EFFECTS OF VARIATIONS IN LOADING CONDITIONS ON MAXIMUM PRESSURE AND MUZZLE VELOCITY WHEN USING COMPOSITE CHARGE

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ABSTRACT

In this paper the effects on maximum pressure and muzzle velocity due to small changes in various quantities defining the loading conditions have been obtained mathematically when using composite charge in guns. Calculations have been done for a particular gun to illustrate these results.

Introduction

In a recent paper the author[1] has discussed the effects of variations in loading conditions (*i.e.* the various conditions which hold after the gun has been loaded but before it is fired) for a single charge. Mathematical expressions were obtained for assessing the percentage changes in maximum pressure and muzzle velocity due to unit percent change in various quantities defining the loading condition. It was shown how muzzle velocity is affected due to the changes in various quantities when maximum pressure is kept constant by varying web size or rate of burning constant or both. In another paper the author[2] further considered this problem and kept maximum pressure constant by varying charge weight alone and compared the two results numerically. The effect of varations in loading conditions on internal ballistics has also been discussed qualitatively in Military College of Service Publication G. M. 8 (1945)[8]. Therein, a particular gun has been considered and a set of presure-space curves is given, showing the variations to be expected when the loading conditions are varied one at a time. Corner[3] has considered the problem of effects of changes in various quantities but he has confined attention to only a particular gun. Thus when using different guns or different shapes of propellants in the same gun, only the results obtained by the author in the above mentioned two papers are to be used. Vickers–Armstrong Ltd. (H.M.S.O.)[4] have also given a table for estimating changes in maximum pressure and muzzle velocity due to one percent increase in the loading conditions separately. It may be mentioned here that Kapur[5] studied this problem and made use of ballistic similitude for the purpose of simplifying the tabulations for the various changes.

Now in guns when maximum pressure has to be brought down to a certain limit and to obtain a desired muzzle velocity we make use of composite charge which consists of a mixture of two or more grains of the same or different compositions, shapes and sizes. The author[6] has already solved the equations of internal ballistics for composite charge using the isothermal model of solution. Since composite charge is employed very often in guns, it is important to discuss the changes in maximum pressure and muzzle velocity due to small changes in the loading conditions. Though the problem can be discussed qualitatively from the fundamental principles yet, as we are more interested in the quantity of these changes, expressions have been obtained for calculating the percentage changes in maximum pressure and muzzle velocity when only one parameter is varied. For the sake of simplicity only the case where the two charges are in tubular form has been discussed in detail and numerical calculations have been done for a particular gun. A more general case where the two charges may be of any form has also been dealt with elsewhere[7].

Suffix $(p_{max})_1$ is used to indicate that maximum pressure occurs during the first stage of burning *i.e.* when both the charges are burning and suffix $(p_{max})_2$ to indicate that maximum pressure occurs during the second stage of burning *i.e.* when one of the charges has burnt out and only the other is burning. It has further been assumed that if maximum pressure with the standard conditions occurs in any one of the two stages then with the changed conditions also it will occur in the same stage.

The following symbols and notations have been used :

C = Charge weight

D = Web size *i.e.* the least thickness for complete combustion of the charge

λ = Force constant

β = Rate of burning Co-efficient

γ = Ratio of specific heats

f = Fraction of D remaining at time t

A = Area of cross-section of the bore

l = Equivalent length of the initial space in the chamber

W = Weight of the projectile

$W_1 = 1{\cdot}06W +$ total charge weight/3

M = Non-dimensional quantity called the Central Ballistic Parameter

p = Mean pressure of the propellant gases at time t

V = Velocity of shot at time t

x = Shot travel at time t

$K = \beta_1 D_2/\beta_2 D_1$

Suffix 1 refers to the first charge, suffix 2 to the second charge; the position of maximum pressure is represented by suffix (*max.*) and the all burnt position by the suffix (B) while suffix (E) gives the conditions at the muzzle of the gun.

Expressions for maximum pressure and muzzle velocity

The author[6] has already obtained the conditions for charge C_1 to burn out first as $K > 1$, *i.e.*, $\beta_1 D_2 > \beta_2 D_1$ and for charge C_2 to burn out first as $K < 1$, *i.e.*, $\beta_1 D_2 < \beta_2 D_1$. Assuming that charge C_1 burns out first, the expression for maximum pressure can be obtained from the general expressions given by the author[6] by taking the limit or by direct calculations as

$$(p_{max})_1 = \frac{\lambda_1 C_1 \left(1 + \frac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)^2}{A\, e\, l\, M_1} \left\{ if\ M_1 > \left(1 + \frac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)\right\} \quad (1)$$

and

$$(p_{max})_2 = \frac{\lambda_2 C_2 / M_2 + \lambda_1 C_1}{A\,(x_m + l)} \quad \left\{ if\ \frac{1}{K} \leqslant \frac{1}{M_2} \leqslant 1 \right\} \quad \ldots \quad (1a)$$

where

$$\log (x_m + l) = \log l + \frac{M_1}{1 + \frac{\lambda_2 C_2}{\lambda_1 C_1 K}} + \frac{M_2 \lambda_1 C_1}{\lambda_2 C_2} \log \frac{\frac{\lambda_1 C_1}{\lambda_2 C_2} + \frac{1}{K}}{\frac{\lambda_1 C_1}{\lambda_2 C_2} + \frac{1}{M_2}} + \left(1 - \frac{M_2}{K}\right) \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Also the muzzle velocity is given by

$$V^2_E = \frac{A^2 D_2^2}{\beta_2^2 W_1} + \frac{2(\lambda_1 C_1 + \lambda_2 C_2)}{W_1} \log \frac{x_E + l}{x_B + l} \quad \ldots \quad (3)$$

where

$$\log (x_B + l) = \log l + \frac{M_1}{1 + \frac{\lambda_2 C_2}{\lambda_1 C_1 K}} + \frac{M_2 \lambda_1 C_1}{\lambda_2 C_2} \times \log \frac{\frac{\lambda_1 C_1}{\lambda_2 C_2} + \frac{1}{K}}{\frac{\lambda_1 C_1}{\lambda_2 C_2} + 1} + M_2 \left(1 - \frac{1}{K}\right) \quad \ldots \quad \ldots \quad (4)$$

Effect of variations in loading conditions

Now in what follows, by using the above expressions, we will calculate the percentage changes in maximum pressure and muzzle velocity due to unit percent change in the following quantities separately,

(a) For Charge :—Charge weights (C_1, C_2); Web Sizes (D_1, D_2); Rate of Burning Constants (β_1, β_2) and Force Constants (λ_1, λ_2).

(b) For Gun :—Chamber Capacity K_o and Shot-travel or length of the bore x_E .

(c) For Shot :—Shot weight W.

Now we proceed on to find the changes.

(I) *Effect of Change in Charge Weights* :—An increase in weight in any one of the two charges means an increase in the total charge weight and thus an increase in the total chemical energy available. Therefore maximum pressure and muzzle velocity must increase. The following expressions give the percentage increase (or decrease) in maximum pressure and muzzle velocity due to unit percent increase (or decrease) in charge weights C_1 and C_2 separately.

(a) Due to a change in charge weight C_1 :—

$$\frac{\partial\,(\log p_{max})_1}{\partial\,(\log C_1)} = \frac{C_1}{A\,l\,\delta_1} + \frac{C_1}{3\,W_1} + \frac{2}{1 + \dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}} \quad \ldots \quad (5)$$

which is obviously positive.

$$\frac{\partial\,(\log p_{max})_2}{\partial\,(\log C_1)} = \frac{C_1}{A\,l\,\delta_1} + \frac{\lambda_1 C_1 + \dfrac{\lambda_2 C_2}{M_2}\,\dfrac{C_1}{3\,W_1}}{\lambda_1 C_1 + \dfrac{\lambda_2 C_2}{M_2}} + \frac{M_1 + \dfrac{M_1 C_1}{3\,W_1}\left(1 + \dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)}{\left(1 + \dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)^2} -$$

$$\frac{M_2 \lambda_1 C_1}{\lambda_2 C_2}\left[\left(1 - \frac{C_1}{3\,W_1}\right) \log \frac{\dfrac{\lambda_1 C_1}{\lambda_2 C_2} + \dfrac{1}{K}}{\dfrac{\lambda_1 C_1}{\lambda_2 C_2} + \dfrac{1}{M_2}} + \frac{\dfrac{\lambda_1 C_1}{\lambda_2 C_2}\left(\dfrac{1}{M_2} - \dfrac{1}{K}\right) - \dfrac{C_1}{3\,M_2\,W_1}\left(\dfrac{\lambda_1 C_1}{\lambda_2 C_2} + \dfrac{1}{K}\right)}{\left(\dfrac{\lambda_1 C_1}{\lambda_2 C_2} + \dfrac{1}{K}\right)\left(\dfrac{\lambda_1 C_1}{\lambda_2 C_2} + \dfrac{1}{M_2}\right)}\right]$$

$$- \frac{M_2 C_1}{3\,W_1 K} \quad \ldots \quad (5a)$$

and

$$\frac{\partial\,(\log V_E)}{\partial\,(\log C_1)} = -\frac{M_2 C_1}{3\,W_1}\;\frac{\lambda_2\,C_2}{W_1\,V^2{}_E} + \frac{\lambda_1 C_1 - \dfrac{C_1}{3\,W_1}(\lambda_1 C_1 + \lambda_2 C_2)}{W_1\,V^2{}_E}\times$$

$$\log\frac{x_E + l}{x_B + l} - \frac{(\,{}_1 C_1 + \lambda_2\,C_2)}{W_1\,V^2{}_E}\left\{\frac{C_1}{A\,\delta_1\,(x_E + l)}\right.$$

$$-\frac{C_1}{A\,l\,\delta_1} - \frac{M_1 + \dfrac{M_1\,C_1}{3\,W_1}\left(1 + \dfrac{\lambda_2\,C_2}{\lambda_1\,C_1\,K}\right)}{\left(1 + \dfrac{\lambda_2\,C_2}{\lambda_1\,C_1\,K}\right)^2}$$

$$+\frac{M_2\,\lambda_1\,C_1\left(1 - \dfrac{C_1}{3\,W_1}\right)}{\lambda_2\,C_2}\log\frac{\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2} + \dfrac{1}{K}}{\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2} + 1}$$

$$-\frac{M_2\,C_1}{3\,W_1}\left(1 - \frac{1}{K}\right) + \frac{M_2\,\lambda_1^2\,C_1^2}{\lambda_2^2\,C_2^2}\times$$

$$\left.\frac{\left(1 - \dfrac{1}{K}\right)}{\left(\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2} + \dfrac{1}{K}\right)\left(\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2} + 1\right)}\right\} \quad .. \quad (6)$$

(*b*) Due to change in charge weight C_2 :—

$$\frac{\partial\,(\log p_{max})_1}{\partial\,(\log C_2)} = \frac{C_2}{A\,l\,\delta_2} + \frac{C_2}{3\,W_1} + \frac{2\;\dfrac{\lambda_2\,C_2}{\lambda_1\,C_1\,K}}{1 + \dfrac{\lambda_2\,C_2}{\lambda_1\,C_1\,K}} \quad .. \quad .. \quad (7)$$

which is again obviously positive.

$$\frac{\partial\,(\log p_{max})_2}{\partial\,(\log C_2)} = \frac{C_2}{A\,l\,\delta_2} + \frac{\dfrac{\lambda_2\,C_2}{M_2}\left(2 + \dfrac{C_2}{3\,W_1}\right)}{\lambda_1\,C_1 + \dfrac{\lambda_2\,C_2}{M_2}}$$

$$+\frac{M_2\,\lambda_1\,C_1\left(1 + \dfrac{C_2}{3W_1}\right)}{\lambda_2\,C_2}\log\frac{\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2} + \dfrac{1}{K}}{\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2} + \dfrac{1}{M_2}}$$

$$+ \frac{\frac{M_2 C_2}{3W_1}\left(1+\frac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)}{\left(1+\frac{{}_2 C_2}{\lambda_1 C_1 K}\right)^2} + \frac{M_2 \lambda_1 C_1}{\lambda_2 C_2} \times$$

$$\frac{\frac{\lambda_1 C_1}{\lambda_2 C_2}\left(\frac{1}{M_2}-\frac{1}{K}\right)+\frac{1}{M_2}\left(1+\frac{C_2}{3W_1}\right)\left(\frac{\lambda_1 C_1}{\lambda_2 C_2}+\frac{1}{K}\right)}{\left(\frac{\lambda_1 C_1}{\lambda_2 C_2}+\frac{1}{K}\right)\left(\frac{\lambda_1 C_1}{\lambda_2 C_2}+\frac{1}{M_2}\right)}$$

$$- \frac{M_2}{K}\left(1+\frac{C_2}{3W_1}\right) \qquad \cdot\cdot \qquad \cdot\cdot \qquad \cdot\cdot \qquad (7a)$$

Also the percentage change in muzzle velocity is

$$\frac{\partial\,(log\, V_E)}{\partial\,(log\, C_2)} = -\frac{M_2 C_2}{3W_1}\,\frac{\lambda_2 C_2}{W_1 V_E{}^2} + \frac{\lambda_2 C_2 - \frac{C_2}{3W_1}\left(\lambda_1 C_1 + \lambda_2 C_2\right)}{W_1 V_E{}^2}$$

$$\times\, log\, \frac{x_E + l}{x_B + l} - \frac{(\lambda_1 C_1 + \lambda_2 C_2)}{W_1 V^2{}_E}\left[\frac{C_2}{A\delta_2\,(x_E + l)} - \frac{C_2}{Al\delta_2}\right.$$

$$- \frac{\frac{M_2 C_2}{3\,W_1}\left(1+\frac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)+\frac{M_1 \lambda_2 C_2}{\lambda_1 C_1 K}}{\left(1+\frac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)^2}$$

$$- \frac{M_2 \lambda_1 C_1\left(2+\frac{C_2}{3W_1}\right)}{\lambda_2 C_2}\; log\, \frac{\frac{\lambda_1 C_1}{\lambda_2 C_2}+\frac{1}{K}}{\frac{\lambda_1 C_1}{\lambda_2 C_2}+1}$$

$$- M_2\left(1+\frac{C_2}{3\,W_1}\right)\left(1-\frac{1}{K}\right) - \frac{M_2 \lambda_1{}^2 C_1{}^2}{\lambda_2{}^2 C_2{}^2} \times$$

$$\left.\left(1-\frac{1}{K}\right)\Big/\left(\frac{\lambda_1 C_1}{\lambda_2 C_2}+\frac{1}{K}\right)\left(\frac{\lambda_1 C_1}{\lambda_2 C_2}+1\right)\right] \qquad (8)$$

(*c*) Due to a change in the proportion of the charges :—

The percentage changes in maximum pressure and muzzle velocity due to the change in the proportion of the charges when the total mass is kept constant can be derived from (*a*) and (*b*) above.

For the purpose of calculations we consider a typical gun for which (C_1 = ·941, C_2 = 1·031); (λ_1 = 1810, λ_2 = 1900); K = 1·05; A = 9·563, K_o = 151; x_E = 77·905, W_1 = 27·158 and (M_1 = 2·264, M_2 = 2·169). From this data it is obvious that the maximum pressure occurs during the first stage of burning. Table 1 gives the percentage changes in maximum pressure and muzzle velocity due to unit percent change in charge weights.

TABLE 1

Percentage changes in max. pressure and muzzle velocity due to unit per cent increase in charge weights

Unit percent increase in	Percentage change in maximum pressure	Percentage change in muzzle velocity
Charge Weight C_1 ..	+1·113	+·357
Charge Weight C_2 ..	+1·218	+·485
Proportion of Weights ..	+0·001	—·137

(*ii*) *Effect of Change in Web Sizes*:—For constant charge weights an increase in web size for any one of the propellants means a decrease in total burning surface area for that propellant and this results in decrease of pressure developed and thereby reducing the rate of burning of that propellant. Therefore the 'burnt' position for that charge is shifted towards the muzzle. This decrease in pressure also affects the generation of the gases from the other propellant and the total effect being that the maximum pressure attained is lowered and the all burnt position is also shifted towards the muzzle. The percentage change in maximum pressure due to unit percent change in D_1 is as follows :

$$\frac{\partial\,(\log p_{max})_1}{\partial\,(\log D_1)} = -\frac{2}{1+\dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}} \quad .. \quad .. \quad .. \quad .. \quad (11)$$

$$\frac{\partial\,(\log p_{max})_2}{\partial\,(\log D_1)} = -\frac{M_1\left\{2+\dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}\right\}}{\left(1+\dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)^2} - \frac{M_2}{K\left(1+\dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)} + \frac{M_2}{K} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (11a)$$

But since

$$\frac{M_1}{M} = \frac{\lambda_2 C_2}{\lambda_1 C_1 K^2}$$

So that (11a) can be put in the form

$$\frac{\partial\,(\log p_{max})_2}{\partial\,(\log D_1)} = -\frac{M_2\,\lambda_2\,C_2}{\lambda_1\,C_1\,K^2\left(1+\dfrac{\lambda_2\,C_2}{\lambda_1\,C_1\,K}\right)^2} = -\frac{M_1}{\left(1+\dfrac{\lambda_2\,C_2}{\lambda_1 C_1 K}\right)^2} \quad \ldots \quad (11b)$$

We notice that, (11) and (11b) being negative quantities, so, as mentioned before, maximum pressure falls (or rises) with an increase (or decrease) in D_1.

Also since in the ballistic equations the rate of burning constant β_1 appears with D_1 as $\frac{\beta_1}{D_1}$, so the changes due to β_1 and D_1 are equal but opposite in sign. Again similarly the changes due to D_2 and β_2 must be equal and opposite in sign and thus the change in maximum pressure is given by

$$\frac{\partial\,(\log p_{max})_1}{\partial\,(\log D_2)} = -\frac{\partial\,(\log p_{max})_1}{\partial\,(\log \beta_2)} = -\frac{2\,\dfrac{\lambda_2\,C_2}{\lambda_1\,C_1\,K}}{1+\dfrac{\lambda_2\,C_2}{\lambda_1\,C_1\,K^2}} \quad \ldots \quad (12)$$

$$\frac{\partial\,(\log p_{max})_2}{\partial\,(\log D_2)} = -\frac{\partial\,(\log p_{max})_2}{\partial\,(\log \beta_2)} = -\frac{2\,\lambda_2\,C_2}{M_2\left(\lambda_1\,C_1+\dfrac{\lambda_2\,C_2}{M_2}\right)}$$

$$-\frac{M_1\,\dfrac{\lambda_2\,C_2}{\lambda_1\,C_1\,K}}{\left(1+\dfrac{\lambda_2\,C_2}{\lambda_1\,C_1\,K}\right)^2}+\frac{M_2}{K}-\frac{2\,M_2\,\lambda_1\,C_1}{\lambda_2\,C_2}\times$$

$$\log\frac{\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2}+\dfrac{1}{K}}{\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2}+\dfrac{1}{M_2}}-\frac{M_2\,\lambda_1\,C_1}{\lambda_2\,C_2}\times$$

$$\frac{\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2}\left(\dfrac{2}{M_2}-\dfrac{1}{K}\right)+\dfrac{1}{K\,M_2}}{\left(\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2}+\dfrac{1}{K}\right)\left(\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2}+\dfrac{1}{M_2}\right)} \quad \ldots \quad (12a)$$

Again we know that the area under pressure-space curve is a measure of the muzzle energy of the shot and since the total energy available is constant and pressure-space curve for an increase in web in any one of the charge is below that of the standard conditions, so the two curves must cross at some point, Thus the area under the curve for larger size is less than that for the standard conditions and so the muzzle velocity is reduced. The percentage change in muzzle velocity due to unit percent change in D_1 or β_1 is

$$\frac{\partial (\log V_E)}{\partial (\log D_1)} = -\frac{\partial (\log V_E)}{\partial (\log \beta_1)} = \frac{(\lambda_1 C_1 + \lambda_2 C_2)}{W_1 V^2_E} \times \left[-\frac{M_1 \left(2 + \dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)}{\left(1 + \dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)^2} + \frac{M_2 \lambda_1 C_1}{\lambda_2 C_2 K \left(\dfrac{\lambda_1 C_1}{\lambda_2 C_2} + \dfrac{1}{K}\right)} - \frac{M_2}{K} \right]$$

$$= -\frac{(\lambda_1 C_1 + \lambda_2 C_2)}{W_1 V_E{}^2} \frac{M_2 \lambda_2 C_2}{\lambda_1 C_1 K^2 \left(1 + \dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)^2}$$

$$= -\frac{M_1 (\lambda_1 C_1 + \lambda_2 C_2)}{W_1 V_E{}^2 \left(1 + \dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)^2} \qquad \ldots \qquad \ldots \quad (13)$$

which is obviously negative.

Similarly the change in muzzle velocity due to unit percent change in D_2 or β_2 is

$$\frac{\partial (\log V_E)}{\partial (\log D_2)} = -\frac{\partial (\log V_E)}{\partial (\log \beta_2)} = \frac{M_2 \lambda_2 C_2}{W_1 V^2_E} - \frac{(\lambda_1 C_1 + \lambda_2 C_2)}{W_1 V^2_E} \times \left[\frac{\dfrac{M_1 \lambda_2 C_2}{\lambda_1 C_1 K}}{\left(1 + \dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)^2} - \frac{M_2}{K \left(1 + \dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)} + \frac{2 M_2 \lambda_1 C_1}{\lambda_2 C_2} \log \frac{\dfrac{\lambda_1 C_1}{\lambda_2 C_2} + \dfrac{1}{K}}{\dfrac{\lambda_1 C_1}{\lambda_2 C_2} + 1} + M_2 \left(2 - \frac{1}{K}\right) \right]$$

$$\ldots \qquad \ldots \qquad \ldots \quad (14)$$

The table given below gives the percentage changes in maximum pressure and muzzle velocity due to unit percent change in D_1 and D_2 for the particular example considered.

TABLE 2

Percentage changes in max. pressure and muzzle velocity due to unit per cent increase in web size

Unit percent increase in	Percentage change in maximum pressure	Percentage change in muzzle velocity
Web size D_1	—·954	—·177
Web Size D_2	—1·045	—·272

(*iii*) *Effect of Changes in Force Constants*:—Here we have the percentage change in maximum pressure due to unit percent change in force constant λ_1 as

$$\frac{\partial\,(\log p_{max})_1}{\partial\,(\log \lambda_1)} = \frac{2}{1+\dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

$$\frac{\partial\,(\log p_{max})_2}{\partial\,(\log \lambda_1)} = \frac{\lambda_1 C_1}{\lambda_1 C_1 + \dfrac{\lambda_2 C_2}{M_2}} + \frac{M_1}{\left(1+\dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)^2}$$

$$- \frac{M_2 \lambda_1 C_1}{\lambda_2 C_2} \log \frac{\dfrac{\lambda_1 C_1}{\lambda_2 C_2}+\dfrac{1}{K}}{\dfrac{\lambda_1 C_1}{\lambda_2 C_2}+\dfrac{1}{M_2}} - \frac{M_2 \lambda_1^2 C_1^2}{\lambda_2^2 C_2^2}$$

$$\times \frac{\dfrac{1}{M_2}-\dfrac{1}{K}}{\left(\dfrac{\lambda_1 C_1}{\lambda_2 C_2}+\dfrac{1}{K}\right)\left(\dfrac{\lambda_1 C_1}{\lambda_2 C_2}+\dfrac{1}{M_2}\right)} \quad \ldots \quad (15a)$$

On using the relation (12) this can be further simplified as

$$\frac{\partial\,(\log p_{max})_2}{\partial\,(\log \lambda_1)} = \frac{M_2\left(1+\dfrac{2\,\lambda_2 C_2}{\lambda_1 C_1 K}\right)}{K\left(1+\dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)^2} - \frac{M_2 \lambda_1 C_1}{\lambda_2 C_2} \times$$

$$\log \frac{\dfrac{\lambda_1 C_1}{\lambda_2 C_2}+\dfrac{1}{K}}{\dfrac{\lambda_1 C_1}{\lambda_1 C_2}+\dfrac{1}{M_2}} \quad \ldots \quad \ldots \quad \ldots \quad (15b)$$

But as $\dfrac{1}{K} < \dfrac{1}{M_2}$, it is obvious that this expression as well as (16) is positive and thus showing that there is an increase (or decrease) in maximum pressure due to an increase (or decrease) in λ_1. Also the percentage change in muzzle velocity due to λ_1 is

$$\frac{\partial\,(\log V_E)}{\partial\,(\log \lambda_1)} = \frac{\lambda_1 C_1}{W_1 V_E^2} \log \frac{x_E + l}{x_B + l} - \frac{(\lambda_1 C_1 + \lambda_2 C_2)}{W_1 V_E^2} \times$$

$$\left[-\frac{M_1}{\left(1+\dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}\right)^2} + \frac{M_2 \lambda_1 C_1}{\lambda_2 C_2} \times \right.$$

$$\log \frac{\dfrac{\lambda_1 C_1}{\lambda_2 C_2}+\dfrac{1}{K}}{\dfrac{\lambda_1 C_1}{\lambda_2 C_2}+1} + \frac{M_2 \lambda_1^2 C_1^2}{\lambda_2^2 C_2^2} \times$$

$$\left. \frac{\left(1-\dfrac{1}{K}\right)}{\left(\dfrac{\lambda_1 C_1}{\lambda_2 C_2}+\dfrac{1}{K}\right)\left(\dfrac{\lambda_1 C_1}{\lambda_2 C_2}+1\right)} \right] \quad \ldots \quad (16)$$

Again the change in maximum pressure due to the force constant λ_2 is given by

$$\frac{\partial\,(log\;p_{max}\,)_1}{\partial\,(log\;\lambda_2)} = \frac{2\,\dfrac{\lambda_2\,C_2}{\lambda_1\,C_1\,K}}{1+\dfrac{\lambda_2\,C_2}{\lambda_1\,C_1\,K}} \quad \cdots \quad (17)$$

$$\frac{\partial\,(log\;p_{max}\,)_2}{\partial\,(log\;\lambda_2)} = \frac{2\,\lambda_2\,C_2}{M_2\left(\lambda_1\,C_1+\dfrac{\lambda_2\,C_2}{M_2}\right)} + \frac{M_1\,\dfrac{\lambda_2\,C_2}{\lambda_1\,C_1\,K}}{\left(1+\dfrac{\lambda_2\,C_2}{\lambda_1\,C_1\,K}\right)^2}$$

$$+\;\frac{2\,M_2\,\lambda_1\,C_1}{\lambda_2\,C_2}\;log\;\frac{\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2}+\dfrac{1}{K}}{\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2}+\dfrac{1}{M_2}} + \frac{M_2\,\lambda_1\,C_1}{\lambda_2\,C_2}\times$$

$$\frac{\dfrac{2\,\lambda_1\,C_1}{\lambda_2\,C_2\,M_2}+\dfrac{1}{K}\left(\dfrac{1}{M_2}-\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2}\right)}{\left(\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2}+\dfrac{1}{K}\right)\left(\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2}+\dfrac{1}{M_2}\right)} - \frac{M_2}{K} \quad \cdots \quad (17a)$$

while the change in muzzle velocity is

$$\frac{\partial\,(log\;V_E\,)}{\partial\,(log\;\lambda_2)} = \frac{\lambda_2\,C_2}{W_1\,V_E^{\,2}}\;log\;\frac{x_E+l}{x_B+l} - \frac{(\lambda_1\,C_1+\lambda_2\,C_2)}{W_1\,V_E^{\,2}}\times$$

$$\left[-\frac{M_1\,\dfrac{\lambda_2\,C_2}{\lambda_1\,C_1\,K}}{\left(1+\dfrac{\lambda_2\,C_2}{\lambda_1\,C_1\,K}\right)^2}\right.$$

$$-\;\frac{2\,M_2\,\lambda_1\,C_1}{\lambda_2\,C_2}\;log\;\frac{\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2}+\dfrac{1}{K}}{\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2}+1}$$

$$-\;\frac{M_2\,\lambda_1^2\,C_1^2\left(1-\dfrac{1}{K}\right)}{{}_2{}^2C_2^2\left(\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2}+\dfrac{1}{K}\right)\left(\dfrac{\lambda_1\,C_1}{\lambda_2\,C_2}+1\right)}$$

$$\left.-\,M_2\left(1-\frac{1}{K}\right)\right] \quad \cdots \quad (18)$$

The percentage changes in maximum pressure and muzzle velocity due to unit percent changes in force constants for the gun considered are given in table 3.

TABLE 3

Percentage changes in max. pressure and muzzle velocity due to unit per cent changes in force constants

Unit percent increase in	Percentage change in maximum pressure	Percentage change in muzzle velocity
Force constant λ_1	$+ \cdot 954$	$+ \cdot 318$
Force constant λ_2	$+1 \cdot 045$	$+ \cdot 432$

(iv) *Effect of change in chamber capacity*:—The larger the chamber for given charges and projectile, the lesser will be the pressure and velocity. It has been shown elsewhere (1958)[7] by the author that the percentage reduction in maximum pressue and muzzle velocity in this case is independent of the shapes of the propellants in use and are given by

$$\frac{\partial\,(\log p_{max})}{\partial\,(\log K_o)} = -\left[1 - \frac{1}{K_o}\left(\frac{C_1}{\delta_1} + \frac{C_2}{\delta_2}\right)\right]^{-1} \quad \ldots \quad \ldots \quad (19)$$

(In whatever stage the maximum pressure may occur)

and

$$\frac{\partial\,(\log V_E)}{\partial\,(\log K_o)} = -\frac{(\lambda_1 C_1 + \lambda_2 C_2)\,K_o}{A\,l\,W_1\,V_E^{\,2}}\left\{1 + \frac{l}{x_E}\right\}^{-1} \quad \ldots \quad (20)$$

For the particular gun chosen these values are $-1 \cdot 307$ and $- \cdot 390$ respectively.

(v) *Effect of change in short-traval*:—An increase in shot-travel results only in a slight increase in muzzle velocity. Maximum pressure and all burnt position are unaffected (except that their relative position from the muzzle are increased). But it is not useful to increase the barrel length beyond a certain limit and it has been shown in an earlier paper (1956)[1] that even by doubling it the increase in muzzle velocity is only about 11 percent. Thus in this case the changes are given by

$$\frac{\partial\,(\log p_{max})}{\partial\,(\log x_E)} = o \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

and

$$\frac{\partial\,(\log V_E)}{\partial\,(\log x_E)} = \frac{(\lambda_1 C_1 + \lambda_2 C_2)}{W_1\,V_E^{\,2}}\left(1 + \frac{l}{x_E}\right)^{-1} \quad \ldots \quad \ldots \quad (22)$$

For the example considered the percentage change in muzzle velocity is $+\cdot 294$.

(*vi*) *Effect of change in shot-weight* :—The effect of an increase in shot weight is that the maximum pressure attained is increased and is given by

$$\frac{\partial\,(\log p_{max})_1}{\partial\,(\log W)} = \left(1 - \frac{C_1 + C_2}{3\,W_1}\right) \quad \cdots \quad (23)$$

$$\frac{\partial\,(\log p_{max})_2}{\partial\,(\log W)} = \left(1 - \frac{C_1 + C_2}{3\,W_1}\right) \left[\frac{\lambda_2 C_2}{M_2 \left(\lambda_1 C_1 + \dfrac{\lambda_2 C_2}{M_2}\right)} + \frac{M_1}{1 + \dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}} + \frac{\dfrac{\lambda_1 C_1}{\lambda_2 C_2}}{\dfrac{\lambda_1 C_1}{\lambda_2 C_2} + \dfrac{1}{M_2}} + \frac{M_2 \lambda_1 C_1}{\lambda_2 C_2} \times \log \frac{\dfrac{\lambda_1 C_1}{\lambda_2 C_2} + \dfrac{1}{K}}{\dfrac{\lambda_1 C_1}{\lambda_2 C_2} + \dfrac{1}{M_2}} - \frac{M_2}{K} \right] \quad \cdots \quad (23a)$$

Now since with the heavier projectile pressure is higher so the pressure space curves for the standard and heavier projectiles must cross over at some point. The area under the curve for the heavy shot is greater than that for the light shot and thus muzzle energy ($\frac{1}{2} W_1 V_E{}^2$) is a little greater. On balance, however, the muzzle velocity is considerably reduced. The percentage reduction in muzzle velocity due to unit percent increase in shot weight is

$$\frac{\partial\,(\log V_E)}{\partial\,(\log W)} = \left(1 - \frac{C_1 + C_2}{3\,W_1}\right) \left[- \frac{M_2 \lambda_2 C_2}{W_1 V_E{}^2} - \frac{(\lambda_1 C_1 + \lambda_2 C_2)}{W_1 V_E{}^2} \times \left\{ \log \frac{x_E + l}{x_B + l} - \frac{M_1}{1 + \dfrac{\lambda_2 C_2}{\lambda_1 C_1 K}} - \frac{M_2 \lambda_1 C_1}{\lambda_2 C_2} \times \log \frac{\dfrac{\lambda_1 C_1}{\lambda_2 C_2} + \dfrac{1}{K}}{\dfrac{\lambda_1 C_1}{\lambda_2 C_2} + 1} - M_2 \left(1 - \frac{1}{K}\right) \right\} \right] \quad \cdots \quad (24)$$

For the gun under consideration the percentage changes in maximum pressure and muzzle velocity due to one percent increase in shot weight are $+\cdot 976$ and $-\cdot 269$ respectively.

For the sake of ready reference we put all the changes due to various quantities in table 4.

TABLE 4

Percentage changes in max. pressure and muzzle velocity due to various changes (see tables 1, 2 and 3)

Percentage increase in	Percentage change in Maximum Pressure	Percentage Change in Muzzle Velocity
Charge weight C_1	+1·113	+·357
Charge weight C_2	+1·218	+·485
Proportion of charge weights	+ ·001	—·137
Web size D_1	— ·954	—·177
Web size D_2	—1·045	—·272
Force constant λ_1	+ ·954	+·318
Force constant λ_2	+1·045	+·432
Chamber capacity K_0	—1·307	—·390
Shot-travel x_E	0	+·294
Shot weight W	+ ·976	—·269

Conclusion

Finally it may be mentioned here that all those problems investigated in the earlier papers for a single charge regarding the changes in muzzle velocity while keeping maximum pressure constant have also been discussed by the author elsewhere for the case of composite charge. The variation of pV which is proportional to the rate of change of kinetic enrgy has been studied for the case of composite change consisting of n charges and condtions have been obtained for its maximum value to occur in or at the end of any stage of burning. The equation of pressure-time curve has also been obtained for this general case of composite charge.

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