SCALING THEORY FOR LIQUID PROPELLANT ROCKET THRUST CHAMBERS

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ABSTRACT

With the advent of the very large liquid propellant rocket, it has become necessary, if possible, to derive a rational scaling theory for combustion chamber design so as to enable relatively simple and economical initial tests to be carried out on small scaled models using scaled parameters of propellant mass flows, pressure etc., and from these tests to predict operating and design data for the full scale rocket. Owing to the complex and interdependent nature of the aerothermo-chemical processes in the chamber involving evaporation, difusion and chemical reaction, the similarity criteria must necessarily extend over several non-dimensional parameters, but it is still possible to evolve relatively simple rules for correlating the design and performance of the model and large scale motors—as shown by Penner-Tsien and Crocco. The paper concludes with a discussion of the accuracy and practical feasibility of such scaling rules.

NOTATIONS

A =area.

a =local speed of sound.

C* =characteristic velocity $\left(= \frac{P_c A_t g}{m} \right)$

D, d =diameter.

E =energy of activation.

F =thrust.

g =gravitational constant.

h =injector orifice diameter.

K =coefficient of thermal conductivity.

K′ =evaporation constant.

L, l =length, characteristic linear dimension.

L* =characteristic length $\left(= \frac{V_c}{A_t} \right)$

m =mass flow rate of propellants.
n =scaling factor.
P =pressure.
r =radius of throat.
t =time.
V =volume.
v =velocity.
x, y, z =constants used for indices.
γ =specific heat ratio.
ρ =density.
μ =coefficient of viscosity.
τ =conversion time.

Suffices

c =chamber conditions.
t =throat conditions.
o =initial conditions.
i =injection conditions.
H =main motor (Hauptas fuhrung)
M_o =scaled down motor (Model).

Introduction

At present there exist only two main, reasonably logically derived theories directly applicable to the scaling of liquid propellant rocket combustion chambers. Certain other contributions and proposals for scaling made for example by Ross[1] and by Barrere[2] do not appear to fit into fully developed, consistent schemes. The latter has proposed a scaling rule based on an empirical expression for the conversion time τ, (*i.e.*, the time between the injection of a propellant element and its conversion into final gaseous products), derived from motor tests with RFNA and furfuryl alcohol, $\tau \propto (\triangle p)^{-x} . P_c^{-y}$. where, $\triangle p$= injector pressure drop, and x and y are + ue indices; whereas Ross's observation on geometrical scaling of chamber shapes by keeping constant L* have only led him to the final conclusion that the rates of mass flows per unit transverse section of chamber $\left(\frac{m_p}{A} \right)$ should decrease when scaling upwards with consequent increase in τ (and diminishing rates of increase of local C* along the length of the chamber)—Which only serves to qualitatively coroborate the results of the two main theories. Finally, bare statements of requirements of chamber volumes for scaling thrusts at different pressure levels [3] such as $V_c = C . F . \frac{1}{\rho_c^x}$ where C = a constant dependent upon the energy of activation (E), and 'x' is an integer greater than two (based empirically upon

experiments with nitric acid and vinyl butyl ethers) are by no means related to similarity scaling criteria.

Similarity Criteria

The attainment of complete similarity of all processes in the combustion chamber during scaling is no doubt the ideal to be aimed at, but as the processes are complex and interdependent (see figure) this may not be absolutely feasible. Also an approach by dimensional similarity analysis cannot be fruitful in the combined presence of :

(1) Fluid mechanical phenomena
(2) Heat transfer phenomena
(3) Chemical composition change and associated liberation of heat

(These two inter-connected by the Arrhenius relation for temperature and reaction velocity).

However it is possible to formulate certain similarity parameters to be aimed at. One of the earliest classical exponents was Damkohler whose five constant ratio groups provided for similarity as shown below:—

Similar local chemical compositions and combustion patterns in H and Mo motors

(*i*) $\frac{\text{Rate of generation of items by chemical reaction}}{\text{Rate of removal by convection}}$

(*ii*) $\frac{\text{Rate of generation of items by chemical reaction}}{\text{Rate of removal by conduction}}$

Hence same turbulent transfer coefficiants

(*iii*) $\frac{\text{Inertial force acting per unit volume}}{\text{Viscous force acting per unit volume}}$

Hence heat generation patterns constant

(*iv*) $\frac{\text{Rate of generation of heat per unit volume by chemical reaction}}{\text{Rate of dissipation by convection}}$

(*v*) $\frac{\text{Rate of generation of heat per unit volume by chemical reaction}}{\text{Rate of dissipation by conduction.}}$

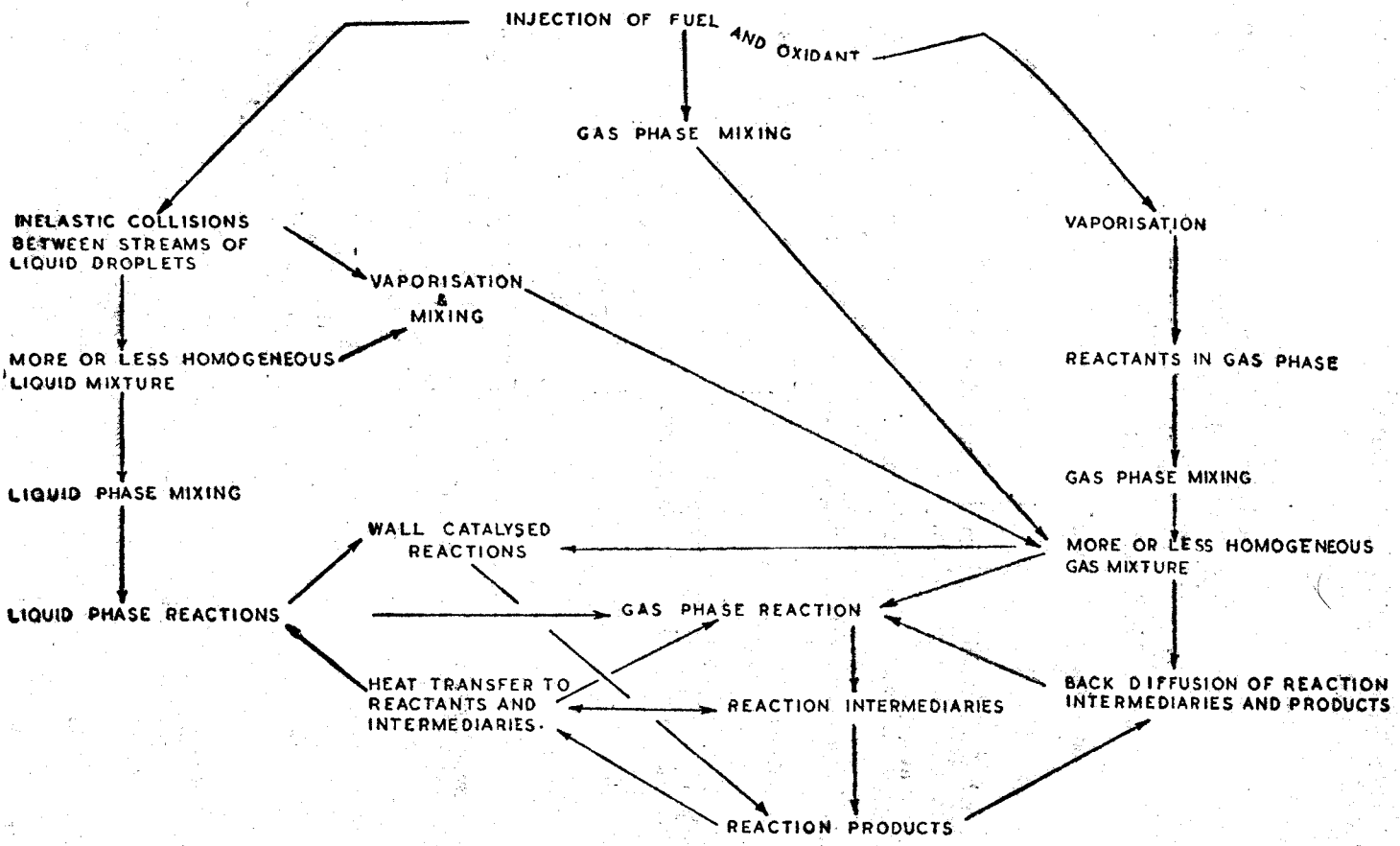

FIG:-SCHEMATIC LINE DIAGRAM OF PROCESSES AND REACTIONS OF LIQUID BIPROPELLANTS IN ROCKET CHAMBER.

Then using the same propellants in the same mixture ratios in geometrically similar combustion chambers,

(*a*) The combustion patterns will be similar.

(*b*) Dynamic behavionr inside the chamber will be similar.

The Mach No $\left(= \sqrt{\frac{\rho v^2}{\gamma P}} \right)$ is not normally an important similarity criterion except for high velocity processes involving oscillations, and since the Schmidt $\left(\frac{\mu}{\rho \times \text{Diffusivity Coeff.}} \right)$ and Prandtl Nos $\left(\frac{c_p \cdot \mu}{K} \right)$ will remain invariant following the maintenance of the same T_c (mixture ratio and propellants the same.....this will however not be constant with the varying pressures in Crocco's rule though he has assumed it to be so) and flow conditions, the major similarity groups for steady aerothermochemical similarity are:—

(1) Reynold's number

$$\therefore \; P_H v_H L_H = P M_\circ v_{M_\circ} L M_\circ \quad .. \quad .. \quad .. \quad (1)$$

(since μ is the same)

(2) Damkohler's 1st constant ratio group

$$\left(\text{basically reducing to } \frac{L/v}{\tau} \right) \quad \therefore \quad \frac{L_H}{v_H \tau_H} = \frac{L M_\circ}{v_{M_\circ} \tau_{M_\circ}} \quad .. \quad (2)$$

$$\text{From (1) and (2),} \quad n_L = \frac{1}{n_p \cdot n_v} \quad .. \quad .. \quad .. \quad (3)$$

and

$$\left[n_v = \frac{v_H}{v_{M_\circ}} \; ; \; n_L = \frac{L_H}{L M_\circ} \text{ (i.e. length, dia. \& p.c.d. of injector orifices geom. scaling factor.} \right] \quad \frac{n_L}{n_V} = n_\tau \quad .. \quad .. \quad (4)$$

$$n_\tau = \frac{1}{n_p \cdot n_v^2} \quad .. \quad (5)$$

Scaling Rules

This brings us to the two main theories postulated by—

(*a*) Crocco and

(*b*) Penner-Tsien.

The two differ only in their assumption of the relations of the conversion time τ with:—

(*i*) Pressure and

(*ii*) Propellant droplet diameters.

(*a*) *Crocco's method*

Rule I—For fixed Mach No.

$$\therefore\ v_H = v_{M_\circ}\ ;\quad n_v = 1 \qquad .. \qquad .. \qquad .. \qquad (6)$$

$$\therefore\ \tau \propto \frac{1}{P_c} \left\{ \begin{array}{l} \therefore \text{ from (4), } n_\tau = n_L \text{ } i.e.\ \tau \propto \text{geom. scaling factor} \quad .. \quad (7) \\ \text{and from (3) } n_p = \dfrac{1}{n_L} \text{ } i.e.\ P_c \propto \dfrac{1}{\text{geom. scaling factor}} \\ .. \quad .. \quad .. \quad (8) \end{array} \right.$$

$$i.e.\ n_\tau = \frac{1}{n_p} \quad .. \quad (7a)$$

Now $\dfrac{F_H}{FM_\circ} = \dfrac{\dot{m}_H}{\dot{m}_{M_\circ}}$ (similarity of combustion, friction and cooling losses) which equates flow rates through chamber, throat and injector orifices,

$$= \frac{(p \,.\, d_i{}^2 v)_H}{(p \,.\, d_i{}^2 v)_{M_\circ}} = \frac{(pr^2 \,.\, a)_H}{(pr^2 \,.\, a)_{M_\circ}} = \frac{(v_i h^2)_H}{(v_i h^2)_{M_\circ}} \qquad .. \qquad .. \qquad (9)$$

combining (1) with the first relation of (9)

$$\frac{\dot{m}_H}{\dot{m}_{M_\circ}} (= n_M) = n_L \text{ } i.e. \text{ mass flow rates of propellants} \propto \text{geom. scaling factor} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (10)$$

Combining (8) with the second relation of (9) and since 'a' is constant,

$$\frac{r_H}{rM_\circ} = n_L$$

i.e., throat radii are scaled directly with rest of geom. scaling factor.

Combining (6) with the 3rd relation of (9)

and since $\dfrac{v i_H}{v i_{M_\circ}} = \dfrac{v_H}{v_{M_\circ}} = n_v$

$$\frac{h_H}{hM_\circ} = n_L^{\frac{1}{2}}$$

i.e., dia of inj. orifices scaled directly as the square root of geom. scaling factor.

Eqns 6, 7, 8, 11 and 12 give Crocco's Rule I.

Rule II—The starting assumption is that

$$\frac{\tau_H}{\tau_{M_\circ}} = \left(\frac{PM_\circ}{P_H} \right)^z$$

i.e. that the conversion time varies inversely as the pressure to some index 'Z' nearly equal to unity. (This is the criterion over which the main arguments arise—*i.e.*, over eqn (7a) of rule I).

It has empirically been found that 'Z' is in fact nearly equal to 1·0 for many non-hypergolic bipropellants of roughly equal volatilities at the lower pressure ranges.

Then since $n_\tau = \dfrac{1}{(n_p)^z}$ .. .. (13)

Then from (4) and (1)

$$n_v = n_L^{\frac{1-z}{1+z}} \; ; \quad n_\tau = n_L^{\frac{2}{1+z}} \; ; \therefore n_p = n_L^{-2/(1+z)}$$

Also the results following from eqn (9) can be readily seen to be:—

$$\frac{\dot{m}_H}{\dot{m}_{M_o}} = n_L \; ; \quad \frac{r_H}{r_{M_o}} = n_L^{\frac{2+z+z^2}{2(1+z)^2}} \; ; \quad \frac{h_H}{hM_o} = n_L^{\frac{z}{z+1}}$$

Penner-Tsien method

For fixed chamber pressure (P_c)

$$\therefore \; n_p = 1$$

and hence from (5) $n_\tau = \dfrac{1}{{n_v}^2} = {n_L}^2$ .. .. .. (14)

and $n_v = \dfrac{1}{n_L}$ .. .. (15)

Also from combinations with the basic equation (9) we get

$\dfrac{\dot{m}_H}{\dot{m}_M} = L \left(= \dfrac{F_H}{FM_o} \right)$ which is seen to be universally true

However, $\dfrac{r_H}{r_{M_o}} = n_L^{\frac{1}{2}}$ .. .. (16)

i.e. there is a distortion from the geom. similarity for the throat.

Whereas the injector orifice diameters are now scaled geometrically linearly,

as $\dfrac{h_H}{hM_o} = n_L$ .. .. (17)

Hence with $n_p = 1$, the other eqns. 14, 15, 16 and 17 give the P-T rule.

Further the conversion time τ is related to the initial droplet diameter to give the practical application of the P-T rule as follows:—

The evaporation law[4] $D^2 = {D^2}_o - K't$ is assumed

where D_o = initial droplet dia

D = final droplet dia

K' = Evaporation constant

t = time

$\therefore$ life time of droplets $= \dfrac{{D^2}_o}{K'}$

This evaporation time is identified in the P-T rule to the conversion time τ.

$$\therefore \frac{\tau_H}{\tau_{M_\circ}} = \frac{D_\circ{}^2{}_H}{D_\circ{}^2 M_\circ}$$

From (14) therefore $\frac{(D_\circ)_H}{(D_\circ)M_\circ} = n_L$

It will hence be seen that the P-T rule will be most applicable where the evaporation rate is the controlling factor (as is usually the case in bipropellant mixtures with widely varying volatilities). The P-T rule will also be the more applicable where the effects of injection and atomisation and mixing have more effect on the general combustion process than the maintenance of similarity of the recirculation patterns as in the Crocco rule.

It must be noted that Crocco's Rule 1 seems to satisfy almost all requirements of the similarity groups τ if $\propto \frac{1}{P^{1.}}$. The complete rule also tends to give the correct scale "length of conversion patterns of droplets from injector face" (l) as,

$$\frac{l_H}{l_{H_\circ}} = \frac{(v_i \times \tau)_M}{(v_i \times \tau)_{M_\circ}} = n_v\, n_\tau = 1 \times n_L = n_L$$ (without considering droplet evaporation intentionally)

$$\left[\text{cf. in } P\text{-}T \text{ rule also } \frac{l_H}{lM_\circ} = n_v \,.\, n_\tau = \frac{1}{n_L} \,.\, n_L{}^2 = n_L \,.\right]$$

At the same time Crocco also takes into account the concurrent chemical conversion rate as being proportional to the pressure, whereas Penner neglects this chemical reaction time in relation to the evaporation time (which is considered realistic since the orders of times are known to be 10^{-7} secs. and 10^{-3} secs. respectively for several propellants and combinations).

The disadvantage, practically, of the Crocco's method is the requirement of operation at very high P_c for small models. In the P-T case there is also a similar requirement of very high injection pressures in the model but this may not pose as great an embarrassment as very high Pc values. The linear scaling of droplets size by altering surface tensions of fluids is no doubt a problem however, it is felt that by knowing the droplet sizes approximately obtaining (*e.g.*, by methods similar to the one by Meiss it may be possible to derive rational relationships of '$D_\circ$' with injection methods, or make slight allowances for known dissimilarities in the scaling based upon the known droplet scaling law (*i.e.*, with the injection pressure drop δP scaled $1/n_L$; v_{inj} scaled $1/n_L{}^2$ and h scaled n_L as required by the rule).

Finally, it is noted that both theories use a form of relation for the conversion time such as follows, and also identify it with total residence time,

$$\tau \propto D_\circ{}^{x} \,.\, P_c{}^{-y},$$

the P-T rule using $X=2$ and $Y=\infty$; the Crocco rule 1 using $x=0$ and $Y=1$. It is however felt that these oversimplifications can be logically modified to represent τ as the sum of the evaporation, diffusion and reaction times, and further to add to τ the time of stay of the final product gases (very small in well designed chambers) to get the true residence time of elements in the volume Vc. A knowledge of the proportions of each (gained by the 'quantitative combustion pattern) will then enable the true and practical scaling of rocket combustion chambers.

References

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(4) Meiss, "Correlation of experimental data on disintegration of liquid jets"—*Ind. Eng. Chem*, 47, 1690.