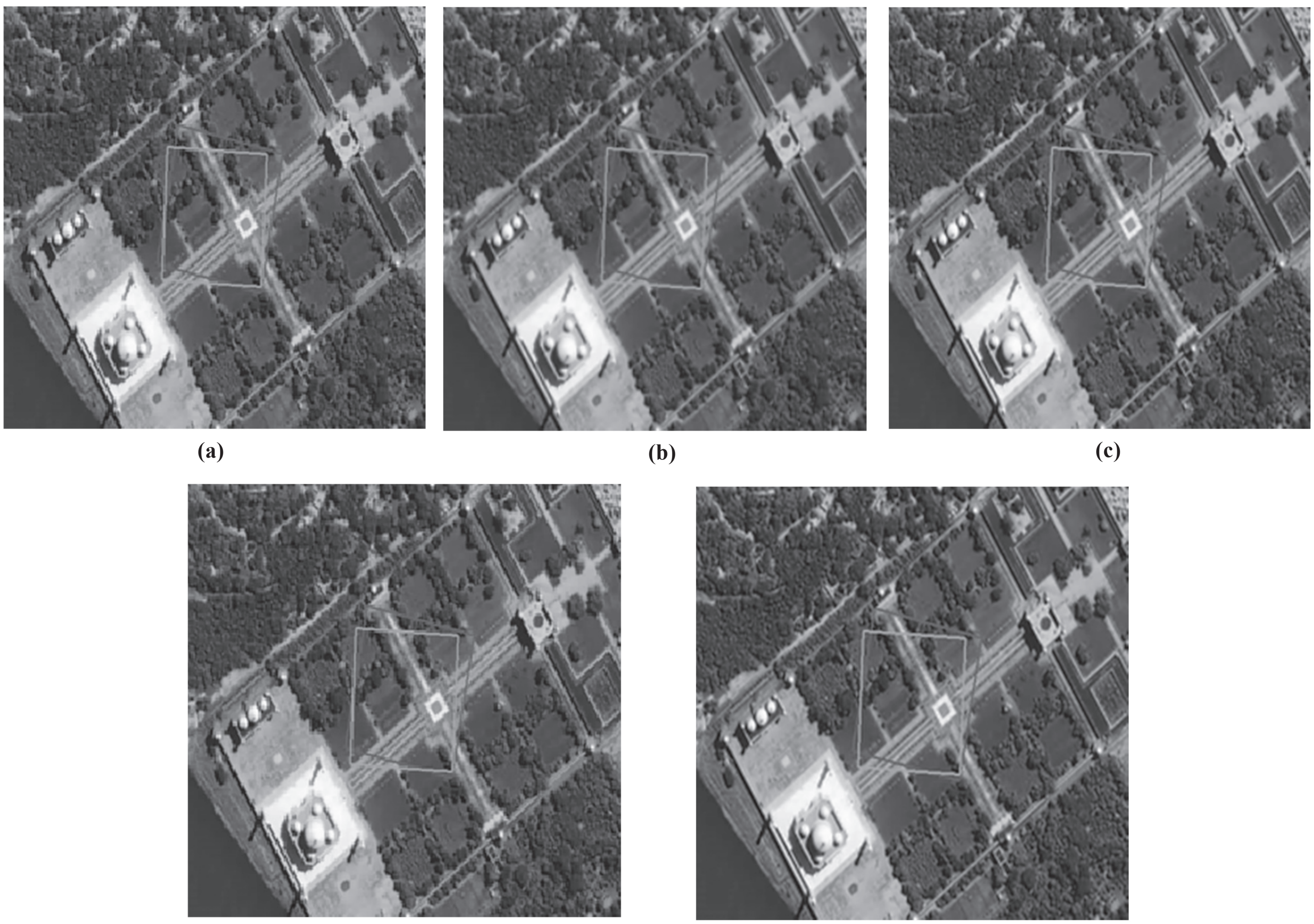


(a) Frame 5 (b) Frame 20

Figure 3. Image registration process for sub-pixel shifted frames with respect to reference frame for road video.

(a) (b) (c)

Figure 4. (a) Original image, (b) SR reconstructed based on10 prior to interpolation, and (c) SR reconstructed based on proposed method prior to interpolation.



(a) (b) (c)

(d) (e)

Figure 5. SR image based on proposed method using (a) nearest neighbour interpolation, (b) bilinear interpolation, (c) bicubic interpolation, (d) box kernel interpolation, and (e) Lanczos-3 interpolation.

5. CONCLUSION

The improvements in the quality of the image have enlarged the use of super resolution image reconstruction into various fields of applications. The analytical image registration uses gradient method to estimate shifted pixels and iterative nearest neighbour method for putting the frame on to a uniform grid. Most of the existing method uses the process of interpolation to increase the resolution, which adds the pixels but does the increase the resolving power. The proposed method chooses singular values precisely from the SVD domain images to reconstructs a high resolution image using fusion technique. The proposed enhanced SVD algorithm is compared with the other methods based on the parameters like mean square error, peak noise to signal ratio and sharpness index. The experimental results shows that enhanced SVD based fusion for super resolution image reconstruction to be highly productive than all current methods giving better edges which is quite evident through the tables and figures shown.

REFERENCE

1. Yang, J.; Wright, J. & Huang, T. Image super-resolution via sparse representation. *IEEE. Trans. Image. Process.,* 2010, **19**(11), 2861-2873. doi: 10.1109/TIP.2010.2050625
2. Glasner, D.; Bagon, S. & Irani, M. Super-resolution from a single image. *In* IEEE 12th International Conference on Computer Vision – 2009, Japan, 2009. doi: 10.1109/ICCV.2009.5459271
3. Adler, A.; Yacov, Helor & Elad, M. A shrinkage learning approach for single image super-resolution with overcomplete representations. *In* Proceedings of ECCV European Conference on Computer Vision 2010: Crete, Greece, 2010.
4. Zeyde, R.; Elad, M. & Protter, M. On single image scale-up using sparse representations. *In* Proceedings of the 7th International Conference on Curves and Surfaces 2010: Avignon, France, 2010. doi: 10.1007/978-3-642-27413-8_47
5. Kim, K.I. & Younghee, Kwon. Example-based learning for single-image super-resolution. *In* Proceedings of the 30th DAGM symposium on Pattern Recognition 2008: Berlin, 2008. doi: 10.1007/978-3-540-69321-5_46
6. Liyakathunisa, L.; Kumar, C.N.R. & Ananthashayana, V.K. Super resolution reconstruction of compressed low resolution images using wavelet lifting schemes. *In* International Conference on Computer and Electrical Engineering, ICCEE 2009, Dubai, 2009. doi: 10.1109/ICCEE.2009.221
7. Joseph, Abraham, Sundar, K.; Vaithiyanathan, V.; Raja, Singh, Thangadurai, G. & Naveen, Namdeo. Design and analysis of fusion algorithm for multi-frame super-resolution image reconstruction using framelet. *Def. Sci. J.,* 2015, **65**(4), 293-299. doi: 10.14429/dsj.65.8265
8. Cernekova, Z.; Kotropoulos, C.; Nikolaidis, N. & Pitas, I. Video shot segmentation using fusion of SVD and mutual information features. *In* Proceedings of IEEE International Symposium on Circuits and Systems, 2005: Kobe, 2005. doi: 10.1109/ISCAS.2005.1465470
9. Wang, J.; Liang, J.; Haihong, Hu; Yan, Li & Bin, Feng. Performance evaluation of infrared and visible image fusion algorithms for face recognition. *In* Proceedings of International Conference on Intelligent System and Knowledge Engineering 2007: Chengdu, China, 2007.
10. Haidawati, Nasir; Vladimir, Stankovic & Stephen, Marshall. Singular value decomposition based fusion for super resolution image reconstruction. *Signal Process. Commun.,* 2011, **27**(6), 180–191. doi: 10.1016/j.image.2011.12.002
11. Izadpanahi, S.; Ozcinar, C.; Anbarjafari, G. & Demirel, H. Resolution enhancement of video sequences by using discrete wavelet transform and illumination compensation. *Turk. J. Elec. Eng. Comp. Sci.,* 2012, **20**(2),1268–1276.
12. Schaum, A. & McHugh, M. Analytic methods of image registration: Displacement estimation and resampling. FOI- Naval Research Laboratory, User Report No. 9298. 1991.
13. Irani, M. & Peleg S. Improving resolution by image registration. *Comput. Vis. Graph. Image Process.,* 1991, **53**(3), 231–239. doi: 10.1016/1049-9652(91)90045-L
14. Hiep, Luong; Bart, Goossens; Aleksandra, Pizurica & Wilfried, Philips. Total least square kernel regression. *J. Visual Commun. Image Repre.,* 2012, **23**(1), 94-99. doi: 10.1016/j.jvcir.2011.09.002
15. Goodman J.W. Introduction to fourier optics. McGraw Hill, USA, 1996.
16. Lee, Wen Li; Yang, Chun Cheng; Wu, Hsien Tsai & Chen, Mei Juan. Wavelet-based interpolation scheme for resolution enhancement of medical images. *J. Sign. Process. Syst.,* 2009, **55**(3), 251-265. doi: 10.1007/s11265-008-0206-6

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