then be associated with the auto-bicoherence and defined as

$$b^2_{xxx}(f_1, f_2) = \frac{|\widehat{\mathrm{B}}^2_{\mathrm{xxx}}(\mathrm{f}_1, \mathrm{f}_2)|^2}{\frac{1}{\mathrm{M}}\sum_{\mathrm{k}=1}^{\mathrm{m}} |\mathrm{X}_{\mathrm{T}}^{(\mathrm{k})}(\mathrm{f}_1)\mathrm{X}_{\mathrm{T}}^{(\mathrm{k})}(\mathrm{f}_2)|^2 \frac{1}{\mathrm{M}}\sum_{\mathrm{k}=1}^{\mathrm{m}} |\mathrm{X}_{\mathrm{T}}^{(\mathrm{k})}(\mathrm{f}_1+\mathrm{f}_2)|^2} \tag{13}$$

By the Schwarz inequality, the value of $b^2_{xxx}(f_1, f_2)$ varies between zero and one.

If no phase relationship exists among the frequency components at (f_1, f_2) and $(f_1 + f_2)$, the value of the auto-bicoherence will be at or near zero (due to averaging effects).

If a phase relationship does exist among the frequency components at (f_1, f_2) and $(f_1 + f_2)$, then the value of the auto-bicoherence will be near unity. Values of the auto-bicoherence between zero and one indicate partial quadratic coupling.

For systems where multiple signals are considered, detection of non-linearities can be achieved by using the cross-spectral moments. For two signals, x(t) and y(t), the cross-bispectrum is estimated as

$$\widehat{B}^2_{yxx}(f_1, f_2) = \frac{1}{\mathrm{M}}\sum_{k=1}^{m} Y_T^{(k)}(f_1+f_2)X_T^{*(k)}(f_1)X_T^{*(k)}(f_2) \tag{14}$$

Where $X_T^{(k)}(f)$and $Y_T^{(k)}(f)$ are discrete FTs of the k^{th} ensemble of the time series x(t) and y(t), respectively, over time T. The cross-bispectrum provides a measure of the non-linear relationship among the frequency components at f_1 and f_2 in x(t) and their summed frequency components $(f_1 + f_2)$ in y(t).

Similar to the auto-bispectrum, the cross-bispectrum of signals x(t) and y(t) is a two dimensional function in frequency and is generally complex-valued. In averaging over many ensembles, the magnitude of the cross-bispectrum will also be determined by the presence, or absence, of a phase relationship among sets of frequency components at f_1, f_2 and $(f_1 + f_2)$. If there is a random phase relationship among the three components, the cross-bispectrum will average to a very small value. If a phase relationship exists among these frequency components, the corresponding cross-bispectral value will have a large amplitude. The cross-bispectrum is thus able to detect non-linear phase coupling among different frequency components in two signals because of its phase preserving effect.