BALLISTIC EFFECTS OF BORE RESISTANCE FOR COMPOSITE CHARGES

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ABSTRACT

In the present paper, the theory of bore resistance for composite charges consisting of n component charges with general form-function for component charges has been given to cover the cases of both simultaneous and non-simultaneous burning out of composite charges and also the possibility that resistance may arise in any stage of burning.

Introduction

Corner[1] made a valuable contribution to the important Internal Ballistics problems of bore resistance, when he gave his 'weighing—factory' method of discussing the ballistic effects of bore resistance. Later Tawakley[2] extended Corner's method for a single charge to a composite charge consisting of two component charges with form-functions in the standard quadratic form of $= (1 - f)(1 + \theta f)$. However, the ballistic effects of bore resistance were not explicitly deduced either by Corner or by Tawakley; their results having been expressed in terms of mathematical formulæ, which, in their treatment are not always very simple.

Recently Kapur[3] (subsequently referred to as paper I) extended Corners method to apply to a charge with the general form-function $z = \phi(f)$ and obtained some interesting and useful general results about the effects of bore resistance on maximum pressure, all-burnt position and muzzle velocity.

In the present paper, we have extended the earlier discussion to the case of composite changes with n components having their form-functions in the general form by using the equivalent charge method of Corner[4] and Clemmow[5] as extended by Kapur[6]. It will be seen that the present study besides being more compact, generalises the earlier discussions in the following directions:

(*i*) the composite charge may consist of n component charges instead of two,

(*ii*) each component charge may have the form-function in the general form,

(*iii*) the resistance may arise in any stage and no separate discussion is necessary for the various possibilities,

(*iv*) no separate discussion is necessary when some or all of the component charges burn out separately,

(*v*) we use throughout non-dimensional variables and constants, so that the results are of general application.

We have also stated conditions for the validity of the earlier results of Kapur[3], when extended to composite charges.

The Form-Function for the Equivalent Charge

Let the composite charge consist of n component charges, and let F_i, C_i, D_i, B_i, z_i, f_i, $z_i = {}'_i\,(f_i)$ refer to the i *th* component charge. We assume that co-volume is equal to specific volume for each component charge and that the ratio of specific heats is the same for all the component charges.

Let F, C, D, β, z, f, $z = \phi(f)$ refer to the equivalent charge, then from results proved in Kapur[6], we get

$$C = C_1 + C_2 + \ldots\ldots\ldots + C_n \quad \ldots \quad (1)$$

$$C = \frac{C_1 F_1 + C_2 F_2 + \ldots\ldots\ldots + C_n F_n}{C_1 + C_2 + \ldots\ldots\ldots + C_n} \quad \ldots \quad (2)$$

$$\frac{D}{\beta} = \frac{Dn}{\beta n} \quad \ldots \quad (3)$$

$$z = {}'_r\,(f)$$

$$= \sum_{i=1}^{r-1} \lambda_i + \sum_{i=r}^{n} \lambda_i\, \phi_i\, [1 - k_i\,(1 - f)] \quad \ldots \quad (4)$$

when $$1 - \frac{1}{k_{\wedge - 1}} \geqslant 1 - \frac{1}{k_r}, \quad \ldots \quad (5)$$

where $$\lambda_i = \frac{C_i\,'\,F_i}{CF},\ k_i = \frac{\beta_i\,'}{\beta'} = \frac{\beta_i\,/D_i}{\beta/D} [i = 1, 2, \ldots. n] \quad (6)$$

and where the charges have been so serially arranged that

$$\frac{D_1}{\beta_1} \leqslant \frac{D_2}{\beta_2} \leqslant \ldots\ldots\ldots\ldots\ldots \leqslant \frac{D_n}{\beta_n} \quad \ldots \quad (7)$$

In the particular case, when

$$\frac{D_1}{\beta_1} = \frac{D_2}{\beta_2} = \ldots\ldots\ldots\ldots\ldots = \frac{D_n}{\beta_n} \quad \ldots \quad (8)$$

all the component charges burn out simultaneously and the composite charge behaves in all respects as a single charge with form-function

$$z = \bar{'}\,(f) = \sum_{i=1}^{n} \lambda_i\ {}'_i\ (f) \quad \ldots \quad (9)$$

In this case no separate discussion is necessary and all the results for a composite charge can be deduced from those for a single charge. The same consideration requires no separate discussion when some of the component charges burn out simultaneously.

In the rest of our discussion, we can assume without loss of generality that

$$\frac{D_1}{\beta_1} < \frac{D_2}{\beta_2} < \frac{D_3}{\beta_3} < \ldots\ldots\ldots\ldots < \frac{D_n}{\beta_n} \quad \ldots \quad (10)$$

In this case there are 'n' distinct stages of burning and in the r *th* stage f goes from f_{r-1} to f_r where

$$f_o = o, \quad f_r = 1 - \frac{1}{k_r} \quad [r = 1, 2, \ldots\ldots\ldots\ldots\ldots, n] \quad \ldots \quad (11)$$

Bore Resistance : General Form-Function for Component charges.

$$\left.\begin{aligned} &\text{Now let } \xi = 1 + \frac{x}{e},\ \eta = \frac{AD}{FC\rho} v, \\ &\zeta = \frac{Al}{FC} p,\ M = \frac{A^2D^2}{FC\rho^2 w_1},\ Al = k_o - \frac{c}{\delta} \end{aligned}\right\} \quad .. \quad (12)$$

$$Z_r (1-f) = \Phi_r (f) - \tfrac{1}{2} (\bar{\gamma} - 1) M (1-f)^2, \quad .. \quad .. \quad .. \quad (13)$$

$$\Phi_r (f) = M \int_k^f \frac{1}{Z_r} df \quad .. \quad .. \quad .. \quad .. \quad .. \quad (14)$$

$$\Psi_r (f) = \frac{1}{1-f_o} \left\{ \int_k^f \frac{df}{Z_r (1-f)} + M (\gamma - 1) \int_k^f \frac{(f_o - f) df}{Z_r^{\,2} (1-f)} \right\} (15)$$

Let the resistance R occur during the l *th* stage, then up to the end of the $(l-1)$ *th* stage, the motion is undisturbed. For the r *th* stage when $r \geqslant l$, (13) of I becomes

$$\int_1^{\xi} \frac{d\xi}{\xi} = - \int_1^{f_1} \frac{Mdf}{Z_1} - \int_{f_1}^{f_2} \frac{Mdf}{Z_2} \cdots\cdots - \int_{f_{r-1}}^{f} \frac{Mdf}{Z_r}$$

$$+ \frac{R}{1-f_o} \Bigg\{ \Bigg[\int_{f_o}^{f_l} \frac{df}{Z_l (1-f)} + \int_{f_l}^{f_{l+1}} \frac{df}{Z_{R+1} (1-f)} + \cdots\cdots$$

$$+ \int_{f_{r-1}}^{f} \frac{df}{Z_r (1-f)} \Bigg] + M (\gamma - 1) \Bigg[\int_{f_o}^{f_l} \frac{(f_o - f) df}{Z_l^2 (1-f)}$$

$$+ \int_{f_l}^{f_{l+1}} \frac{(f_o - f) df}{Z_{l+1}^2 (1-f)} + \cdots\cdots\cdots + \int_{f_{r-1}}^{f} \frac{(f_o - f) df}{Z_r^{\,2} (1-f)} \Bigg] \Bigg\} (16)$$

$$\therefore \log \xi = - \Big\{ \Phi_1 (f_1) - \Phi_1 (1) + \Phi_2 (f_2) - \Phi_2 (f_1) + \cdots\cdots\cdots\cdots$$

$$\cdots\cdots + \Phi_r (f) - \Phi_l (f_{r-1}) \Big\}$$

$$+ R \Big\{ \Psi_l (f_l) - \Psi_l (f_o) + \Psi_{l+1} (f_{l+1}) - \Psi_{l+1} (f_l)$$

$$+ \cdots\cdots + \Psi_r (f) - \Psi_r (f_{r-1}) \Big\}, \quad .. \quad .. \quad .. \quad (17)$$

and

$$\triangle (\log \xi) = R \Big\{ \Psi_l (f_l) - \Psi_l (f_o) + \Psi_{l+1} (f_{l+1}) - \Psi_{l+1} (f_l) + \ldots\ldots + \Psi_r (f) - \Psi_r (f_{r-1}) \Big\} \quad (18)$$

In particular

$$\triangle (\log \xi_{\bar{2}}) = R \Big\{ \Psi_l (f_l) - \Psi_l (f_o) + \ldots\ldots + \Psi_n (f) - \Psi_n (f_{n-1}) \Big\} \quad (19)$$

where $\bar{2}$ is the suffix corresponding to all-burnt.

using (37) of paper I, we see that the maximum pressure occurs during the $r\,th$ stage, if

$$\phi_r (f) + \bar{\gamma} M (1-f) + \frac{R}{1-f_o} \Big\{ -(\bar{\gamma}-1) - (\gamma - 1) \frac{f_o - f}{Z_r (1-f)'} \Big[\phi_r{}^1 (1-f) + (\gamma - 1) M (1-f) \Big] - 1 - \frac{M(\gamma - 1)(f_o - f)}{Z_r} = o \quad \ldots \quad \ldots (20)$$

has a root lying between f_{r-1} and f_r

When $R = O$, it gives for $f_{\bar{m}}$ ($\bar{m}$ is the suffix corresponding to the instant of maximum pressure:

$$\phi_r{}'(f_{\bar{m}}) + \bar{\gamma} M (1 - f_{\bar{m}}) = o \quad \ldots \quad \ldots \quad \ldots (21)$$

With bore resistance, maximum pressure occurs when

$$f_m = f_{\bar{m}} \; A_r \; R, \quad \ldots \quad \ldots \quad \ldots \quad \ldots (22)$$

where, using (40) of I,

$$A_r = \frac{\bar{\gamma}}{(1-f_o)[\Phi_\wedge{}''(f_{\bar{m}}) - {}_{,}M],} \quad \ldots (23)$$

provided $$1 - \frac{1}{k_{r-1}} \geqslant f_{\bar{m}} + A_r R \geqslant 1 - \frac{1}{k_r} \quad \ldots (24)$$

Also from (35), (36) of I and (18) above

$$\triangle (\log \zeta) = -\frac{(\gamma-1)R}{1-f_o} \Big\{ \frac{f_o - f_l}{\Phi_l (f) - \frac{1}{2}(\gamma - 1) M (1-f)^2} + \frac{f_l - f_{l+1}}{\Phi_{l+1}(f) - \frac{1}{2}(\gamma-1)M(1-f)^2} + \ldots + \frac{f_{r-1} - f}{\Phi_r (f) - \frac{1}{2}(\gamma-1)M(1-f_r)^2} \Big\}$$
$$- R \Big\{ \Psi_l (f_l) - \Psi_l (f_o) + \Psi_{l+1} (f_{l+1}) - \Psi_{l+1} (f_l) + \ldots + \Psi_r (f) - \Psi_r (f_{r-1}) \Big\} \quad \ldots \quad \ldots (25)$$

In particular to obtain $\frac{\Delta \zeta_{\bar{m}}}{\zeta_{\bar{m}}}$, we replace f by $f_{\bar{m}}$ in (25), and to obtain $\frac{\Delta \zeta_{\bar{2}}}{\zeta_{\bar{2}}}$ we replace r by n and f by zero.

For $\Delta\left(\eta_{\bar{3}}^{\ 2}\right)$ (46 of I is modified to

$$\Delta(\eta_{\bar{3}}^{\ 2}) = -2MR\left\{1 + \frac{f_o}{1-f_o}\left(\frac{\xi_{\bar{2}}}{\xi_{\bar{3}}}\right)^{-1} + \left(\frac{\xi_{\bar{2}}}{\xi_{\bar{3}}}\right)^{-1}\left[1 - \tfrac{1}{2}(\,_{\prime} - 1)M\right]\left[\Psi_n(_o) - \Psi_n(f_{n-1}) + \Psi_{n-1}(f_{n-1}) - \Psi_{n-1}(f_{n-2}) + \ldots + \Psi_l(f_l) - \Psi_l(f_o)\right]\right. \quad (26)$$

where suffix $\bar{3}$ refers to the muzzle position

4. Particular cases of Quadratic and Cubic form-Functions :

If the form-function for the i th component charge is

$$z_i = (1 - f_i)\ (1 + \theta_i f_i),$$

the form-function for the equivalent change for the case of simultaneous burning out is

$$z = (1 - f)(1 + \theta f), \quad \ldots \quad (27)$$

where

$$\theta = \frac{C_1 F_1 \theta_1 + C_2 F_2 \theta_2 + \ldots\ldots + C_n F_n \theta_n}{C_1 F_1 + C_2 F_2 + \ldots\ldots + C_n F_n}. \quad \ldots \quad (28)$$

and all the results of Tawakley (2) for this case can be obtained by making the substitutions (1), (2), (3), (27), (28) with $n = 2$ in Corner's results for a single charge.

For non-simultaneous burning out, the form-function for the r th stage is [Kapur[6]].

$$z = \Phi_r(f) = A_r + B_r(1-f) - E_r(1-f)^2, \quad \ldots \quad (29)$$

where

$$A_r = \sum_{r-1}^{r-1} \lambda_i,\ B_r = \sum_{i=r}^{n} \lambda_i k_i (1 + \theta_i),\ E_r = \sum_{i=r}^{n} \lambda_i k_i^{\ 2} \theta_i \quad \ldots \quad (30)$$

Also in this case

$$\Phi_r(f) = \frac{M}{k_r(c_r + l_r)} \log\left[(a_r - 1 + f)^{a_r}(b_r + 1 - f)^{b_r}\right] \quad (31)$$

$$\Psi_r(f) = \frac{1}{1-f_o}\left\{\frac{1}{k_r(a_r + l_r)}\log\frac{a_r - 1 + f}{b_r + 1 - f} + \frac{M(\,.\,-1)}{k_r^{\ 2}(a_r^{\ 2} + b_r^{\ 2})}\left[\left(\frac{2a_r b_r}{a_r + b_r} + 1 - f_o\right)\log\frac{a_r - 1 + f}{b_r + 1 - f} - \left(\frac{(1-f_o)a_r - a_r^{\ 2}}{a_r - 1 + f} - \frac{(1-f_o)b_r - b_r^{\ 2}}{b_r + 1 - f}\right)\right]\right\} \quad \ldots \quad (32)$$

where

$$K_r\, a_r\, b_r = A_r\ ,\ K_r\,(a_r - b_r) = B_r\ ,\ K_r = l_r + \tfrac{1}{2}(\gamma - 1)\,M \quad .. \quad (33)$$

Making the substitutions (1), (3), (6), (27)—(32) in (18), (19), (21), (24), (25), (26), we get the results obtained for the particular case $n=2$ by Tawakley[2],

If the form-function for the i *th* component charge is

$$z_i = (1 - f_i)\ (1 + \theta_i\, f_i + \psi_i\, f_i^{\,2}) \quad .. \quad .. \quad .. \quad (34)$$

then for simultaneous burning out [Kapur[6]] .. .. .. (35)

$$\phi\,(f) = (1 - f)\,(1 + \theta f + \psi f^2),$$

where

$$\theta = \frac{\sum_{i=1}^{n} C_i\, F_i\, \theta_i}{\sum_{i=1}^{n} C_i\, F_i}\ ,\ \psi = \frac{\sum_{i=1}^{n} C_i\, F_i\, \psi_i}{\sum_{i=1}^{n} C_i\, F_i} \quad .. \quad .. \quad (36)$$

and for non-simultaneous burning out

$$\phi_r\,(f) = \Delta_r^{\,1} + B_r^{\,1}(1 - f) - E_r^{\,1}(1 - f)^2 + D_r^{\,1}(1 - f)^3 \qquad (37)$$

where

$$\left.\begin{array}{l} A_r = \sum_{i=1}^{r-1} \lambda_i\ ,\ B_{r-1} = \sum_{i=r}^{n} \lambda_i\, k_i\,(1 + \theta_i + \psi_i) \\ E_r^{\,1} = \sum_{i=r}^{n} \lambda_i\, k_i^{\,2}\,(\theta_i + 2\,\psi_i),\ D_r^{\,1} = \sum_{i=r}^{n} \lambda_i\, k_i^{\,3}\,\psi_i \end{array}\right\} \quad .. \quad (38)$$

and closed expressions for $\Phi_r\ (f)$, $\psi_r\ (f)$, $\log(\xi)$, $\Delta\,(\log \xi)$, $\Delta\,(\log \xi_{\bar{2}})$ $\Delta\,(\log \zeta)$, $\Delta\,(\log \zeta_{\bar{m}})$, $\Delta\,(\eta_3^{\,2})$, can be easily written down and are not being written here for reasons of space

5. Conditions for the Results for a singal charge when applied to composite charges.

Some important results about the ballistic effects of bore resistance on internal ballistics have been proved by [Kapur[3]] under certain assumptions. The corresponding assumptions for composite charges would be

(*i*) $1 - \tfrac{1}{2}(\gamma - 1)\,M > o$ .. .. (39*a*)

(*ii*) $\phi_r\ (f) \geqslant (1 - f)^2$ for $f_{r-1} \geqslant f \geqslant f_r$ $[r = 1, 2 \ldots n,]$ (39*b*)

(*iii*) $\phi_r\,''\,(f) - \bar{\gamma}\,M < o\ [r = 1, 2, \ldots n]$ .. .. .. (39*c*)

All the results of I would hold for composite charges subject to the above assumptions. In particular, we see that if without bore resistance, maximum pressure occurs in the r *th* stage of burning, then, with bore resistance, it may occur in the r *th* stage or $(r + 1)$ th stage or in crossing from the r *th* stage to the $(r + 1)^{th}$, but not in the $(r - 1)$ stage.

Also precisely the same sort of arguments as used for a single charge show that for an overwhelming majority of cases likely to arise in practice, the above conditions would be satisfied.

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