A NEW HOS-BASED MODEL FOR SIGNAL DETECTION IN NON-GAUSSIAN NOISE: AN APPLICATION TO UNDERWATER ACOUSTIC COMMUNICATIONS

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Abstract - In the context of digital signal processing addressed to underwater acoustic communications, this work focuses attention on the optimization of *detection of weak signals* in presence of additive independent stationary *non-Gaussian noise*. In order to detect signals in the case of low SNR values, the selected binary statistical testing approach consists in a Locally Optimum Detector (LOD), designed on the basis of a new proposed HOS-based model of non-Gaussian noise probability density function (pdf). In particular, an *"asymmetric Gaussian"* pdf model is introduced, in order to describe realistically non-Gaussian noise in a very simple way. The resulting test has been compared with the Gaussian-hypothesis LOD test. Experimental results have shown significant advantages in modelling noise pdf on the basis of the proposed pdf function; they derive from the application of the LOD test for detecting known deterministic signals corrupted by real acoustic ship-traffic-radiated noise.

1. INTRODUCTION*

Conventional digital signal processing algorithms and detection criteria, based on the Second Order Statistics (SOS), and optimised in presence of Gaussian noise, may degrade their performances in non-Gaussian environments. In this case, Higher Order Statistics (HOS) [1][2] is selected for analysing noise and building efficient detection tests. This paper presents an innovative HOS-based noise pdf model applied for detection in presence of additive, independent and identically distributed (iid), stationary, non-Gaussian noise, under the critical conditions of weak signals (e.g., for values of the Signal-to-Noise Ratio - SNR - belonging to the range [-20÷-5] dB).

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The detection problem is faced with reference to a general digital communication system, as shown in Fig. 1.

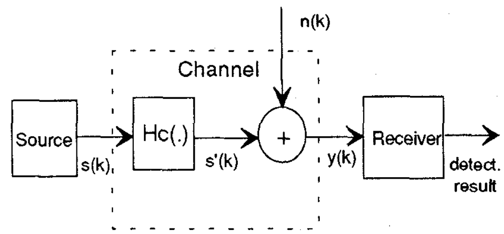


Fig. 1. Mathematical block diagram of a general communication system.

The system is made up of three main blocks:

a. a *source* of deterministic or stochastic signals, $\{s(k), k=1, .., K\}$;
b. a *propagation channel* that uses an input/output function, $H_C(\cdot)$, and adds to the transmitted signal independent, stationary, generally non-Gaussian noise, $\{n(k), k=1, ..., K\}$, consisting of iid samples;
c. a *receiver* of the resulting observation, $\{y(k), k=1, ..., K\}$, on the basis of which to decide between the two hypotheses of presence (hypothesis H1) and absence (hypothesis H0) of a transmitted signal $\{s(k)\}$ [3][4].

Attention is focused on the *receiver block*; the other modules are simplified:

a) the signals emitted are deterministic and have simple known shapes (e.g., impulses, sinusoids, etc.);
b) the transfer function can only attenuate the signals transmitted ($H_C(w)=G$, G is constant, $G\leq 1$).

Detection is dealt with as *binary hypothesis testing* in the context of *statistical inference* [3].

Under the aforesaid assumptions, detection optimization can be reached by selecting the most suitable:

1. binary hypothesis statistical detection criteria,
2. signal processing techniques for noise characterization as a basis for designing detection algorithms.

For improving detection performances with generalized noise environments and low SNR values, the *Locally Optimum Detection class* is selected [3]; attention is focused on signal processing techniques for noise modelling aimed at LOD design.

1.1 Selection of HOS-based signal processing techniques

Conventional processing of deterministic or stochastic signals is based on SOS theory [5], which allows one to describe completely only linear minimum-phase processes, for they can be realistically modelled as *linear combinations of statistically uncorrelated harmonic components* in time and in frequency.

Under the assumptions of deviations from Gaussianity and linearity, the SOS parameters are not sufficient for a complete signal description. HOS-based techniques [1][2] have been selected in the present work as a powerful means to characterise non-Gaussian noise and build efficient and robust detectors on the basis of more complete characterization and more realistic statistical modelling. The main reasons for using HOS to overcome spectral inefficiency are the following analytical properties:

a) deviation from Gaussianity is detected if some values in the domains of spectra or cumulants of higher orders than the second one are significantly different from zero; additive independent Gaussian components are suppressed in the HOS cumulant/spectra domain; some HOS operators can model deviation from Gaussianity;
b) HOS aids in detecting the presence of non-linearities in stochastic processes;
c) HOS allow the detection and the analysis of non-minimum-phase signals.

This paper is focused on property a).

Various recent works have described the use of Higher Order Statistics as the signal-processing basis for non-Gaussian signal detection in Gaussian and non-Gaussian noise: third-order-based algorithms were presented by Hinich and Wilson [6], Kletter and Messer [7], and others; the advantages of a fourth-order statistical parameter (i.e., kurtosis) in noise modelling and signal detection were particularly stressed by Dwyer [8] and Webster [9].

Higher-order moments, cumulants and spectra can be defined as extensions of SOS operators; an extensive and overall presentation of their definitions, properties and possible applications is provided in [1][2].

1.2 Application of detection to a real underwater acoustic channel characterized by non-Gaussian noise

An example of application of the developed methods to real data is given: an *underwater acoustic communication system* [10] is considered. Data sequences were acquired during an oceanographic campaign conducted in the Southern Adriatic Sea in May 1993 (within the framework of the CEC MAST1 SNECOW Project) [11][12]. The test data were acquired by using a single hydrophone (i.e., a receiving passive underwater acoustic sensor), dropped 30 meter deep from an oceanographic ship, and describe background ship-traffic-radiated noise.

The application target consists in detecting deterministic known signals in presence of the acquired noise.

The experiments were performed under the critical conditions of *shallow-water* (from 100 to 600 meter deep), and *coastal* waters, so with much ship traffic (on average, about 6-10 ships in a circular area of 12-nm-radius around the fixed receiver).

Traffic intensity is associated with the following conditions:

a. ship-radiated noise components overcome the other ambient-noise components over a narrow frequency range (from few Hz up to about 1000 Hz);
b. traffic ships can be assumed to be uniformly distributed in the area surrounding the receiver.

Ship-radiated noise is a complex random process, resulting from the combination (either linear or non linear) of many components. As the noise emitted by a ship is the combination of periodical, almost periodical and slowly varying non-linear components, it is expected to present a series of spikes in its power spectrum, hence to be non-Gaussian.

dominant noise components characterized by slowly varying almost periodic time shapes
(at least) weakly-stationary
additive
approximately iid ⇒ univariate samples
non Gaussian

Table 1. Main characteristics generally expected about acoustic ship traffic noise.

In the studied application, noise is due to both oceanographic and traffic ships, that is, N independent random sources with very different powers, in particular, few strong sources (the closest ships) and many weak sources (background ships passing far from the sensor). Under these conditions, the resulting noise cannot satisfy the Central Limit

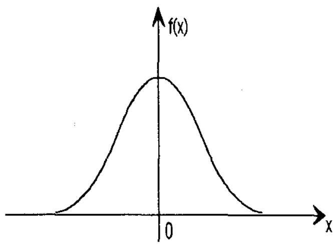
Theorem [9], and is expected to be non Gaussian. Table 1 summarizes the main expected noise characteristics.

2. DESCRIPTION OF THE METHOD USED AND ITS APPLICATION TO A REAL CASE

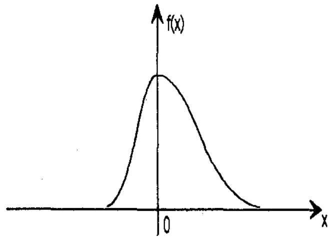
If noise is tested to be iid, stationary, non Gaussian (with preliminary statistical analysis, as presented in [11][12]), its pdf has to be modelled analytically in a realistic way for LOD design. To this end, an HOS parameter has been selected as it can aid in quantifying deviation from symmetry, and, so, from Gaussianity; it is the skewness (third order).
The **skewness** for a stationary zero-mean iid process $\{X(k)\}$ can be expressed as:

$$c_3^x(0,0) = E\{X^3\} \tag{1}$$

where $c_3^x(\tau_1, \tau_2)$ is the third order cumulant and is defined as the corresponding third-order moment for zero-mean processes (E is the expectation operator).
Skewness values different from zero imply an *asymmetric* (hence non-Gaussian) *pdf* (see Fig. 2).



$c_3^x(0,0) = 0$
(a)



$c_3^x(0,0) \neq 0$
(b)

Fig. 2. Pdf models differentiated by skewness values.

In the present work a new asymmetric pdf model is presented: it is called "asymmetric Gaussian" pdf, as it directly derives from the Gaussian shape (with consequent advantages in employing it in detector design). It depends on *two second-order parameters* (deriving from the definition of variance), σ_l^2 and σ_r^2, called respectively "left and right variances" and estimated from discrete process sequences in accordance with the following formulas:

$$\sigma_l^2 = \frac{1}{N_l - 1}\sum_{\substack{i=1\\ n_i<\mu}}^{N_l}(n_i - \mu)^2 \quad \text{and} \tag{2}$$

$$\sigma_r^2 = \frac{1}{N_r - 1}\sum_{\substack{i=1\\ n_i>\mu}}^{N_r}(n_i - \mu)^2 \tag{3}$$

where μ is the mean value, and N_l (N_r) is the number of n samples $<\mu$ ($>\mu$). The model expression follows:

$$p_{aG}(n) = \begin{cases} \dfrac{2}{\sqrt{2\pi}(\sigma_l + \sigma_r)} e^{-\frac{(n-\mu)^2}{2\sigma_l^2}} & n < \mu \\ \dfrac{2}{\sqrt{2\pi}(\sigma_l + \sigma_r)} e^{-\frac{(n-\mu)^2}{2\sigma_r^2}} & n \geq \mu \end{cases} \tag{4}$$

It is very general (in the particular case of $\sigma_l^2 = \sigma_r^2$, the model expression coincides with the Gaussian function) and simple. As you can deduce from appendix A, in the case of an iid process with zero mean value, the left and right variances are linked with the skewness parameter as follows:

$$\begin{aligned} c_3^n(0,0) &= E\{n^3\} = \\ &= \int_{-\infty}^{+\infty} n^3 p_{nG}(n)dn = \\ &= -2\sigma_l^4 + 2\sigma_r^4 \end{aligned} \tag{5}$$

Hence they maintain the same level of information as provided by the 3th order statistics. Fig. 2(b) is an example of asymmetric Gaussian pdf where $\sigma_l<\sigma_r$. As consisting of two Gaussian parts, it is particularly suitable for easily expressing the LOD characteristic function (which is *piece-wise linear* in this case):

$$\begin{aligned} g_{lo,aG}(y_k) &= -\frac{p'_{aG}(y_k)}{p_{aG}(y_k)} s_k = \\ &= \begin{cases} \dfrac{y_k - \mu}{\sigma_l^2} s_k & y_k < \mu \\ \dfrac{y_k - \mu}{\sigma_r^2} s_k & y_k \geq \mu \end{cases} \end{aligned} \tag{6}$$

(note that it is linear in the case of the Gaussian model).

When K samples $\{y_k\}$ are collected by the receiver, the LOD test is applied to the stochastic variable λ_{lo}, computed as:

$$\lambda_{lo} = \sum_{k=1}^{K} g_{lo}(y_k) \tag{7}$$

and compared with a suitable threshold T_α, fixed in terms of a certain value of $P_{FA}=\alpha$ (given by the user) and of the selected noise pdf model $p(\cdot)$ [1]:

$$\lambda_{lo} \underset{no\ signal}{\overset{signal}{\gtrless}} T_\alpha \tag{8}$$

The test is applied as based on the asymmetric Gaussian and the Gaussian (used as reference hypothesis) pdf models, in order to compare resulting performances as SNR varies. In Fig. 4 the block diagram of the developed detection test.

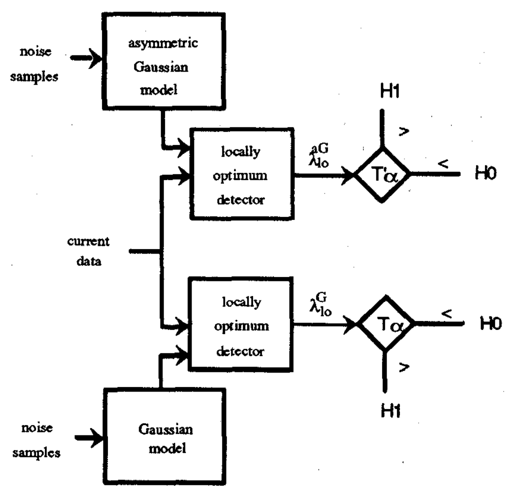


Fig. 4. Block diagram of the developed detection tests.

3. EXPERIMENTAL RESULTS

Acquired data were extensively used in order to evaluate the advantages of using the presented HOS-based pdf model vs. the Gaussian hypothesis in detector design. Performances were evaluated in terms of P_{det}, for different SNR values, given a fixed P_{FA}. The results in the two different cases are summarized in Fig. 5. A large set of signal shapes with increasing degrees of complexity were selected (from impulses to combined sinusoids).

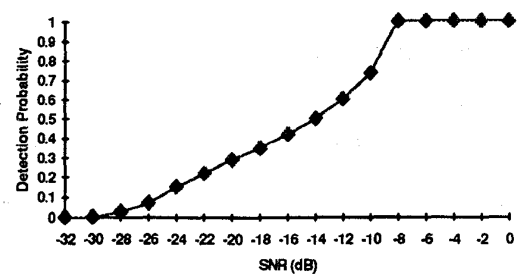
The next investigation step, concerning the model of propagation through a real shallow-water channel under specified conditions (in terms of channel identification, models of Doppler, non-linearity and distortion effects, etc. [10][11]), is going to be carried out in order to disregard the constraint of a-priori knowledge of the signal shape at the receiver input.

The tests were carried out under the following working conditions:

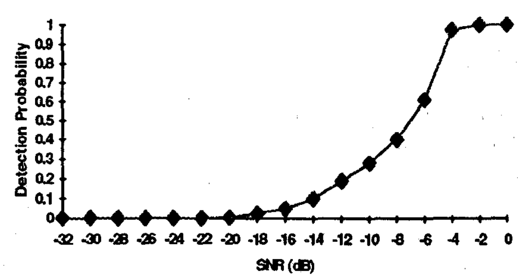
- Fixed probability of false alarm: $P_{FA}=\alpha=5\%$.
- real underwater acoustic noise, characterized by $\mu\sim 0$, $\sigma_l= 1860$, $\sigma_r=1500$.
- Fixed time-window width (equal for signal and receiver): K=1000 samples.

It is easy to notice that in these tests the asymmetric Gaussian model allows a strong improvement of detection performances as compared with the Gaussian model; this proofs that taking into account noise deviation from symmetry is particularly useful for detection aims; the asymmetric Gaussian model provides $P_{det}\neq 0$ even for very low SNR values and allows P_{det} to reach 1 around -8 dB (by using the Gaussian model, P_{det}=1 is reached only when SNR is at least -4 dB).

A notable characteristic of the asymmetric Gaussian model is its simplicity in both the phases of estimation and application.



(a)



(b)

Fig. 5. LOD test results from *asymmetric Gaussian* (a), and *Gaussian* (c) *hypotheses* of noise pdf.

4. CONCLUSIONS

The paper has been focused on a new pdf model for generalized iid asymmetric noise. Results have shown the advantages provided by its use in a LOD test design for optimizing detection performances under the conditions of non-Gaussian noise and weak signals. The research will be developed by including an analytical model of the channel and, so, by testing detection performances on real acoustic signals; moreover, the pdf model will be further generalized by inserting fourth-order parameters (e.g., kurtosis), modelling different sharpness of the pdf shape.

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APPENDIX A

In the following how the left and the right variances are linked to the skewness is shown, with reference to the asymmetric Gaussian pdf model (see formula (5)).

It is presented in the case of an iid stochastic process $\{X(n)\}$ with zero mean value. The proof starts from the definition of skewness in the continuous case, in terms of the process pdf analytical expression $p(\cdot)$:

$$c_3^x(0,0) = E\{X^3\} = \int_{-\infty}^{+\infty} X^3 p(X)dX \tag{9}$$

By substituting $p(\cdot)$ with the asymmetric Gaussian expression under the integration operator, one can obtain:

$$\begin{aligned} c_3^x(0,0) = & \int_{-\infty}^{0} X^3 \frac{2}{(\sigma_l + \sigma_r)\sqrt{2\pi}} e^{-\frac{X^2}{2\sigma_l^2}} dX + \\ & + \int_{0}^{+\infty} X^3 \frac{2}{(\sigma_l + \sigma_r)\sqrt{2\pi}} e^{-\frac{X^2}{2\sigma_r^2}} dX \end{aligned} \tag{10}$$

The integration can be solved by considering general expression:

$$\int_0^{+\infty} x^m e^{-ax^2} dx = \frac{\Gamma[(m+1)/2]}{2a^{(m+1)/2}} \tag{11}$$

where the function $\Gamma(\cdot)$ is defined as follows:

$$\Gamma(k) = \int_0^{+\infty} e^{-x} x^{k-1} dx \tag{12}$$

and, if k is integer,

$$\Gamma(k) = (k-1)! \tag{13}$$

In this particular case, the value of the parameter m is 3, so $[(m+1)/2] = 2$ is integer and $\Gamma(2) = 1! = 1$. Hence,

$$\int_0^{+\infty} X^3 e^{-\frac{X^2}{2\sigma_r^2}} dX = \frac{1}{2\left(\frac{1}{2\sigma_r^2}\right)^2} = 2\sigma_r^2 \tag{14}$$

In conclusion,

$$c_3^x(0,0) = -2\sigma_l^2 + 2\sigma_r^2 \tag{15}$$

It is easy to notice that if $\sigma_l^2 \neq \sigma_r^2$ (i.e., deviation from symmetry is detected by analysing sequences of noise), then $c_3^x(0,0) \neq 0$, as confirmed by the well-known skewness properties about deviation from symmetry [1][2].