AN APPROXIMATE METHOD FOR ESTIMATING ROCKET TRAJECTORY ELEMENTS

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ABSTRACT

A simple approximate method is explained for deriving trajectory elements from firing data. The method is based on the use of "the principle of the rigidity of the trajectory".

Introduction

The motion of a rocket after the charge is all burnt is the same as that of a shell of the same mass and ballistic coefficient, having for its initial velocity and direction of motion those for the rocket at the instant of "all-burnt". However, there appears to be no simple method of calculating the horizontal range *on the ground* of a rocket of given ballistic characteristics fired at a given elevation or conversely of deriving the value of the ballistic coefficient from the observed ground range realised at a given elevation. The difficulty may be appreciated by a reference to Fig. 1.

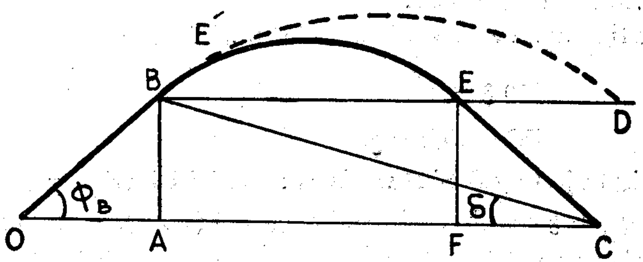

FIG. 1

The rocket is fired from O on the ground in the direction OB, B being the position of "all-burnt". It is assumed that during the period of burning, the effect of gravity may be neglected so that the trajectory up to the point B is the straight segment OB. C is the impact-point on the ground, *i.e.*, on the horizontal through the point of projection. The quantity of interest is the ground range OC. The velocity of the rocket at B can be calculated from the internal ballistics. From B onwards the trajectory of the rocket is the ballistic trajectory of a shell. Now the available ballistic tables (*e.g.*, the British 1910-law and 1940-law Tables) enable us to calculate only BE, the horizontal range through B and there is no means of calculating the portion FC corresponding to the portion EC of the trajectory, unless one resorts to a small-arc calculation of the trajectory BEC. In their recent book "*Exterior Ballistics of Rockets*", Davis, Follin and Blitzer have explained a method, called "*the method of the equivalent shell*" for calculating the range and time of flight to the impact point. Essentially, the method consists in calculating, from the state of the rocket at the end of burning, the initial conditions that a shell (of the same mass and drag as the rocket) must be given in order that it may have the same trajectory as the rocket has after burning is over. The impact conditions are then calculated from the range tables or formulae for the shell. However since the method is based on a drag function of the form cv^2 with c constant, it is restricted to cases where the all-burnt velocity is less than about 800 ft/sec. In this note we indicate an alternative approximate procedure which is applicable even when the drag function is of the general form that is usually employed in external ballistics.

The method

The basis of the present method is the observation that in nearly all cases of interest, the angle ABC will be small, of the order of about 1°. Let this angle be denoted by δ and let φ_B denote the angle that the tangent to the trajectory at B makes with the horizontal. (In view of the assumption* of the neglect of gravity during burning, φ_B will be the same as the initial angle of departure at O).

Applying the principle of the rigidity of the trajectory we see that the slant range AC for the actual trajectory BEC is the same as the horizontal range BED of a trajectory with the same initial velocity (at B) but with an angle of departure $\varphi_B + \delta$. This remarks enables us to solve the problems mentioned in the introduction, as will be explained immediately.

We now consider two types of problems:

(*i*) Suppose the problem is to determine the ballistic coefficient C_0 from the observed ground range OC.

From the known values of OA, OC & AB we deduce

$$\tan \delta = \frac{AB}{AC}$$

and $$BC = AC \sec \delta.$$

Thus for the trajectory B E′ D we know: the initial velocity v_B, the angle of departure $\varphi_B + \delta$ and the horizontal range BD (=BC). These data enable us to derive the value of C_0 by the usual methods using ballistic tables.

(*ii*) Next suppose the problem is to determine the ground range and time of flight for a rocket fired at a given elevation.

Here we do not know either the angle δ or the slant range BC. However since δ is usually small, of the order of 1°, we assume a provisional value δ_0 (δ_0 =1°, say) for δ and calculate the horizontal range BD corresponding to v_B and the angle of departure $\varphi_B + \delta_0$. By our principle above, BC=BD and we now calculate the angle ACB by sin ACB=AB/BC. If our initial choice of δ_0 as the value of δ was correct then the angle ACB must be equal to δ_0. If it is different, as we may expect it will be, we choose a slightly different value δ_1 of δ and proceed as before. This process is repeated till the calculated value of ACB and the estimate δ_i of δ coincide. This gives the correct value of δ and then

*Note that if we do not neglect the effect of gravity the conditions at B, viz. the values of φ_B, v_B, OA and OB may be deduced from the equations during the burning.

AC=BD cos δ, so that finally OC = OA + AC gives the desired range. To obtain the time of flight from B to C we may use the "cosine rule."

$$Sin E = Sin\ \varphi \cos S$$

where E & φ are the angles of projection for the same time of flight at an angle of sight S and zero respectively. Here $S = -\delta$ (with δ calculated as above) and $E = \varphi_B + \delta$ so that

$$Sin\varphi = Sin\ (\varphi_B + \delta)\ sec\ \delta$$

The time of flight from B to C is then the time of flight in the trajectory $BE'D$ corresponding to (v_B, φ and C_o).

Illustrative Examples

(*i*) Given : $\varphi_B = 9°$, $v_B = 1450$ ft/*sec*, $OB = 367$ yds.

and ground range $OC = 3800$ yds.

To derive a value for the ballistic coefficient C_o. Here

	OA	$= OB \cos 9°$	$= 363$ yds.
	AC	$= OC - OA$	$= 3437$ yds.
and	AB	$= OA Sin\ 9°$	$= 57{\cdot}41$ yds.
$\therefore$	$tan\delta$	$= AB/AC$	$= \frac{57{\cdot}41}{3437}$
whence	δ	$= 58' 25''$	
and	BC	$= AC\ sec\delta$	$= 3438$ yds.

Hence for the trajectory $BE'D$ we have

Range = BD= BC=3438 yds.

Initial velocity = v_B = 1450 ft/sec.

Angle of departure = $\varphi_B + \delta$=9° 58′ 25″.

Using the 1910-Law Tables we derive from these data the value of the ballistic coefficient:

$$\underline{C_o = 0{\cdot}86}\ (1910\text{-law})$$

(*ii*) Given: $v_B = 1450$ *ft/sec*, $\varphi_B = 9°$, $C_o = 0{\cdot}86$ (1910-law)

OB=367 yds.

To find the ground range OC.

We make a first guess for $\delta : \delta = \delta_o = 1°$.

The corresponding angle of departure for the trajectory BED is $\varphi_B + \delta_o =$ 10° . The horizontal range BD comes out to be 3456 yds. This gives

$$Sin\ ACB = Sin\delta_{cal} = AB/BC = AB/BD = \frac{57{\cdot}41}{3456}$$

so that $\delta_{cal} = 57'7''$

We try next $\delta = \delta_1 = 58'$ which gives $BD = 3449$ yds.

$$\text{and } Sin\ AOB = Sin\ \delta_{cal} = \frac{57 \cdot 41}{3449} \quad \delta_{cal} = 57'13''$$

Now try $\delta = \delta_2 = 57'25''$. We then get $BD = 3445$ yds.

$$\text{and } Sin\ \delta_{cal} = \frac{57 \cdot 41}{3445},\ \delta_{cal} = 57'18''$$

The estimated and calculated values of δ now agree fairly closely. So we take $\delta = 57'25''$,

giving $BC = BD \cos \delta = 3444$ yds.

and ground range $= OC = OA + AC = 3807$ yds.