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Unsteady MHD Flow of Elastico-Viscous Incompressible Fluid through a Porous Medium between Two Parallel Plates under the Influence of a Magnetic Field

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ABSTRACT

An unsteady flow of elastico-viscous incompressible and electrically conducting fluid through a porous medium between two parallel plates under the influence of transverse magnetic field is examined. Initially, the flow is generated by a constant pressure gradient parallel to the bounding fluids. After attaining the steady state, the pressure gradient is suddenly withdrawn and the resulting fluid motion between the parallel plates under the influence of magnetic field is then to be investigated. The problem is solved in two stages: the first stage is a steady motion between the parallel plates under the influence of a constant pressure gradient and the magnetic parameter. The momentum equation of steady state does not involve the elastic-viscosity parameter; however, the influence Darcian friction would appear in it. The solution of the momentum equation at this stage will be the initial condition for the subsequent flow. The second stage concerns with an unsteady motion for which the initial value for the velocity will be that obtained in stage one together with the no-slip condition on the boundary plates. The problem was solved employing Laplace transformation technique. It was found that the effect of the applied transverse magnetic field has significant contribution on the velocity profiles.

Keywords: Elastico-viscous fluid, second order fluid, elastico-viscous parameter, porous media, magnetic parameter

NOMENCLATURE

ϕ_1	Coefficient of viscosity
$g_a(s)$	Retarded history
A_i	Acceleration component in the i^{th} coordinate
L	Characteristic length
ϕ_3	Coefficient of cross-viscosity
ϕ_2	Coefficient of elastico-viscosity
ρ	Density of the fluid
a_i	Dimensionless acceleration component in the i^{th} direction
υ_c	Dimensionless cross viscosity parameter
L_1	Dimensionless elastico-viscosity parameter
F	Dimensionless external force applied
m	Dimensionless magnetic parameter
p	Dimensionless indeterminate hydrostatic pressure
L_2	Dimensionless porosity factor
u_i	Dimensionless velocity component along the i^{th} coordinate
$g(s)$	Given history
M	Magnetic parameter
P	Indeterminate hydrostatic pressure
α	Retardation factor
S	Stress tensor
T	Time parameter
U_i	Velocity component in the i^{th} direction

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1. INTRODUCTION

Flow through porous medium has been the subject of considerable research activity in recent years because of its several important applications, notably in the flow of oil through porous rocks, the extraction of geothermal energy from the deep interior of the earth to the shallow layers, the evaluation of the capability of heat removal from particulate nuclear fuel debris that may result from a hypothetical accident in a nuclear reactor, the filtration of solids from liquids, flow of liquids through ion-exchange beds, drug permeation through human skin, chemical reactor for economical separation or purification of mixtures, and so on.

In many chemical processing industries, slurry adheres to the reactor vessels and gets consolidated. As a result of this, the chemical compounds within the reactor vessel percolates through the boundaries causing loss of production and the consuming more reaction time. In view of such technological and industrial importance wherein the heat and mass transfer takes place in the chemical industry, the problem by considering the permeability of the bounding surfaces in the reactors attracted the attention of several investigators. An important application is in the petroleum industry, where crude oil is tapped from natural underground reservoirs in which oil is entrapped. Since the flow behaviour of fluids in petroleum reservoir rock depends, to a large extent, on the properties of the rock, techniques that yield new or additional information on the characteristics of the rock would enhance the performance of the petroleum reservoirs. A related bio-mechanical application is the flow of fluids in the lungs, blood vessels, arteries, and so on, where the fluid is bounded by two layers which are held together by a set of fairly regularly spaced tissues.

Viscous fluid flow over wavy wall has attracted the attention of relatively few researchers although the analysis of such flows finds applications in different areas, such as transpiration cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vapurisation in combustion chambers, etc. Especially, where the heat and mass transfer takes place in the chemical processing industry, the problem by considering the permeability of the bounding surface in the reactors assumes greater significance. Many materials such as drilling muds, clay coatings and other suspensions, certain oils and greases, polymer melts, elastomers and many emulsions have been treated as non-Newtonian fluids. Because of the difficulty to suggest a single model, which exhibits all properties of non-Newtonian fluids, these cannot be described simply as Newtonian fluids and there has been much confusion over the classification of non-Newtonian fluids. However, non-Newtonian fluids may be classified as

(i) fluids for which the shear stress depends only on the rate of shear;

(ii) fluids for which the relation between shear stress and shear rate depends on time;

(iii) the visco-elastic fluids, which possess both elastic and viscous properties.

Because of the great diversity in the physical structure of non-Newtonian fluids, it is not possible to recommend a single constitutive equation as the equation for use in the cases described in (i) – (iii). For this reason, many non-Newtonian models or constitutive equations have been proposed and most of these are empirical or semi-empirical. For more general three-dimensional representation, the method of continuum mechanics is needed[1]. Although many constitutive equations have been suggested, many questions are still unsolved. Some of the continuum models do not give satisfactory results in accordance with available experimental data. For this reason, in many practical applications, empirical or semi-empirical equations have been used.

It has been shown that for many types of problems in which the flow is slow enough in the visco-elastic sense, the results given by Oldroyd's constitutive equations will be substantially equal to those of the second-or third-order Rivilin- Ericksen constitutive equations[2]. Thus, if this is the sense in which the solutions to which problems are to be interpreted, it would seem reasonable to use the second-or third-order constitutive equations in carrying out the calculations. This is particularly in view of the fact that, the calculation will generally be still simpler. For this reason, in this paper, the second-order fluid model is used. The constitutive equation for the fluids of second-grade (or second-order fluids) is a linear relationship between the stress, the first Rivlin - Ericksen tensor, its square and the second Rivlin-Ericksen tensor. The constitutive equation has three coefficients. There are some restrictions on these coefficients due to the Clausius – Duhem inequality and the assumption that the Helmholtz free energy is minimum in equilibrium. A comprehensive discussion on the restrictions for these coefficients has been given by Dunn[3,4], *et al.* One of these coefficients represents the viscosity coefficient in a way similar to that of a Newtonian fluid in the absence of the other two coefficients. The restrictions on these two coefficients have not been confirmed by experiments and the sign of these material moduli is the subject of much controversy[5]. In general the equation of the motion of incompressible second-grade fluids, is of higher order than the Navier-Stokes equation. The Navier-Stokes equation is second-order partial differential equation, but the equation of motion of a second-order fluid is a third-order partial differential equation. A marked difference between the case of the Navier-Stokes theory and that of fluids of second-grade is that ignoring the nonlinearity in the Navier- Stokes equation does not lower the order of the equation; however, ignoring the higher order nonlinearities in the case of the second-grade fluid, reduces the equation. Exact solutions are very important for many reasons. These provide a standard for checking the accuracies of many approximate methods such as numerical and empirical. Although computer techniques make the complete numerical integration of the nonlinear equations feasible, the accuracy of the results can be established by a comparison with an exact solution. Many attempts to collect the exact solution of the nonlinear equations for unsteady flow of second-grade fluid have been by different researcher for different geometries.

Several studies of industrial and technological importance[6], to find the solution for the problem of the exact solutions of two dimensional flows of a second-order incompressible fluid by considering the rigid boundaries were taken up. Later, a linear analysis of the compressible boundary layer flow over a wall was presented by Lekoudis[7], *et al*. Shankar[8], *et al*. studied the problem of Rayleigh for wavy wall while Lessen[9], *et al*. examined the effect of small amplitude wall waviness upon the stability of the laminar boundary layer. Further, the problem of free convective heat transfer in a viscous incompressible fluid confined between vertical wavy wall and a particle flat wall was examined by Vajravelu[10] and Das[11], *et al*. Later Patidar[12], *et al*. studied the free convective flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls. Taneja[13], *et al*. had examined the problem of MHD flow with slip effects and temperature-dependent heat source in a viscous incompressible fluid confined between a long vertical wall and a parallel flat plate. Murthy[14], *at el*, examined the problem of elastico-viscous fluid of second-order type where the bounding surface is porous and subjected to sinusoidal disturbances. Kulkarni[15-16], *et al*. studied the unsteady poiseuille flow of second-order fluid in a tube of elliptical and spherical cross-section on the porous boundary. Kulkarni[17], *et al*. examined unsteady flow of an incompressible viscous electrically conducting fluid in tube of elliptical cross-section under the influence of magnetic field.

In all the above investigations, the fluid under consideration was viscous incompressible fluid and one of the bounding surfaces has a wavy character or bounding surface subjected to sinusoidal disturbances or circular or elliptical cross-section on the porous boundary. In all of the above situations, not much attention has been paid to the study of unsteady MHD flow of elastico-viscos incompressible fluid through a porous medium between two parallel plates under the influence of magnetic field. Therefore, an attempt was made to study the effects of the transverse magnetic field on the flow of incompressible viscous electrically conducting fluid of second-order type between two parallel plates is considered under constant pressure gradient on the porous boundary. Hence, this aspect is also studied and during the course of discussion both non-magnetic and magnetic cases have been compared. The results are expressed in terms of a non-dimensional porosity parameter, which depends on the non-Newtonian coefficient.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

This paper deals with an investigation of an unsteady flow of a second-order visco-elastic fluid between two impermeable horizontal parallel plates under the influence of magnetic field, separated by a distance 2 h. The flow is governed by a generalised momentum equation which takes care of shear stress generated in the flow medium and also the inertial convective acceleration, apart from conventional Darcy's resistive force. Initially the flow is under a constant pressure gradient down the bounding plates. After attaining the steady state, the pressure gradient is suddenly withdrawn and the subsequent fluid motion is investigated by employing Laplace transform technique.

The problem is solved in two stages; the first stage is a steady motion between the parallel plates under the influence of a constant pressure gradient and the magnetic parameter. The momentum equation is free of the visco-elastic parameter while the Darcian friction would find its place in it. The solution of the momentum at this stage will be the initial condition for subsequent flows. The second stage is an unsteady motion for which the initial velocity is taken the same as that obtained in earlier stage together with the no slip condition on the boundary plates.

Cartesian coordinates (*X*,*Y*,*Z*) with the *X* -axis parallel to the plates placed midway between these. The fluid velocity can be taken as (*U*(*Y*,*T*)0,0).

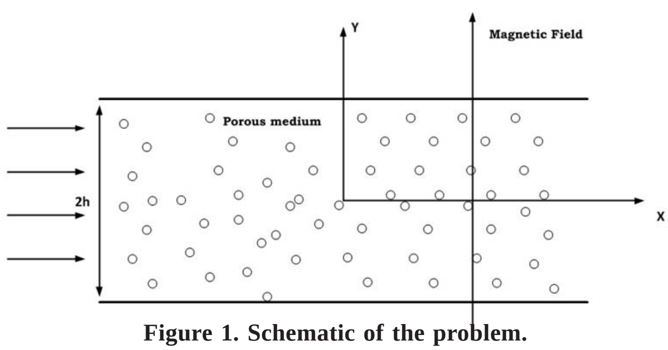

Figure 1. Schematic of the problem.

In the sense of Noll[18] a simple material is a substance for which stress can be determined with entire knowledge of the history of the strain. This is called simple fluid, if it has property that at all local states, with the same mass density, are intrinsically equal in response, with all observable differences in response being due to definite differences in the history. For any given history $g(s)$, a retarded history $g_\alpha(s)$ can be defined as:

$$g_\alpha(s) = g(\alpha s)\ ;\ 0 < s < \infty\ ,\ 0 < \alpha \le 1 \tag{1}$$

where α being termed as a retardation factor. Assuming that the stress is more sensitive to recent deformation than to the deformations at distant past, it has been established by Coleman[19], *et al*. that the theory of simple fluids yields the theory of perfect fluids as $\alpha \to 0$ and that of Newtonian fluids as a correction (up to the order of α) to the theory of the perfect fluids. Neglecting all the terms of the order of higher than 2 in α, we have incompressible elastic-viscous fluid of second-order type whose constitutive relation is governed by:

$$S = -PI + \varphi_1 E_{ij}^{(1)} + \varphi_2 E_{ij}^{(2)} + \varphi_3 E_{ij}^{(1)^2} \tag{2}$$

where

$$E_{ij}^{(1)} = U_{i,j} + U_{j,i} \tag{3}$$

and

$$E_{ij}^{(2)} = A_{i,j} + A_{j,i} + 2U_{m,i}U_{m,j} \tag{4}$$

The constitutive relation for general Rivlin[20], *et al*. fluid also reduces to Eqn (2) when the squares and higher orders of $E^{(2)}$ are neglected, while the coefficients being constant. Also the non-Newtonian models considered by Reiner[21] could be obtained from Eqn (2) when $\phi_2 = 0$ and naming ϕ_3 as the coefficient of cross viscosity. With reference to the[20] fluids, ϕ_2 be called as the coefficient of elastico- viscosity.

The Clausius - Duhem inequality and the assumption that the Helmholtz free energy is minimum in equilibrium provides the following restriction[3].

$$\phi_1 \ge 0\ ,\ \phi_2 \ge 0\ ,\ \phi_1 + \phi_2 = 0$$

The condition $\phi_1 + \phi_2 = 0$ is consequence of the Clausius-Duhem inequality and the condition $\phi_2 \ge 0$ follows the requirement that the Helmholtz free energy is minimum in equilibrium. A comprehensive discussion on the restrictions for ϕ_1, ϕ_2 and ϕ_3 can be found in the work by Dunn[4], *et al*. The sign of the material moduli ϕ_1, ϕ_2 is the subject of much controversy[5]. In the experiments on several non-Newtonian fluids, the experimentalists have not confirmed these restrictions ϕ_1 and ϕ_2.

If $V(U_1, U_2, U_3)$ is the velocity component and F (F_x, F_y, F_z) are the body forces acting on the system, then the equation of motion in *X*, *Y* and *Z* directions is given by:

$$\rho\frac{DU_1}{DT} = \rho F_X + \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} + \frac{\partial S_{XZ}}{\partial Z} \tag{5}$$

$$\rho\frac{DU_2}{DT} = \rho F_Y + \frac{\partial S_{YX}}{\partial X} + \frac{\partial S_{YY}}{\partial Y} + \frac{\partial S_{YZ}}{\partial Z} \tag{6}$$

$$\rho\frac{DU_3}{DT} = \rho F_Z + \frac{\partial S_{ZX}}{\partial X} + \frac{\partial S_{ZY}}{\partial Y} + \frac{\partial S_{ZZ}}{\partial Z} \tag{7}$$

where

$$\frac{D}{DT} = \frac{\partial V}{\partial T} + V \cdot \nabla V$$

If the bounding surface is porous, then the rate of percolation of the fluid is directly proportional to the cross-sectional area of the filter bed and the total force, say the sum of the pressure gradient and the gravity force. In the sense of Darcy[2].

$$q = CA(\frac{P_1 - P_2}{H_1 - H_2} + \rho G) \quad (8)$$

where A is the cross-sectional area of the filter bed, $C = \frac{k}{\mu}$ in which k is the permeability of the material and μ is the coefficient of viscosity and q is the flux of the fluid. A straightforward generalisation of Eqn (8) yields. ηV

$$V = -\frac{k}{\mu}[\nabla P + \rho G\eta] \quad (9)$$

where the velocity vector V and η is the unit vector along the gravitational force. If other external forces are acting on the system, instead of gravitational force, then one has

$$V = -\frac{k}{\mu}[\nabla P - \rho F] \quad (10)$$

In the absence of external forces, $V = -\frac{k}{\mu}\nabla P$ this gives $\nabla P = -\frac{\mu}{k}V$

Therefore, the net resulting equations (in the dimensional form) of motions in the *X, Y* and *Z* directions and when the bounding surface is porous, are given by

$$\rho\frac{DU_1}{DT} = \rho F_X + \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} + \frac{\partial S_{XZ}}{\partial Z} - \frac{\mu}{k}U_1 \quad (11)$$

$$\rho\frac{DU_2}{DT} = \rho F_Y + \frac{\partial S_{YX}}{\partial X} + \frac{\partial S_{YY}}{\partial Y} + \frac{\partial S_{YZ}}{\partial Z} - \frac{\mu}{k}U_2 \quad (12)$$

$$\rho\frac{DU_3}{DT} = \rho F_Z + \frac{\partial S_{ZX}}{\partial X} + \frac{\partial S_{ZY}}{\partial Y} + \frac{\partial S_{ZZ}}{\partial Z} - \frac{\mu}{k}U_3 \quad (13)$$

Introducing the following non-dimensional variables as:

$$U_i = \frac{\phi_1 u_i}{\rho L} \quad T = \frac{\rho L^2 t}{\phi_1} \quad \phi_2 = \rho L^2 L_1 \quad P = \frac{\phi_1^2 p}{\rho L^2}$$

$$\frac{X_i}{L} = x_i \quad \frac{Y_i}{L} = y_i \quad \phi_3 = \rho L^2 \upsilon_c \quad A_i = \frac{\phi_1^2 a_i}{\rho^2 L^3}$$

$$S_{i,j} = \frac{\phi_1^2 s_{i,j}}{\rho L^2} \quad E_{i,j}^{(1)} = \frac{\phi_1 e_{i,j}^{(1)}}{\rho L^2} \quad E_{i,j}^{(2)} = \frac{\phi_1^2 e_{i,j}^{(2)}}{\rho^2 L^4} \quad k = \frac{\rho L^3}{\phi_1^2 L_2}$$

$$F_i = \frac{\phi_1^2}{\rho L^3} f_i \quad M = \frac{\phi_1 m}{L^2}$$

We consider a class of plane flows given by the velocity components

$$u_1 = u(y,t) \text{ and } u_2 = 0 \quad (14)$$

In the directions of rectangular Cartesian coordinates *x* and *y*, the velocity field given by Eqn (14) identically satisfies the incompressibility condition. The stress can now be obtained in the non-dimensional form as:

$$s_{xx} = -p + \upsilon_c(\frac{\partial u}{\partial y})^2 \quad (15)$$

$$s_{yy} = -p + (\upsilon_c + 2\beta)(\frac{\partial u}{\partial y})^2 \quad (16)$$

$$s_{xy} = \frac{\partial u}{\partial y} + \beta\frac{\partial}{\partial y}(\frac{\partial u}{\partial t}) \quad (17)$$

In view of the above, the equations of motion in the present case of porous boundary will yield

X - Component:

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + L_1\frac{\partial}{\partial t}(\frac{\partial^2 u}{\partial y^2}) - (\frac{1}{L_2} + m)u \quad (18)$$

Y - Component

$$0 = -\frac{\partial p}{\partial y} + (2\beta + \upsilon_c)\frac{\partial}{\partial y}(\frac{\partial u}{\partial y})^2 \quad (19)$$

Z - Component

$$0 = -\frac{\partial p}{\partial z} + (2\beta + \upsilon_c)\frac{\partial}{\partial z}(\frac{\partial u}{\partial z})^2 \quad (20)$$

The Eqn (18) shows that $-\frac{\partial p}{\partial x}$ must be independent of space variables, and hence, may be taken as $\xi(t)$. Eqn (19) now yields.

$$p = p_0(t) - \xi(t)x + (\upsilon_c + 2\beta)(\frac{\partial u}{\partial y})^2$$

$$\frac{\partial p}{\partial y} = 0 \text{ and } \frac{\partial p}{\partial z} = 0$$

Showing that $p = p(x)$. Therefore Eqns (18), (19) and (20) reduce to single equation. The flow characterised by the velocity is given by:

$$\frac{\partial u}{\partial t} = -\frac{dp}{dx} + \frac{\partial^2 u}{\partial y^2} + L_1\frac{\partial}{\partial t}(\frac{\partial^2 u}{\partial y^2}) - (\frac{1}{L_2} + m)u \quad (21)$$

where L_2 is the non-dimensional porosity parameter, it may be noted that the presence of L_1 changes the order of differential from two to three.

Comparing the terms from the above Eqn (21) one gets steady and unsteady problems. The steady state is given by the following equation.

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \left((\frac{1}{k} + M)\right)u = 0 \quad (22)$$

The unsteady state problem from the Eqn (21) is given by following equation.

$$\frac{\partial u}{\partial t} = L_1\frac{\partial^3 U}{\partial T \partial Y^2} - \left((\frac{1}{L_2} + m)\right)u \quad (23)$$

The boundary conditions are

$$u = 0 \text{ when } y = 1 \text{ and } u = 0 \text{ when } y = -1 \quad (24)$$

3. SOLUTION OF THE PROBLEM

Case (i) Steady State: On solving the Eqn (22) by applying Laplace transformation with the boundary conditions are given by Eqn (24), we get the initial velocity (as the visco-elastic parameter L_1 disappears)

$$u(y,0)=\frac{pL_2}{(1+L_2m)}\left[1-\frac{\cosh\frac{y}{\sqrt{\frac{1+mL_2}{L_2}}}}{\cosh\frac{1}{\sqrt{\frac{1+mL_2}{L_2}}}}\right] \qquad (25)$$

Case (ii) Unsteady State: The unsteady state problem is given by Eqn (23). The presence of L_1 increase in the order of differential Eqn (23) from two to three. Three boundary conditions are required to solve Eqn (23). Therefore, the solution of steady state is used as the initial condition along with the boundary conditions. On applying Laplace transformation to the Eqn (23) with the initial condition as given by Eqn (25) together with boundary conditions as given in Eqn (24), then Eqn (23) becomes;

$$\frac{\partial^2\bar{u}}{\partial y^2}-r^2\bar{u}=-\frac{pL_2}{(1+L_1s)(1+L_2s)}+p\left[\frac{L_2}{(1+L_1s)(1+L_2s)}-\frac{L_1}{(1+L_1s)}\right]\left[\frac{\cosh\frac{y}{\sqrt{\frac{1+mL_2}{L_2}}}}{\cosh\frac{1}{\sqrt{\frac{1+mL_2}{L_2}}}}\right] \qquad (26)$$

where $r^2=\frac{L_2(s+m)+1}{L_2(1+L_1s)}$ on solving Eqn (26) together with the boundary conditions, $\bar{u}(y=\pm1,s)=0$ one gets.

$$\bar{u}=c_1e^{ry}+c_2e^{-ry}+\frac{pL_2^2}{(1+L_2m)(L_2(s+m)+1)}-\frac{pL_2}{s(1+L_2m)}\left[\frac{\cosh\frac{y}{\sqrt{\frac{1+mL_2}{L_2}}}}{\cosh\frac{1}{\sqrt{\frac{1+mL_2}{L_2}}}}\right] \qquad (27)$$

where $c_1=\frac{pL_2}{s(L_2(s+m)+1(e^r+e^{-r})}=c_2$ on applying the inverse Laplace transformation for the Eqn (27), one gets the velocity as

$$u(y,t)=\frac{pL_2}{(1+L_2m)}\{[1-e^{-\left(\frac{m+1}{L_2}\right)t}]\frac{\cosh ry}{\cosh r}+e^{-\left(\frac{m+1}{L_2}\right)}\left[\frac{\cosh\frac{y}{\sqrt{\frac{1+mL_2}{L_2}}}}{\cosh\frac{1}{\sqrt{\frac{1+mL_2}{L_2}}}}\right]\} \qquad (28)$$

Mass flow rate is given by $\bar{Q}=\int_{-1}^{1}u\,dy$

4. CONCLUSIONS

In this paper, research was carried out to show the effect of the applied pressure gradient and magnetic field on unsteady flow of a fluid of second-order type with bounding surface is porous under the influence of magnetic field between two parallel plates. The solution was obtained by applying Laplace transform and Inverse Laplace transform method. As $m\to 0$, the results obtained for the velocity field are in agreement to that of Gnana[23], *et al*. The case of Newtonian fluid can be realised as $\beta\to 0$ and $m\to 0$.

Initially the flow was under a constant pressure gradient, and hence, the velocity profiles were parabolic type and symmetric about the channel – centralline. The pressure gradient was suddenly withdrawn and subsequent flow was investigated.

5. RESULT AND DISCUSSION

The effect of magnetic parameter on the velocity profile has been illustrated in the Fig. 2. For a nonzero value of elastico-viscosity, porosity and time parameter, as the magnetic parameter increases, velocity decreases. This is due to the fact that with the introduction of transverse magnetic field normal to the flow direction, have tendency to create a drag like Lorenz force which tends to retard the flow, hence velocity decreases.

Figure 3 represents the effect of different values of porosity on velocity profile. As the porosity increases, velocity decreases in the boundary layer which is in agreement with the natural phenomenon.

In the Fig. 4, the effect of porosity on velocity profile is shown. For nonzero values of elastico-viscosity, magnetic parameter and zero value of time parameter. The backword flow is noticed in the boundary layer. This is because of the Darcian friction that would appear in it.

It has been noticed in Fig. 5, as time parameter increases, the velocity profiles are significantly distributed and the flow settles down in the core region.

The effects of various parameters on the mass flow rate have been illustrated in Fig. 6. For a constant value of the porosity factor, as the elastico-viscosity increases, the flow rate

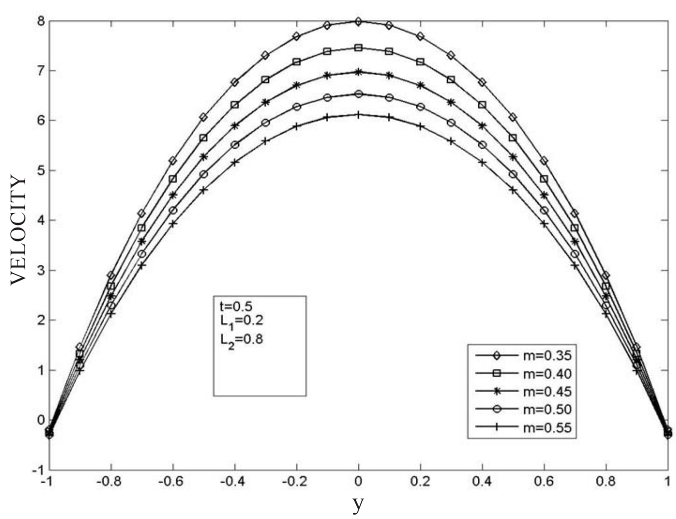

Figure 2. Unsteady state velocity for different values of magnetic parameter *m* with time.

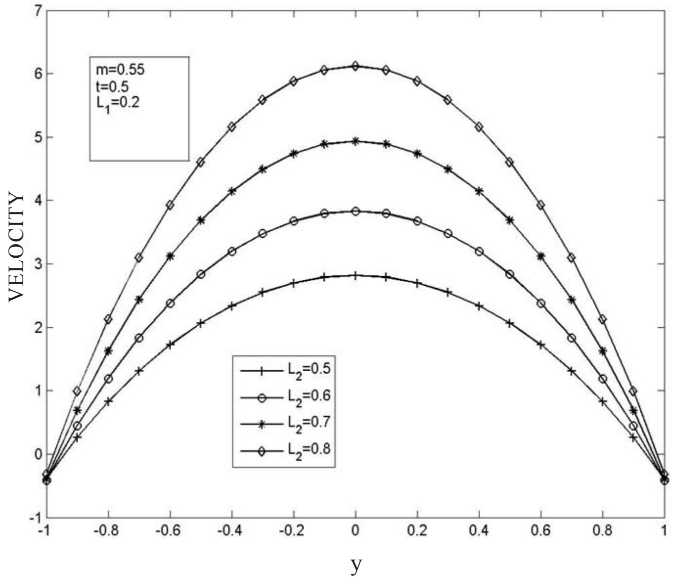

Figure 3. Unsteady state velocity for different values of L_2 with time t=0.5.

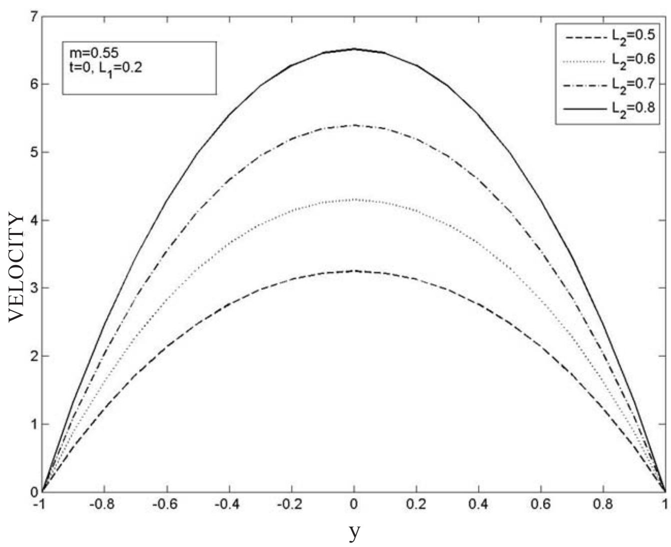

Figure 4. Steady state velocity for different values of L_2 with time t=0.

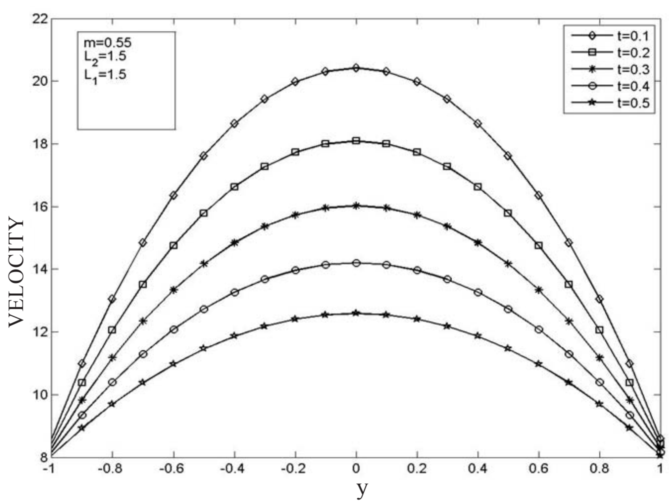

Figure 5. Unsteady state velocity for different values of time.

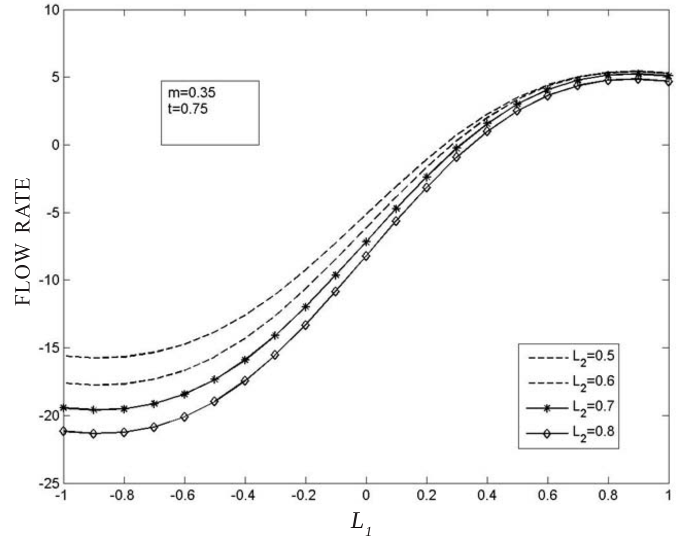

Figure 6. Flow rate for different values of L_2.

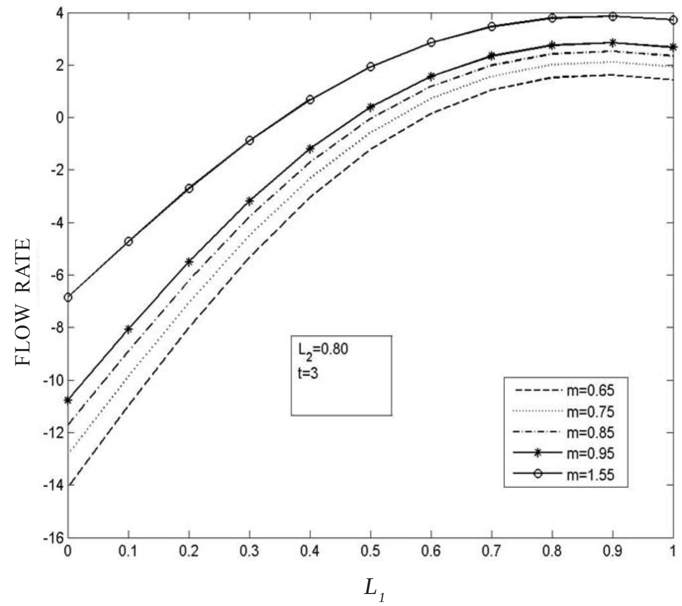

Figure 7. Flow rate for different values of magnetic parameter.

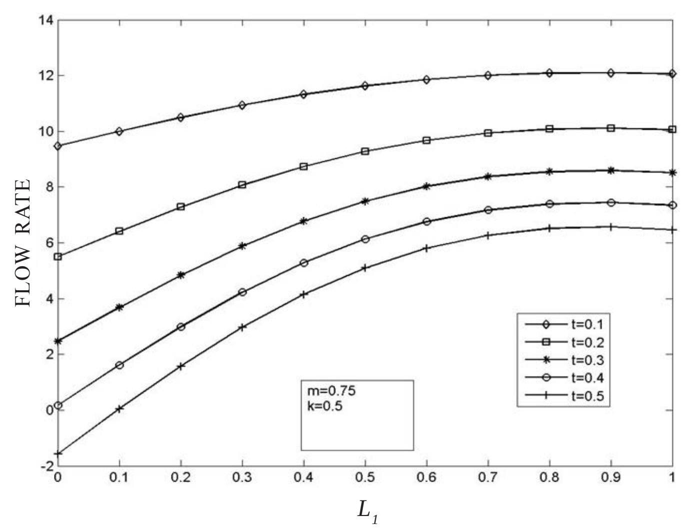

Figure 8. Mass flow rate for different values of time.

decreases at the lower plate and the reverse trend is observed in the upper plate.

It is observed from the Fig. 7, that the magnetic parameter increases and flow rate decreases. It agrees with the realistic nature of the fluid motion.

The effect of the mass flow rate on time parameter is noticed in Fig. 8. As the time parameter increases, the flow rate decreases after $t = 0.5$ and backward flow is observed. Such an effect can be attributed to the increase in density of the medium and the resistance offered by the bounding surfaces.

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