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Damages Identification in the Cantilever-based on the Parameters of the Natural Oscillations

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ABSTRACT

An approach to parametric identification of damages such as cracks in the rod cantilever construction is described. The identification method is based on analysis of shapes of the natural oscillations. The analytic modelling is performed in the Maple software on the base of the Euler-Bernoulli hypothesis. Crack is modelled by an elastic bending element. Transverse oscillations of the rod are considered. We take into account first four eigen modes of the oscillations. Parameters of amplitude, curvature and angle of bends of the waveforms are analysed. It was established that damage location is revealed by 'kink' on corresponding curves of the waveforms. The parameters of oscillation shapes are sensitive to the crack parameters in different degree. The novelty of the approach consists in that the identification procedure is divided into two stages: (a) it is determined the crack location, and (b) it is determined the crack size. Based on analytical modelling, an example of determination of dependence of the crack parameters on its size in the cantilever rod is presented. Study of features of the waveforms during identification of the fracture parameters shows that the features found in the form of 'kinks' and local extreme a of the angle between the tangent and curvature of waveforms for different modes of bending oscillations, define the crack location in cantilever. They can serve as one of diagnostic signs of crack identification and allow us to determine its location.

Keywords: Cantilever, Maple software, flexural vibration waveform, Euler-Bernoulli, modelling, transverse oscillations

1. INTRODUCTION

One of the main tasks of monitoring and evaluation of buildings and structures, as well as their individual components is the identification the parameters of damages and stress state. Solving the problem of identification of damage in rods, beams and more complex structures analysis of these studies is described in reviews[1-9].

Studies investigating problems of parameter identification of damages and their location in the structural elements have been conducted using a variety of analytical and finite element models[10], combined *c* using artificial neural networks[11,12], as well as the base of field experiments using a strain gauge, optical and vibrodiagnostic techniques[13,14].

Different algorithms for solving the problems based on the influence of damages parameters to change the natural oscillation frequencies, modal characteristics of the waveforms, dynamic calculations on compliance and damping matrices of the damaged structures[8,9]. The difference between the latter algorithm is in contrast to the equations for the objective functions and the scheme used for optimisation. In some algorithms for solving the problems of identification of damage based on the change in the shape of various oscillation modes used warranty modal criterion (modal assurance criterion)[9].

Analysis of different approaches to solving the problems of identification of damages shows that the study of the parameters causes significant damage, including computational difficulties[7]. In this connection, it selects the most simple and at the same time a reliable diagnostic sign of identification, based on the analysis of the frequency characteristics[10]. In this method for the synthesis of natural frequencies and mode shapes of the system given by the Eigen functions of subsystems can be used for theoretical justification of monitoring structures in parts by measuring its vibrations in the form of a linear combination of the responses of the system to test the impact[15].

2. RESEARCH OBJECTIVES

Modelling of damage identification parameters of the cantilever based on the analysis of the shape parameter of the transverse oscillations.

As an example the identification of damages in the core design based on analytical modelling[10].

3. SIMULATION OBJECT

Simulation object was cantilever rod (length L = 250 mm, height of the cross section of h = 8 mm, a = 4 mm thick) with a damage (the cross cut width 1 mm, and the absolute depth of

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h_d= 4 mm) arranged in spaced apart from the pinch point of the rod distance $\overline{L}_d$ = 0.25 where $\overline{L}_d = L_d / L$, L_d - location of the damage. Next, authors introduce the dimensionless coordinate $\overline{x} = x / L$. The relative depth of the damage taken as $\overline{t} = h_d / h$. Rod model had a damage with the following values:

$\overline{t}$ = 0; 0.25; 0.5; 0.75; 0.85.

Authors consider the transverse vibrations of the rod.

4. MODELLING

In the analytical modelling, consider the differential equation of forced oscillations in the model of the Euler-Bernoulli:

$$\frac{\partial^2}{\partial x^2}\left[EJ(x)\frac{\partial^2 u_i}{\partial x}\right] - m(x)\frac{\partial^2 u_i}{\partial t^2} + F(t)\delta(x - L_F) + p(x,t) = 0 \quad (1)$$

where $u_i(x,t), i = 1,2,3$ is the displacement of the beam axis of the points where the subscript indicates the number of area of the beam, as shown in Fig. 1. E is modulus of elasticity; $J(x)$ is the moment of inertia; $m(x)$ is linear density; $F(t)\delta(x - L_F)$ is the force applied at the point L_F ; $p(x,t)$ is distributed load.

Boundary conditions for the composite rod construction are of the form:

$$at\ x = 0: \quad u_1(0) = u_1'(0) = 0;$$

$$\begin{aligned} at\ x = L_c:\ & u_1(L_c) = u_2(L_c); \\ & u_1''(L_c) = u_2''(L_c); \\ & u_1'''(L_c) = u_2'''(L_c); \\ & -EJ\cdot u_1''(L_c) = K_t[u_1'(L_c) - u_2'(L_c)]; \end{aligned} \quad (2)$$

$$\begin{aligned} at\ x = L_F:\ & u_2(L_F) = u_3(L_F); \\ & u_2'(L_F) = u_3'(L_F); \quad u_2''(L_F) = u_3''(L_F); \\ & u_2'''(L_F) - u_3'''(L_F) = F_0 / EJ; \end{aligned}$$

$$at\ x = L: \quad u_3''(L) = u_3'''(L) = 0;$$

where K_t is stiffness of the elastic element.

To determine the relationship between the magnitude of damage to full-bodied *FE* model and the stiffness of the elastic element analytical model was obtained by the relation[10]:

$$\overline{t} = 1.186 - 0.135 K_t^{0.23} \quad (3)$$

Counting results by applying the Eqn. (1), the value of flexural rigidity of the elastic element to the analytical model as shown in Table 1. To determine the natural modes at different values of stiffness of the elastic element and its location $\overline{L}_{\tilde{n}}$ = 0.25 based on analytical modelling of the package have been computed natural frequencies (Table 1).

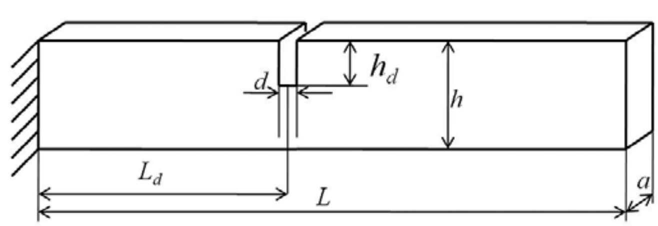

Figure 1. Scheme of the cantilever with the location of the damage.

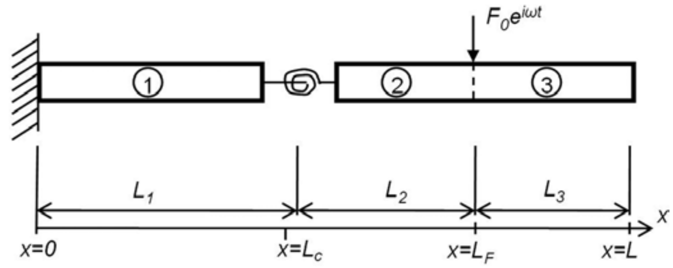

Figure 2. Model of a cantilever beam with an elastic element.

Table 1. Natural frequencies of the cantilever at different values of the stiffness of the elastic element and its location $\overline{L}_{\tilde{n}}$ = 0.25

Stiffness K_t	Natural frequencies ω_i (Hz)			
	ω_1	ω_2	ω_3	ω_4
12684	107.8	676	1892	3709
4531	105.1	675.9	1856	3633
1173	98.1	674	1772	3486
163	67,9	667	1533	3216
52	44,9	664	1440	3147

Solved the problem of the natural vibrations of the cantilever. Get the shape of the cantilever when different stiffness of the damage. Figure 3 shows the normalised eigen modes of the rod at different values of the damage

Analysis charts waveforms shows that the damage in the place has a characteristic pronounced ‘kink’ in varying degrees depending on the size of the damage. As the exponential characteristic of a damage in a cantilever beam can be used at an angle α formed by the tangent to the curve shape of the oscillations.

The parameter of the angle α between the tangents at the points of waveform can be calculated using the discrete approach associated with the process of measuring the amplitudes of the oscillations in a finite number of points. Figure 4 shows a diagram of the location of points on the curve section waveforms. In the Fig. 4, the symbols marked: U_i - displacement of natural modes in the i -th point of the rod; $\tilde{o}_i$ - coordinate of the point with the number i , $i \in 1..N$, N - the total number of points.

In the presence of discrete information about the mode shape, the angle α_i at the point x_i corresponding to the vibration modes can be calculated as follows:

$$\alpha_i = \arccos\left(\frac{(\overline{AB})(\overline{BC})}{\left\|\overline{AB}\right\|\left\|\overline{BC}\right\|}\right) \quad (4)$$

where $\overline{AB}$ and $\overline{BC}$ - the vector representation of the two segments, respectively, between the points of the normalised waveform numbers $[i-1, i]$ and $[i, i+1]$.

The curvature of the waveform can be considered as an additional feature that allows to specify the parameters of the damage.

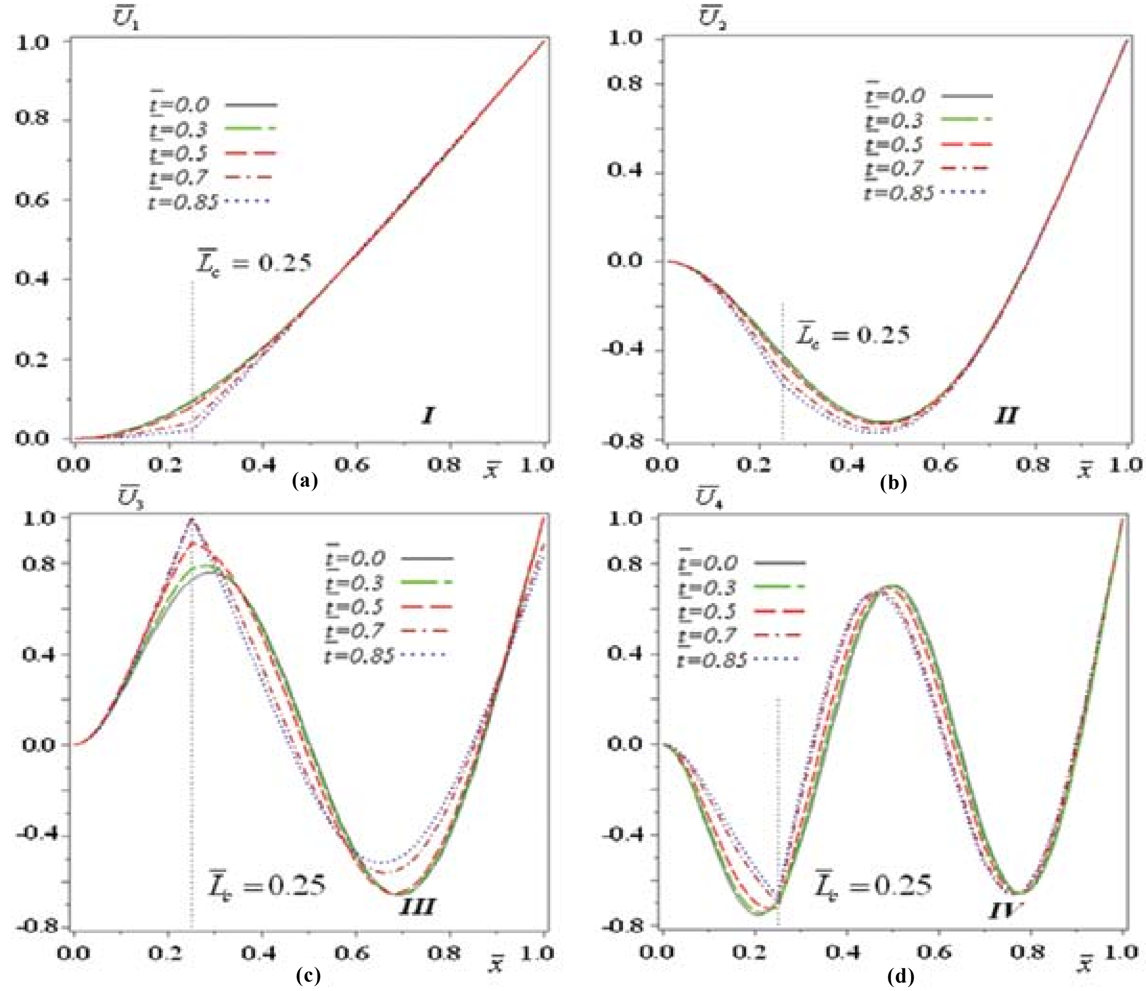

Figure 3. The normalised values of the transverse displacement of the cantilever with the damage of various size for the first four transverse modes of vibration. Accordingly, the oscillation modes: (a) 1, (b) 2, (c) 3, and (d) 4.

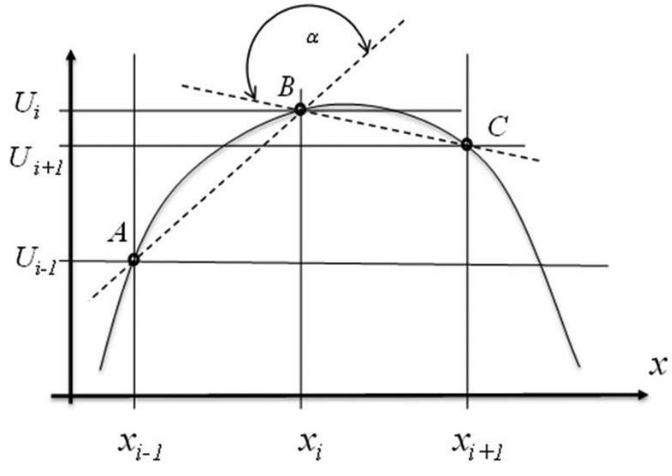

Figure 4. Arrangement of points in the calculation of the parameter of the angle α between the tangents.

When using discrete measurements, including small amplitude oscillations, the curvature U_i'' at the point i of the beam can be calculated as follows:

$$U_i'' = \frac{U_{i-1} - 2U_i + U_{i+1}}{\Delta x^2} \tag{5}$$

where Δx is the distance between the points of measurement.

Upon receipt and processing of the data collection necessary for the organisation of the oscillation amplitudes of the vectors waveforms at various frequencies. This requires a normalisation of the data on the amplitudes to the interval [0,1]. Each point of the waveform amplitude readings normalised to the maximum value of deflection:

$$\bar{U}_i = \frac{U_i}{|U_{\max}|} \tag{6}$$

where $\bar{U}_i$ is the normalised magnitude of the displacement in the i point of the rod; $U_{\max}$ is the value of the maximum deviation of points waveforms.

The curvature at the point i when the normalised amplitude of the waveform will be calculated as:

$$\bar{U}_i'' = \frac{\bar{U}_{i-1} - 2\bar{U}_i + \bar{U}_{i+1}}{\Delta x^2} \tag{7}$$

The values of curvature at different points of the Eqn. (5) and the angles between the tangents at various points along the length of the cantilever by the Eqn. (4). The magnitude of the discrete segment Δx assumed $1/60$ to be equal to the length of the cantilever. Graphs of curvature at different points of waveforms shown in Fig. 5, the angles between the tangents in the various points of the curve waveform shown in Fig. 6.

Analysis parameters waveforms shows that the injury site has a characteristic pronounced 'kink' in varying degrees depending on the size of the damage. In the analysis of the graphs of curvature and angle between the tangents at the location of the damage has outliers 'kink'.

To assess the influence on cut depth variation of the amplitude, waveform curvature and the angle in the 'kink' zone, were considered the relative values of these parameters from different rigidity values of the elastic element at its location.

For this case the location of the elastic element relative magnitude of the changes of the normalised amplitude is calculated as follows:

$$\Delta\bar{U}_i = \frac{\left|\bar{U}_i^d - \bar{U}_i^0\right|}{\bar{U}_i^0}100\% \qquad (8)$$

where $\bar{U}_i^d$ and $\bar{U}_i^0$ are the normalised amplitude of the curve shape oscillations of an elastic rod in the i-*th* point along the length of the rod when it is damaged and intact, respectively.

Values of the transverse displacement for various oscillation modes and their relative values at a location of damage are as shown in Table 2.

For comparison, the curvature of the waveforms in this cases the location of the elastic element addressed by:

$$\bar{U}_i'' = \left|\bar{U}_{id}''\right| \qquad (9)$$

where $\bar{U}_{id}''$ is the curvatures waveforms in i^{th} point along the length of the rod.

The curvatures $\bar{U}''$ waveform at a location of the damage are presented in Table 3.

The relative magnitude of changes in the angle α between the tangent at the point waveform in this case the location of the elastic element is calculated as follows:

$$\Delta\alpha_i = \frac{\alpha_i^d - \alpha_i^0}{\alpha_i^0}100\% \qquad (10)$$

where α_i^d and α_i^0 are the corners in i th point waveforms rod in the presence of the damage and intact, respectively.

Angles 'kink' waveforms and their relative quantities for the points to the location of the damage are presented in Table 4.

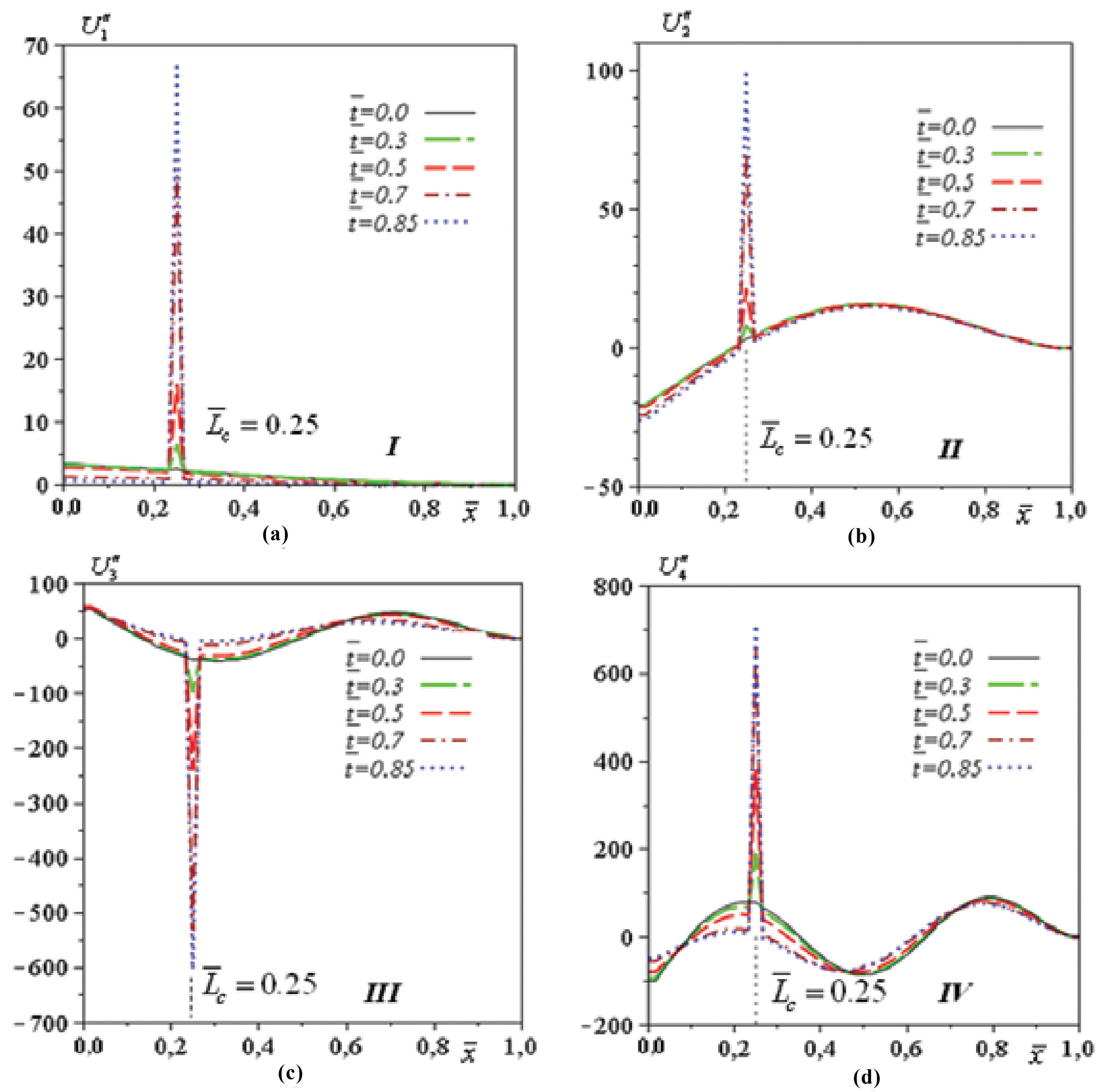

Figure 5. The curvature of the graph of the first four forms of the transverse oscillations of the cantilever with the damage of various size. Accordingly, the oscillation modes: (a) 1, (b) 2, (c) 3, and (d) 4.

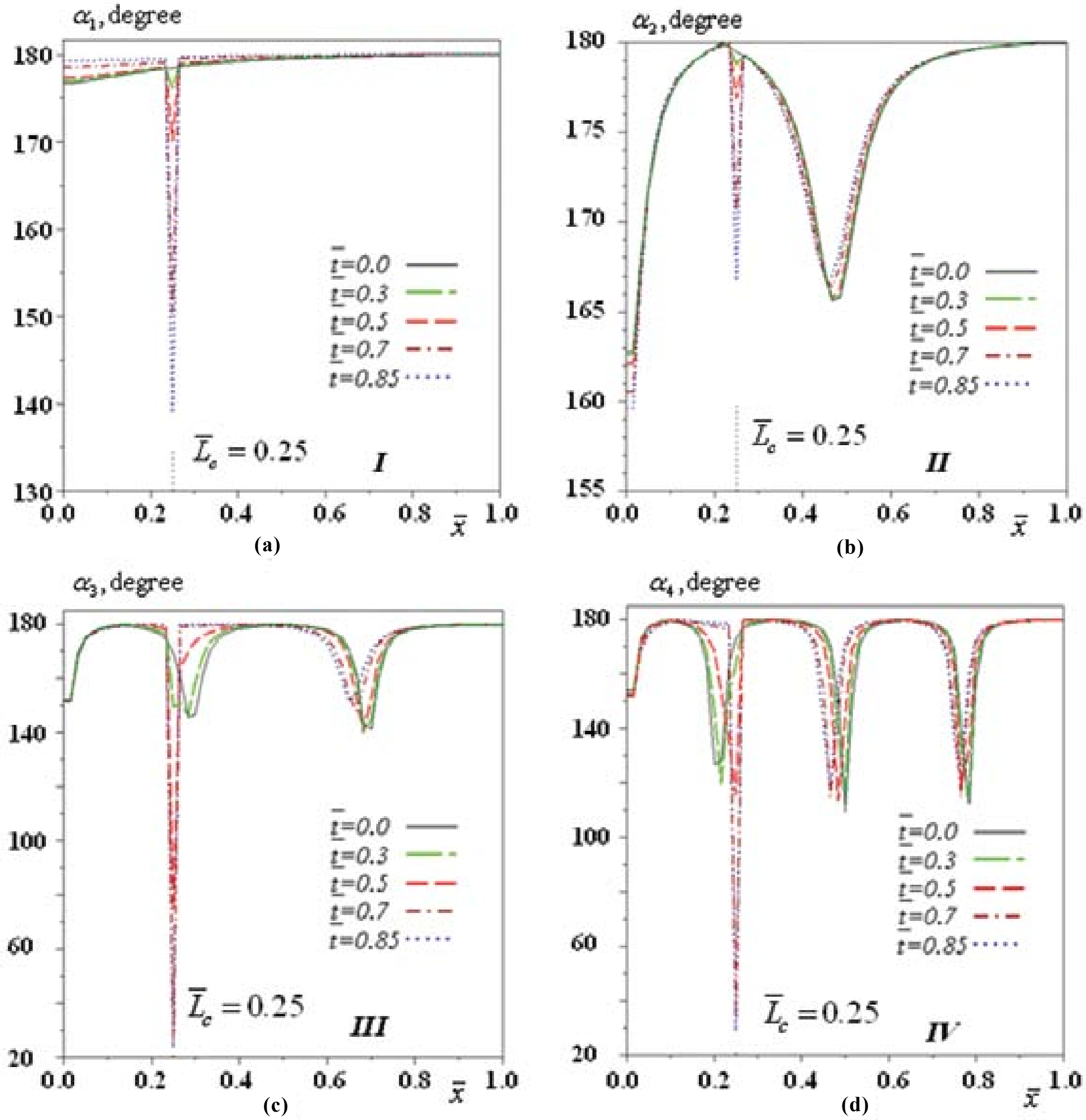

Figure 6. The angles between the tangents to the waveforms at various points of the cantilever with the damage of various sizes. Accordingly, the oscillation modes: (a) 1, (b) 2, (c) 3, and (d) 4.

Table 2. The value of the transverse displacements $\bar{U}$ and their relative values $\Delta\bar{U}$ at a point to the location of the damage for different oscillation modes, depending on the stiffness of the cantilever

K_t	$\bar{t}$	1 mode		2 mode		3 mode		4 mode	
		$\bar{U}$	$\Delta\bar{U}$,%	$\bar{U}$	$\Delta\bar{U}$,%	$\bar{U}$	$\Delta\bar{U}$, %	$\bar{U}$	$\Delta\bar{U}$,%
1	2	3	4	5	6	7	8	9	10
12684	0	0.097	0.0	-0.417	0.0	0.727	0.0	-0.686	0.00
4531	0.25	0.093	4.1	-0.424	1.7	0.774	6.5	-0.703	2.48
1173	0.50	0.081	16.5	-0.444	6.5	0.889	22.3	-0.723	5.39
163	0.75	0.043	55.6	-0.510	22.3	1	37.6	-0.695	1.31
52	0.85	0.022	77.3	-0.551	32.1	1	37.6	-0.670	2.33

5. ANALYSIS

Analysis of the graphs of waveforms, angles at points between the tangent and curvature shows that the location of the elastic element $\bar{L}_{\tilde{n}} = 0.25$ relative variation of displacement compared with intact waveforms model is when the value of damage $\bar{t} = 0.75$ for the 1st oscillation modes $\Delta\bar{U} = 55.6$ per cent, for the 2nd oscillation modes $\Delta\bar{U} = 22.3$ per cent, for the 3rd oscillation modes $\Delta\bar{U} = 37.6$ per cent, for the 4th oscillation modes $\Delta\bar{U} = 1.31$ per cent. For the value of the angle α between the tangent at points along the length of the rod to the schedule

Table 3. Curvatures U'' waveform at a damage location to various modes in dependence on the stiffness K_t of the cantilever

K_t	$\bar{t}$	1 mode	2 mode	3 mode	4 mode
		$\bar{U}''$	$\bar{U}''$	$\bar{U}''$	$\bar{U}''$
1	2	3	4	5	6
12684	0	2.5	3.2	38.8	80.8
4531	0.25	6.4	8.2	97.7	187
1173	0.50	15.9	21.1	237	381
163	0.75	49.3	70.3	529	660
52	0.85	66.6	98.8	602	713

of waveforms changes the corresponding coefficients at the same location of the elastic element are: for the 1st oscillation modes $\Delta\alpha$ = 15.7 per cent for the 2nd oscillation modes $\Delta\alpha$ = 5 per cent, for the 3rd oscillation modes $\Delta\alpha$ = 83.9 per cent, for the 4th oscillation modes $\Delta\alpha$ = 83.5 per cent. For values of curvature waveforms location $\bar{L}_{\tilde{n}}$ = 0.25 and the size of damage $\bar{t}$ = 0.75 for different modes of oscillation: for the 1st oscillation modes $\bar{U}''$ = 49.3, for the 2nd oscillation modes $\bar{U}''$ = 70.3, for the 3rd oscillation modes $\bar{U}''$ = 529, 4th oscillation modes $\bar{U}''$ = 660.

The dependency analysis of the relative magnitude of the transverse displacement and the magnitude of damage at the point of location for different modes of vibrations shows that this dependence is well manifested in this case the location of the damage for the 1st, 2nd, and 3rd modes and less sensitive at the 4th oscillation modes.

Analysis of the relative magnitude of the curvature mode shapes at the point of location of the damage from its size for different modes of oscillation shows that all the graphics are monotonically increasing. This dependence in this case the location of the damage is well shown for the 1st and 2nd modes, 3rd and 4th modes slightly less sensitive to the magnitude of the damage.

Analysis of the dependences of the relative change value of angle 'kink' mode shapes between the tangent at the point of location of the damage from its size for different modes of oscillation shows that all the graphics are monotonically decreasing in varying degrees. In this case the location of the damage, this dependence is well manifested in its dimensions $\bar{t} > 0.3$ to the 3rd and 4th modes of oscillation, and is not pronounced for the 1-st and 2-nd.

Table 4. The angles α between the tangent to the graph of the waveforms at the point of location of the damage and their relative values $\Delta\alpha$ depending on the stiffness K_t for different modes of vibrations

K_t	$\bar{t}$	1 mode		2 mode		3 mode		4 mode	
		α	$\Delta\alpha$,%	α	$\Delta\alpha$,%	α	$\Delta\alpha$,%	α	$\Delta\alpha$,%
1	2	3	4	5	6	7	8	9	10
12684	0	178.4	0.0	179.5	0.0	169.1	0.0	173.3	0.0
4531	0.25	176.1	-1.3	178.8	-0.4	149.3	-11.7	163.3	-5.8
1173	0.50	170.1	-4.7	176.9	-1.4	74.5	-55.9	115.8	-33.2
163	0.75	150.4	-15.7	170.6	-5.0	27.2	-83.9	34.2	-80.3
52	0.85	138.7	-22.3	166.7	-7.1	23.3	-86.2	28.6	-83.5

Analysis of the characteristics of mode shapes of a cantilever with damage shows the following:

- Graphics mode shapes have a point 'kinks' in the location of the damage. If the location of the damage (elastic element) is localised in the zone of inflection of the curve of the waveform or in its surroundings, the 'kinks' curve mode shapes are poorly identified, and thus this fashion will be weakly sensitive to the identification of the location of the damage in design.
- Parameters of the angle α at points between the tangent and curvature $\bar{U}''$ mode shapes are more sensitive compared with the amplitude $\bar{U}$ of the oscillations to identify the location of the damage.

6. CONCLUSION

(i). Studies of the characteristics of the waveforms in the process to identify the parameters of the damages show that the features found in the form of 'kinks' and local extrema of the angle α between the tangent and curvature $\bar{U}''$ on the forms of the different modes of Flexural vibrations, which coincides with the coordinate location of the damage in the cantilever, can serve as one of the diagnostic characteristics identification of the damage and identify its location.

(ii). The angle α between the tangent and curvature $\bar{U}''$ mode shapes of the first 4 modes in the point damage location can serve as a diagnostic sign of identification of its size.

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