

process typical of a non-linear dynamical system of high order [1], let us consider single bubble dynamical motion to underpin the rationale behind HOS analysis. Growth and collapse of a bubble can be equated to forced bubble oscillator resonant response as a function of hydrodynamic driving pressure.

The resonant frequency  $f_0$  of a bubble with radius  $R_0$  at equilibrium can be given by Eq(1):[3]

$$f_0 \approx \frac{1}{2\pi} \sqrt{3\gamma \frac{P_0 + \frac{2\sigma}{R_0}}{\rho_0 R_0^2} - \frac{2\sigma}{\rho_0 R_0^3}} \quad (1)$$

where  $\gamma$  is the polytropic index ( $\gamma = 4/3$  for adiabatic pulsations),  $P_0$  is the hydrostatic pressure ( $P_0 = 101325 \text{ N m}^{-2}$ ),  $\sigma$  is the surface tension ( $\sigma = 0.072 \text{ Nm}^{-1}$ ),  $R_0$  is the equilibrium bubble radius,  $\rho_0$  is the equilibrium density of water ( $\rho_0 = 1000 \text{ kg m}^{-3}$ ).

From (1) it can be found that the resonant value of the bubble radius for  $f_0$  equal to the driving frequency (40kHz) is approximately 80.64  $\mu\text{m}$ ; 40kHz has been chosen as this represents the acoustic drive frequency of the cleaning vessel used to test the cavitation sensor concept [4]. In order to predict the variation of the radius of a cavitating bubble in a liquid when subject to a sinusoidally varying forcing pressure, the Gilmore equation was implemented. This equation is of the form [5]:

$$R \left( 1 - \frac{c}{U} \right) \frac{d^2 R}{dt^2} + \frac{2}{3} \left( 1 - \frac{c}{U} \right) \left( \frac{dR}{dt} \right)^2 - \left( 1 + \frac{c}{U} \right) H - \frac{c}{U} \left( 1 - \frac{c}{U} \right) R \frac{dH}{dR} = 0, \quad (2)$$

where  $R$  is the bubble radius as a function of time  $t$ ,  $U = \frac{dR}{dt}$  is the bubble wall velocity,  $c$  is the sound velocity at the bubble wall,  $H$  is the specific enthalpy of the liquid.  $H$  and  $c$  are, respectively, given by [5]:

$$H = \int_{P(R)}^{P(\infty)} \frac{dP}{\rho} \quad (3)$$

and

$$c = [c_0^2 + (m-1)H]^{\frac{1}{2}} \quad (4)$$

where  $P$  is the time varying pressure of the liquid,  $\rho$  is the time-varying density of the liquid,  $c_0$  is the velocity of sound in the unperturbed liquid,  $P(R)$  is the pressure