A NOTE ON THE STABILITY CONDITION FOR A SPINNING SHELL

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ABSTRACT

It is proved that in the absence of any cross-Magnus-effects, the motion of a spinning artillery shell is stable if,

$$s \geqslant (\nu + h)^2 (1 + \gamma_2)/4\nu h.$$

where S is the usual stability factor, γ, h and γ_2 are factors of certain aerodynamic forces acting on the projectile. The above condition is obtained by applying Fowler's method of approximation to Nielsen and Synge stability conditions. If the cross force due to cross spin be neglected (i.e. $\gamma_2 = 0$), this goes over to the usual Nielsen and Synge condition of stability.

It is well known that the motion of a spinning shell has very close resemblance with that of a spinning top. For the top a condition of stable motion about the vertical exists, when,

$$s = (\text{AN})^2/4\text{B}\mu > 1 \qquad \ldots \qquad (1)$$

where, A and B are the axial and transverse moments of inertia, N the axial spin and μ the moment factor of the top with assumed symmetry about its axis. For the shell having rotational symmetry about its axis a similar condition of stability as (1) holds good. In case of the shell the spin N at the muzzle is given by $2\pi v_0/nd$, where v_0 is the muzzle velocity and n the number of calibers per one turn of rifling, and μ in case of the shell arises as moment coefficient due to air forces. A stability condition which is more general than the classical condition[1] implicit in the work of Fowler, Gallop, Lock and Richmond[2] and pointed out by Kent[3,1] is

$$s \geqslant (k + h)^2/4kh \qquad \ldots \qquad (2)$$

where h and k are the coefficients of the yawing couple and the lift force acting on the shell. In the early part of the trajectory one has h=3k approximately and therefore $s \geqslant 4/3$ which is very nearly true. Condition (2) is valid when the shell is free from Magnus forces, acting normally to the axis of the shell. This condition was further modified by Nielsen and Synge[4] as

$$s \geqslant (\nu + h)^2/4\nu h \qquad \ldots \qquad (3)$$

where $\nu = (11/10)k$ and is the usual normal force factor.

In the absence of cross-Magnus forces, we have found out in the present note, a condition of stability which is more general than the usual condition of Nielsen and Synge (i.e. inequality (3)). This condition of ours fulfills the expectations of Nielsen and Synge[4*] that the stability condition must contain the cross force due to cross spin which *theirs* does not—as we shall see later—due to the stringency of their approximations.

A set of necessary and sufficient condition for the stability of a spinning shall as laid down by Nielsen and Synge[4] is given by

$$K_1 \leqslant 0 \quad \ldots \quad (4)$$

$$K_1^2 + K_2^2 + 4K_3^2 \geqslant 0 \quad \ldots \quad (5)$$

$$K_1^2 K_3 + K_1 K_2 K_4 - K_4^2 \geqslant 0 \quad \ldots \quad (6)$$

where,

$$K_1 = X_1 + Y_1' \quad \ldots \quad (7)$$

$$K_2 = \Omega + X_2 + Y_2' \quad \ldots \quad (8)$$

$$K_3 = -\Omega X_2 + \omega X_2' + X_1 Y_1' - X_2 Y_2' - X_1' Y_1 + X_2' Y_2 \quad (9)$$

$$K_4 = \Omega X_1 - \omega X_1' + X_1 Y_2' + X_2 Y_1' - X_1' Y_2 - X_2' Y_1 \quad (10)$$

and the X and Y's are defined by the relations

$$X = P/m,\ Y = Q/m,\ X' = P'/B,\ Y' = Q'/B \quad \ldots \quad (11)$$

with, $X = X_1 + iX_2 \ldots\ldots\ldots Y = Y_1 + iY_2 \ldots\ldots$ etc. .. (12)

and $P = P_1 + iP_2 \ldots\ldots\ldots Q = Q_1 + iQ_2 \ldots\ldots$ etc. .. (13)

It may be noted that m is the mass of the shell and A, B are the usual axial and transverse moments of inertia in terms of which we define,

$$\Omega = (A/B)\, N. \quad \ldots \quad (14)$$

'Ω' is roughly of the order of 100 radians per second for the normal shell, and ω is the axial component of the velocity of the c.m. of the shell.

When the cross Magnus forces are absent we have

$$P_2 = Q_1 = P_1' = Q_2' = o \quad \ldots \quad (15)$$

whence,

$$K_1 = P_1/m + Q'/B, \quad \ldots \quad (16)$$

$$K_2 = \Omega \quad \ldots \quad (17)$$

$$K_3 = \omega P_2'/B + (1/mB)(P_1 Q_1' + P_2' Q_2) \quad \ldots \quad (18)$$

$$K_4 = \Omega P_1/m \quad \ldots \quad (19)$$

and the corresponding stability conditions are

$$P_1/m + Q_1'/B \leqslant o \quad \ldots \quad (20)$$

$$(P_1/m + Q_1'/B)^2 + \Omega^2 + 4\left\{ \omega P_2'/B + (1/mB)(P_1 Q_1' + P_2' Q_2) \right\} \geqslant o \quad (21)$$

$$(P_1/m + Q_1'/B)^2 \left\{ \omega P_2'/B + (1/mB)(P_1 Q_1' + P_2' Q_2) \right\} + (P_1/m)(P_1/m + Q_1'/B)\,\Omega^2 - \Omega^2 (P_1/m) \geqslant o \quad \ldots \quad (22)$$

* Cf. Ref. (4) pp. 217.

Using certain auxiliary parameters of Fowlder et al[2] for the aerodynamic forces we write

$$P_1 = -m\nu, \; P_2' = -\mu/\omega, \; Q_1' = -Bh \quad \ldots \quad (23)$$

and from (1) and (14) we have

$$\mu/B = \Omega^2/4s \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

One can further write,

$$Q_1 = m\omega\gamma_1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

$$Q_2 = m\omega\gamma_2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

Now with (23), (24), (25) and (26) the stability conditions, (20), (21) and (22) may be written as

$$\nu + h \geqslant o \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

$$(\nu + h)^2 + \Omega^2 - (\Omega^2/s)(1 + \gamma_2) + 4\,\nu h \geqslant o \quad \ldots \quad \ldots \quad (28)$$

$$(\nu + h)^2 \left\{ \nu h - \frac{\Omega^2}{4s}(1 + \gamma_2) \right\} + \Omega^2 \nu h \geqslant o \quad \ldots \quad \ldots \quad (29)$$

We know from Fowler et al[2] that h and k and therefore ν are of the order of unity compared to Ω, and it can be seen—in fact we have proved elsewhere that γ_1 and γ_2 are $O(1/\Omega)$. Now in (28) and (29) if we neglect such terms which are $O(1/\Omega^2)$ compared to the highest order terms in Ω retained, we find that (28) and (29) respectively give us

$$s \geqslant 1 + \gamma_2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

and

$$s \geqslant (\nu + h)^2 (1 + \gamma_2)/4\nu h \quad \ldots \quad \ldots \quad (31)$$

Obviously (30) is included in (31).

We then have (31) as out final stability condition, which goes over to the Nielsen-Synge condition when $\gamma_2 = 0$. It may be noted that the cross force, due to cross spin which is absent in Nielsen-Synge condition, appears through γ_2 in (31).

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References

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(5) Rath, P. C., On the motion of a spinning artillery shell (in Press).