INTERNAL BALLISTICS OF COMPOSITE HEPTA-TUBULAR POWDERS

by

M. C. Gupta,

Maharaja's College, Jaipur

ABSTRACT

In this paper the problem of using composite hepta-tubular charges in conventional guns for a linear law of burning has been discussed. A solution of the basic equations of Internal Ballistics has been derived under certain conditions, *viz.*, the value of the ratio of specific heats, has been assumed to be the same for both the charges and the covolume corrections have not been neglected.

Introduction

The problem of using composite charges in a gun has been discussed in the theory of 'The Interior Ballistics of Guns' by Corner[1] and in the "Internal Ballistics[2]" by reducing the problem to one of an equivalent single charge. In latter only a particular case of two charges with different shapes and web sizes of the same propellant has been considered and an approximate solution has been given by making use of modified form factor.

Corner[1] has considered the more general problem of two charges of different shapes, sizes, and compositions by reducing the problem to that of a single equivalent charge with adjusted parameters. Patni and Venkatesan[3] have given a direct treatment of the general problem based on the Hunt-Hinds system under the assumptions that the covolume correction is negligible *i.e.*, covolumes of the gases equal their specific volumes, for each propellant and that the ratio of the two specific heats is the same for both the propellants. Venkatesan[4] has discussed the problem by taking into account the covolume correction and also by taking a cubic form function in the form

$$z = (1-f)\ (1+\theta f+\theta' f^2).$$

In the present paper an attempt is made to treat the problem by taking a more general cubic form function in the form

$$z = (1-y)\ (a-by-cy^2).$$

This form function holds good also in the case of the hepta-tubular powders upto the rupture of the grains, *i.e.*, during the first phase of the combustion of the hepta-tubular powder as shown by Tavernier[5, 6]. We have further discussed the solution of the equations of Internal Ballistics of composite hepta-tubular powders beyond the rupture of the grains, during which stage the form function is very complicated one.

Principal Notations

The following principal notations have been used here:—

μ = The fictitious mass of the projectile, taking into consideration the different passive resistance as well the differences of pressure between the base and the breech.

v = Velocity of the projectile.

σ = Cross-section of the arm taking into consideration the rifling.

P = Mean-pressure under which the powder burns.

n_i = Ratio of the two specific heats of the gases for the ith powder.

$$\nu_i = \frac{n_i - 1}{2}$$

z_i = Fraction of the ith powder burnt at any instant 't'.

y_i = Fraction of the thickness (web size) of the powder grains remaining at time 't'.

η_i = Covolume of the gases.

$\frac{1}{\delta_i}$ = Specific volume of the gases.

f_i = Force constant of the powder.

$\bar{\omega}_i$ = Weight of the powder.

$$r_i = \frac{\nu_i \, \mu \, v^2}{f_i \, \bar{\omega}_i}$$

A_i = Vivacity of the powder.

D'_i = Web-size.

B'_i = Linear velocity of the combustion of the powder under the unit pressure.

S_{io} = Surface of the powder initially exposed to the combustion.

S_i = Surface of the combustion at the instant 't'.

W_{io} = Initial volume of the powder.

$\varphi_i\ (z_i) = \frac{S_i}{S_{io}}$ = Function of the progressivity of the powder.

x = Shot-travel at the instant 't'.

C^* = Internal volume of the gun upto the base of the projectile at the instant 't', (*i.e.*, equal to $C' + \sigma x$)

C' = Internal volume at the instant when $t = 0$.

Equation of Internal Ballistics

The equations of Internal Ballistics using composite charges can be written as:—

(1) Resal's Equation of Energy:—

$$z_1 + Lz_2 - r_1 = \left[\frac{P\left(c' - \frac{\bar{\omega}_1}{\delta_1} - \frac{\bar{\omega}_2}{\delta_2}\right)}{f_1 \bar{\omega}_1} \right] \left\{ \frac{c^* - \frac{\bar{\omega}_1}{\delta_1} - \frac{\bar{\omega}_2}{\delta_2}}{c' - \frac{\bar{\omega}_1}{\delta_1} - \frac{\bar{\omega}_2}{\delta_2}} - \frac{\bar{\omega}_1\left(\eta_1 - \frac{1}{\delta_1}\right) z_1 + \bar{\omega}_2\left(\eta_2 - \frac{1}{\delta_2}\right) z_2}{c' - \frac{\bar{\omega}_1}{\delta_1} - \frac{\bar{\omega}_2}{\delta_2}} \right\}$$

or $z_1 + Lz_2 - r_1$ .. .. (1)

$$= \frac{P\left(c' - \frac{\bar{\omega}_1}{\delta_1} - \frac{\bar{\omega}_2}{\delta_2}\right)}{f_1 \bar{\omega}_1} [y - B_1 z_1 - B_2 z_2] \quad \text{.. (1A)}$$

(2) Equation of Inertia :

$$\mu \frac{dv}{dt} = c^*_s P \quad \text{.. .. .. (2)}$$

or $\frac{1}{2} d\,(\mu v^2) = P dc^*$ .. .. (2A)

(3) Equation of combustion—

$$\frac{dz_i}{dt} = A_i\, P\, \varphi_i\,(z_i) \quad \text{.. .. .. (3)}$$

The equations of combustion can also be taken in the form

$$D'_i \frac{dy_i}{dt} = -\beta'_i\, P, \quad \text{.. .. .. (4)}$$

where the form function in the form (z, y), is given by equations (9) and (11) of Gupta for the two phases of combustion of a hepta-tubular powder, it can easily be seen that,

$$A_i = \frac{a_i - b_i - c_i}{D'_i} \beta'_i \quad \text{.. .. .. (5)}$$

$$= (a_i - b_i - c_i)\, \beta''_i \quad \text{.. .. .. .. (5A)}$$

where

$$L = \frac{f_2 \bar{\omega}_2}{f_1 \bar{\omega}_1} \quad \text{.. .. .. (6)}$$

$$r_i = \frac{\nu_i\, \mu\, v^2}{f_i\, \bar{\omega}_i} \quad \text{.. .. .. (7)}$$

$$B_1 = \frac{\bar{\omega}_1\left(\eta_1 - \frac{1}{\delta_1}\right)}{C' - \frac{\bar{\omega}_1}{\delta_1} - \frac{\bar{\omega}_2}{\delta_2}} \quad \text{.. .. .. (8)}$$

$$B_2 = \frac{\bar{\omega}_2 \left(\eta_2 - \frac{1}{\delta_2} \right)}{C' - \frac{\bar{\omega}_1}{\delta_1} - \frac{\bar{\omega}_2}{\delta_2}} \quad \ldots \quad (9)$$

$$Y = \frac{C^* - \frac{\bar{\omega}_1}{\delta_1} - \frac{\bar{\omega}_2}{\delta_2}}{C' - \frac{\bar{\omega}_1}{\delta_1} - \frac{\bar{\omega}_2}{\delta_2}} \quad \ldots \quad (10)$$

$$\beta''_i = \frac{\beta'_i}{D'_i} \quad \ldots \quad (11)$$

Solutions of the Equations

The initial conditions for the solution of the equations are

$$\left. \begin{array}{l} t = o, \quad P = P_o, \; v = o, \\ \text{and} \\ z_i = z_{io}, \; y_i = y_{io} \end{array} \right] \quad \ldots \quad (12)$$

Let us put

$$P_c = \frac{f_1 \bar{\omega}_1 + f_2 \bar{\omega}_2}{C' - \frac{\bar{\omega}_1}{\delta_1} - \frac{\bar{\omega}_2}{\delta_2}} \quad \ldots \quad (13)$$

$$V_i \; (z_i) = \int_{z_{io}}^{z_i} \frac{dz_i}{\varphi_i \; (z_i)} \quad \ldots \quad (14)$$

$$\xi_i = \frac{\nu_i \; \sigma^2}{f_1 \bar{\omega}_1 A^2_i \; \mu} \quad \ldots \quad (15)$$

From (4) we get

$$\frac{dy_1}{dy_2} = \frac{\beta''_1}{\beta''_2} \quad \ldots \quad (16)$$

which on integration gives

$$\beta''_2 - \beta''_1 = \beta''_2 \, y_1 - \beta''_1 \, y_2$$

since $y_1 = y_2 = 1$, when combustion starts.

Also when shot starts $y_1 = y_{10}$ and $y_2 = y_{20}$,

$$\therefore \quad \beta''_2 - \beta''_1 = \beta''_2 \, y_1 - \beta''_1 \, y_2 = \beta''_2 \, y_{10} - \beta''_1 \, y_{20} \quad \ldots \quad (17)$$

$$y_2 = 1 - \frac{\beta''_2}{\beta''_1} \; (1 - y_1) \quad \ldots \quad (17A)$$

From equations (3), (4) and (14) we get

$$V_i \; (z_i) = \frac{A_i}{\beta''_i} \; (y_{io} - y_i) \quad \ldots \quad (18)$$

Also from (2), (3) and (14), we have

$$v = \frac{\sigma}{A_i \, \mu} . V_i \, (z_i) \quad \ldots \quad (19)$$

so that from (18) and (19) we get

$$v = \frac{\sigma}{\mu \, \beta''_i} \, (y_{io} - y_i) \quad \ldots \quad (20A)$$

$$\text{or } y_i = y_{io} - \frac{\mu \, \beta''_i}{\sigma} \, v \quad \ldots \quad (20B)$$

From (19), we get

$$\frac{V_1 (z_1)}{A_1} = \frac{V_2 (z_2)}{A_2} \quad \ldots \quad (21)$$

From (7), (19) and (15), we easily get

$$r_i = \xi_i \, V^2_i \, (z_i) \quad \ldots \quad (22)$$

Now the Equation of Energy (1) can be written with the help of (13) and (6) as

$$z_1 + L z_2 - \xi_1 V^2_1 = \frac{P\,(1 + L)}{P_c} \, [y - B_1 \, z_1 - B_2 \, z_2] \quad \ldots \quad (23)$$

whence

$$P = \frac{P_c}{1 + L} \left[\frac{z_1 + L z_2 - \xi_1 V^2_1}{y - B_1 \, z_1 - B_2 \, z_2} \right] \quad \ldots \quad (24)$$

where

$$V_1 = V_1 \, (z_1) \quad \ldots \quad (24A)$$

Since the pressure cannot be negative, also the quantities $B_1 \; z_1$ and $B_2 \, z_2$ involving covolume corrections are small in comparison with Y, so that the denominator is positive, the equation (24) is valid only,

if $\quad z_1 + L z_2 - \xi_1 \, V_1^2 \geqslant o$

i.e. if $\quad \xi_1 \leqslant \dfrac{z_1 + L z_2}{V_1^2}$

for all values of z_1 and z_2 lying in the interval (0, 1).

In order that the complete combustion of the powders is achieved even in infinitely long tube, we must have

$$\xi_1 \leqslant \frac{z_1 + L z_2}{V_1^2} = \xi_M$$

where ξ_M is the minimum value of $\dfrac{z_1 + L z_2}{V_1^2}$ in the ranges of z_1 and z_2 considered.

Substituting the values of v and P from equations (19) and (24) in the Equation of Inertia (2A) and with the help of (13) and (14), we get

$$\frac{dy}{dz_1} - \left(\frac{\xi_1}{\nu_1} \cdot \frac{V_1}{\varphi_1} \cdot \frac{1}{z_1 + L z_2 - \xi_1 \, V_1^2} \right) Y$$
$$= - \frac{\xi_1}{\nu_1} \cdot \frac{V_1}{\varphi_1} \cdot \frac{B_1 \, z_1 + B_2 \, z_2}{z_1 + L z_2 - \xi_1 \, V_1^2} \quad \ldots \quad (25)$$

The integrating factor

$$H = e^{-\int_{z_{10}}^{z_1} \frac{\xi_1}{\nu_1} \cdot \frac{V_1}{\varphi_1} \cdot \frac{1}{z_1 + Lz_2 - \xi_1 V_1^2} dz_1} \qquad \ldots \quad (26)$$

i.e. $$\log H = -\frac{\xi_1}{\nu_1} \int_{01_z}^{z_1} \frac{V_1}{\varphi_1} \cdot \frac{dz_1}{z_1 + Lz_2 - \xi_1 V_1^2}$$

$$= -\frac{D}{\nu_1} \qquad \ldots \quad (27)$$

where

$$D = \xi_1 \int_{z_{10}}^{z_1} \frac{V_1}{\varphi_1} \cdot \frac{dz_1}{z_1 + Lz_2 - \xi_1 V_1^2} \qquad \ldots \quad (27\text{A})$$

$$\therefore \quad H = e^{-\frac{D}{\nu_1}} \qquad \ldots \quad (28)$$

It is clear that for $z_1 = z_{10}$, $D = O$, $H = 1$ and $Y = 1$.

Then the solution of (25) is

$$\Big[Y.H \Big]_{z_{10}}^{z_1} = -\frac{\xi_1}{\nu_1} \int_{z_{10}}^{z_1} H . \frac{V_1}{\varphi_1} . \frac{B_1 z_1 + B_2 z_2}{z_1 + Lz_2 - \xi_1 V_1^2} dz_1 \qquad \ldots \quad (29)$$

$$= I \ (say) \qquad \ldots \quad (29\text{A})$$

This is a fundamental equation giving Y as a function of z_1.

(*i*) Now when a charge is burning before its rupture, the value of z_i ($i = 1,2$) can be expressed in terms of v with the help of (20B) as:—

$$z_i = z_{io} + \frac{\mu}{\sigma} \beta''_i \, v \, [b_i + a_i - 2 \, y_{io} (b_i - c_i) - 3 \, c_i \, y^2_{io}]$$

$$+ \left(\frac{\mu}{\sigma} \beta''_i \right)^2 v^2 \, [b_i - c_i + 3 \, c_i \, y_{io}]$$

$$- c_i \left(\frac{\mu}{\sigma} \beta''_i \right)^3 v^3 \qquad \ldots \quad (30)$$

$$= T_{io} + T_{i1} v + T_{i2} v^2 - T_{i3} v^3$$

$$= F_i \, (v), \ (say), \qquad \ldots \quad (31)$$

where

$$T_{io} = z_{io} \qquad \ldots \quad (31\text{A})$$

$$T_{i1} = \frac{\mu}{\sigma} \beta''_i \, [b_i + a_i - 2 \, y_{io} (b_i - c_i) - 3 \, c_i \, y^2_{io}] \qquad \ldots \quad (31\text{B})$$

$$T_{i2} = \left(\frac{\mu}{\sigma}\,\beta''_i\right)^2 \left[b_i - c_i + 3\,c_i\,y_{io}\right] \quad \ldots \quad (31C)$$

$$T_{i3} = c_i \left(\frac{\mu}{\sigma}\,\beta''_i\right)^3 \quad \ldots \quad (31D)$$

(*ii*) Also after the rupture of the grains, we have with the help of (20B),

$$y_i = \frac{m_i + 1}{m_i - 3}\left(1 - sec\,\omega_i\right) = y_{io} - \frac{\mu}{\sigma}\,\beta''_i\,v$$

so that

$$sec\,\omega_i = \left[1 - \frac{m_i - 3}{m_i + 1}\left\{y_{io} - \frac{\mu}{\sigma}\,\beta''_{iv}\right\}\right] \quad \ldots \quad (32)$$

Now after the rupture of the grains of the powder, since z_i is a function of ω_i, z_i can be expressed in terms of v with the help of (32), so that we can write

$$z_i = F_i^*(v), \;\; say \quad \ldots \quad (33)$$

Thus the equations (29) and (24) give Y (defining shot-travel) and pressure P in terms of the velocity v as z_1, z_2, V_1, φ_1, and H all can be expressed ultimately in terms of v.

From (17) we see that

(1) If $\beta_2'' > \beta_1''$, then $\beta_2''\,y_1 - \beta_1''\,y_2$ is positive, so that y_1 cannot become zero before y_2. Hence the second charge should have the rupture first.

(2) If $\beta_2'' < \beta_1''$ then $\beta_2''\,y_1 - \beta_1''\,y_2$ is negative, so that y_2 cannot become zero before y_1. Hence the first charge should have the rupture first.

(3) If $\beta_2'' = \beta_1''$, then $\beta_2''\,y_1 - \beta_1''\,y_2$ is zero, so that y_1 is always equal to y_2. Hence the two charges have simultaneous ruptures and also the simultaneous all-burnt positions.

Let us call for the sake of convenience that charge (propellant) which will have its rupture first as $\bar{\omega}_2$. Then the following different cases may arise:—

Case A—When the charges have their ruptures at different times and do not burn out simultaneously. This is to be dealt with in six parts:—

(*i*) Both the charges are burning before their respective ruptures.

(*ii*) The second charge is burning after its rupture and the first charge is burning before its rupture.

(*iii*) Both the charges are burning after their respective ruptures.

(*iv*) The second charge has been burnt out while the first charge is still burning before its rupture.

(*v*) The second charge has been burnt out while the first charge is still burning after its rupture.

(*vi*) Both the charges have been completely burnt out.

Case B—When the charges have their ruptures at the same time and burn out simultaneously. This is to be dealt with in three parts:—

(*i*) When the charges are burning before their ruptures.

(*ii*) When the charges are burning after their ruptures.

(*iii*) When the charges have been burnt out.

The above cases are now dealt with in somewhat details:—

Case A(*i*)—In this case we get from (31) and (19),

$$z_1 + Lz_2 - \xi_1 V_1^2 = F_1(v) + LF_2(v) - \xi_1 \left(\frac{A_1 \mu}{\sigma}\right)^2 v^2$$
$$= R_o + R_1 v + R_2 v^2 - R_3 v^3 \quad \ldots \quad (34)$$

where

$$R_o = z_{10} + Lz_{20} \quad \ldots \quad (34A)$$

$$R_1 = \frac{\mu}{\sigma}\left[\beta_1'' \left\{b_1 + a_1 - 2y_{10}(b_1 - c_1) - 3c_1 y^2_{10}\right\} + L\beta_2'' \left\{b_2 + a_2 - 2y_{20}(b_2 - c_2) - 3c_2 y^2_{20}\right\}\right] \quad \ldots \quad (34B)$$

$$R_2 = \frac{\mu^2}{\sigma^2}\left[\beta_1''^2 \left\{b_1 - c_1 + 3c_1 y_{10}\right\} + L\beta_2''^2 \left\{b_2 - c_2 + 3c_2 y_{20}\right\} - \xi_1 A_1^2\right] \quad \ldots \quad (34C)$$

$$R_3 = \frac{\mu^3}{\sigma^3}\left[c_1 \beta_1''^3 + L c_2 \beta_2''^3\right] \quad \ldots \quad (34D)$$

Let further

$$z_1 + Lz_2 - \xi_1 V_i^2 = R_o + R_1 v + R_2 v^2 - R_3 v^3$$
$$= (K_1 - v)\ (K_2 + v)\ (K_3 - Kv) \quad \ldots \quad (35)$$

where

$$K_1 K_2 K_3 = R_o \quad \ldots \quad (35A)$$
$$K_1 K_3 - K_2 K_3 - K K_1 K_2 = R_1 \quad \ldots \quad (35B)$$
$$K K_2 - K K_1 - K_3 = R_2 \quad \ldots \quad (35C)$$
$$K = -R_3 \quad \ldots \quad (35D)$$

Also from (31), we have

$$B_1 z_1 + B_2 z_2 = B_1 F_1(v) + B_2 F_2(v)$$
$$= N_o + N_1 v + N_2 v^2 - N_3 v^3 \text{ say} \quad \ldots \quad (36)$$

where

$$N_o = B_1 z_{10} + B_2 z_{20} \quad \ldots \quad (36A)$$

$$N_1 = \frac{\mu}{\sigma}\left[B_1 \beta_1'' \left\{ b_1 + a_1 - 2y_{10}(b_1 - c_1) - 3c_1 y_{10}^2 \right\} + B_2 \beta_2'' \left\{ b_2 + a_2 - 2y_{20}(b_2 - c_2) - 3c_2 y^2_{20} \right\}\right] \quad \ldots \quad (36B)$$

$$N_2 = \frac{\mu^2}{\sigma^2}\left[B_1 \beta_1''^2 \left\{b_1 - c_1 + 3c_1 y_{10}\right\} + B_2 \beta_2''^2 \left\{b_2 - c_2 + 3c_2 y_{20}\right\}\right] \quad \ldots \quad (36C)$$

$$N_3 = \frac{\mu^3}{\sigma^3}\left[B_1 c_1 \beta_1''^3 + B_2 c_2 \beta_2''^3\right] \quad \ldots \quad \ldots \quad \ldots \quad (36D)$$

$\therefore$ the equation (29) becomes with the help of (35), (36), (14) and (19)

$$\Big[Y.H\Big]_o^v = -M_1 \int_o^v H.\frac{v\,(N_o + N_1 v + N_2 v^2 - N_3 v^3)}{(K_1 - v)\,(K_2 + v)\,(K_3 - K\,v)}\,dv \quad \ldots \quad (37)$$

where

$$\log H = -\frac{D}{\nu_1} = -M_1 \int_o^v \frac{vdv}{(K_1 - v)\,(K_2 + v)\,(K_3 - Kv)} \quad (38)$$

with

$$M_1 = \frac{\xi_1}{\nu_1}\left(\frac{A_1 \mu}{\sigma}\right)^2 \quad \ldots \quad \ldots \quad \ldots \quad (38A)$$

$$\text{i.e. } H = \left[(K_1 - v)^{\frac{M_1 K_1}{(K_2 + K_1)\,(K_3 - KK_1)}}\,(K_2 + v)^{\frac{M_1 K_2}{(K_1 + K_2)\,(K_3 + KK_2)}}\,(K_3 - Kv)^{\frac{M_1 K_3}{(KK_1 - K_3)\,(KK_2 + K_3)}}\right] \div$$

$$\left[K_1^{\frac{M_1 K_1}{(K_2 + K_1)\,(K_3 - KK_1)}}\,K_2^{\frac{M_1 K_2}{(K_1 + K_2)\,(K_3 + KK_2)}}\,K_3^{\frac{M_1 K_3}{(KK_1 - K_3)\,(KK_2 + K_3)}}\right] \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

Initially when shot starts, $Y = 1$, $H = 1$.

$\therefore$ Equation (37) becomes

$$Y\,(K_1 - v)^{\frac{M_1 K_1}{(K_2 + K_1)\,(K_3 - KK_1)}}\,(K_2 + v)^{\frac{M_1 K_2}{(K_1 + K_2)\,(K_3 + KK_2)}}\,(K_3 - Kv)^{\frac{M_1 K_3}{(KK_1 - K_3)\,(KK_2 + K_3)}} - K_1^{\frac{M_1 K_1}{(K_2 + K_1)\,(K_3 - KK_1)}}\,K_2^{\frac{M_1 K_2}{(K_1 + K_2)\,(K_3 + KK_2)}}\,K_3^{\frac{M_1 K_3}{(KK_1 - K_3)\,(KK_2 + K_3)}}$$

$$= -M_1 I_1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (40)$$

where

$$I_1 = \int_o^v v\,(N_o + N_1 v + N_2 v^2 - N_3 v^3) \left[(K_1 - v)^{\frac{M_1 K_1}{(K_2 + K_1)(K_3 - KK_1)} - 1} (K_2 + v)^{\frac{M_1 K_2}{(K_1 + K_2)(K_3 + KK_2)} - 1} (K_3 - K v)^{\frac{M_1 K_3}{(KK_1 - K_3)(KK_2 + K_3)} - 1} \right] \quad .. \quad (40A)$$

Also from (24)

$$P = \frac{P_c}{1 + L} \left[\frac{(K_1 - v)\ (K_2 + v)\ (K_3 - Kv)}{Y - (N_o + N_1 v + N_2 v^2 - N_3 v^3)} \right] \quad .. \quad (41)$$

These equations (40) and (41) give Y (defining the shot travel x) and P in terms of v (and therefore in terms of y_1 and y_2) and are valid only before the rupture of the grains of both the charges.

Similar expressions may be obtained in other cases.

Case B—When $\beta_2'' - \beta_1''$, i.e. when the charges attain their ruptures simultaneously and burn out at the same time.

(*i*) This case has already been dealt with in case A(*i*).

(*ii*) When the charges are burning after their simultaneous ruptures.

Let the instant of simultaneous ruptures, be denoted by the suffix rr.

Then from (20B),

$$y_{10} - \frac{\mu\, \beta_1''}{\sigma}\, v_{rr} = o = y_{20} - \frac{\mu\, \beta_2''}{\sigma}\, v_{rr}$$

$$\therefore \quad v_{rr} = \frac{\sigma}{\mu\, \beta_1''}\, y_{10} = \frac{\sigma}{\mu\, \beta_2''}\, y_{20} \quad .. \quad .. \quad .. \quad .. \quad (42)$$

$\therefore$ the values of Y_{rr} and P_{rr} are given from (40) and (41) as:—

$$Y_{rr}\, (K_1 - v_{rr})^{\frac{M_1 K_1}{(K_2 + K_1)(K_3 - KK_1)}} (K_2 + v_{rr})^{\frac{M_1 K_2}{(K_1 + K_2)(K_3 + KK_2)}} (K_3 - K v_{rr})^{\frac{M_1 K_3}{(KK_1 - K_3)(KK_2 + K_3)}} - K_1^{\frac{M_1 K_1}{(K_2 + K_1)(K_3 - KK_1)}} K_2^{\frac{M_1 K_2}{(K_1 + K_2)(K_3 + KK_2)}} K_3^{\frac{M_1 K_3}{(KK_1 - K_3)(KK_2 + K_3)}}$$

$$= - M_1 I_{rr} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (43)$$

where I_{rr} is obtained from (48A) by taking the upper limit $v = v_{rr}$, and

$$P_{rr} = \frac{P_c}{1+L}\left[\frac{(K_1 - v_{rr})\ (K_2 + v_{rr})\ (K_3 - K\,v_{rr})}{Y_{rr} - (N_o + N_1 v_{rr} + N_2 v^2_{rr} - N_3 v^3_{rr})}\right] \quad \ldots \quad (44)$$

In this case from (24) and (29) the values of Y and P, shall be given by

$$\Big[Y.H\Big]_{v_{rr}}^{v} = -M_1 \int_{v_{rr}}^{v} \frac{H\,v[B_1 F_1^*(v) + B_2 F_2^*(v)]\,dv}{F_1^*(v) + L F_2^*(v) - M_1 \nu_1 v^2} \quad \ldots \quad (45)$$

with

$$\log H = -M_1 \int_{o}^{v_{rr}} \frac{v dv}{F_1(v) + L\,F_2(v) - M_1 \nu_1 v^2}$$

$$- M_1 \int_{v_{rr}}^{v} \frac{v dv}{F_1^*(v) + L\,F_2^*(v) - M_1 \nu_1 v^2} \quad \ldots \quad (45A)$$

and

$$P = \frac{P_c}{1+L}\left[\frac{F_1^*(v) + L\,F_2^*(v) - M_1 \nu_1 v^2}{Y - B_1 F_1^*(v) - B_2 F_2^*(v)}\right] \quad \ldots \quad (46)$$

Case B (*iii*)—When both the charges are completely burnt out.

Let the instant when both the charges are just burnt out simultaneously be denoted by suffix BB.

Here

$$y_{1_{min}} = y_{2_{min}}.$$

Then from (20B), we have

$$v_{BB} = \frac{\sigma}{\mu\,\beta_1''}\Big[y_{10} - y_{1_{min}}\Big] = \frac{\sigma}{\mu\,\beta_2''}\Big[y_{20} - y_{2_{min}}\Big] \quad \ldots \quad (47)$$

$\therefore$ the value of Y_{BB} and P_{BB} are given from (45) and (46) as:—

$$[Y.H]_{v_{rr}}^{v_{BB}} = -M_1 \int_{v_{rr}}^{v_{BB}} \frac{H.\,v\,[B_1 F_1^*(v) + B_2 F_2^*(v)]\,dv}{F_1^*(v) + L\,F_2^*(v) - M_1 \nu_1 v^2} \quad \ldots \quad (48)$$

with log H given by (45A)

and

$$P_{BB} = \frac{P_c}{1+L}\left[\frac{F_1^*(v_{BB}) + L\,F_2^*(v_{BB}) - M_1 \nu_1 v^2_{BB}}{y_{BB} - B_1 F_1^*(v_{BB}) - B_2 F_2^*(v_{BB})}\right] \quad \ldots \quad (49)$$

Therefore in this case after all-burnt position the values of Y and P shall be given as:—

$$(Y - B_1 - B_2)\ (1 + L - M_1 \nu_1 v^2)^{\frac{1}{2\nu_1}}$$

$$= (Y_{BB} - B_1 - B_2)\,(1 + L - M_1 \nu_1 v^2_{BB})^{\frac{1}{2\nu_1}} \quad \ldots \quad (50)$$

or $$[\, Y - B_1 - B_2 \,]^{2v_1} \, [1 + L - M_1 \nu_1 v^2 \,] = \varphi^{**}, \qquad \ldots \quad (50A)$$

where

$$\varphi^{**} = [\, Y_{BB} - B_1 - B_2 \,]^{2v_1} \, [1 + L - M_1 \nu_1 v^2{}_{BB} \,] \qquad \ldots \quad (50B)$$

and

$$P = \frac{P_e}{1 + L} \left[\frac{1 + L - M_1 \nu_1 v^2}{Y - B_1 - B_2} \right] \qquad \ldots \quad (51)$$

Also from equation (87), the value of v shall be given by

$$v^2 = \frac{1}{M_1 \nu_1} \left\{ (1+L) - \frac{(Y_{BB} - B_1 - B_2)^{2v_1} \, (1 + L - M_1 \nu_1 v^2{}_{BB})}{(Y - B_1 - B_2)^{2v_1}} \right\} \qquad (52)$$

In this case also, the muzzle velocity v_3 can be derived by putting $Y = y_3$.

Acknowledgements

The author is highly grateful to Dr. G. C. Patni, for his interest, encouragement and guidance in the preparation of this paper and to Dr. R. S. Varma, F. N. I. for making valuable suggestions.

References

1. Corner, J., *Theory of Internal Ballistics of Guns.* (John Wiley and Sons) 1950.
2. H. M. S. O. Publication, "*Internal Ballistics*" (1951).
3. Patni, G. C. & Venkatesan, N. S., *Def. Sci. J.* **3**, 51, 1953.
4. Venkatesan N. S., *Def. Sci. J.* **7**, 93, 1957.
5. Taveriner, P. *Memorial de L' Artillerie Francaise*, Tome, 30, Fas. 4, 1015—1065.
6. Tavernier, P. *Memorial de L' Artillerie Francaise*, Tome 30, 1 Fas; 117, 1956.
7. Gupta, M. C., *Def. Sci. J.* (in press) (1959).