STUDY OF MAXIMUM PRESSURE FOR COMPOSITE HEPTA-TUBULAR POWDERS

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ABSTRACT

In this paper the expressions for maximum pressure occurring at different positions in the case of composite hepta-tubular powders used in conventional guns and the corresponding conditions have been derived under certain conditions, viz., the value of η, the ratio of specific heats, has been assumed to be the same for both the charges and the covolume corrections have not been neglected.

Introduction

The problem of using composite charges in an orthodox gun has been discussed by the various authors Corner[1], Patni and Venkatessan[2], Patni[3], Aggarwal[4], Kapur[5], Venkatessan[6] under different conditions. The author of this paper[7] has discussed the Internal Ballistics of composite hepta-tubular powders for a linear law of burning under the assumption that the ratios of the specific heats of the component charges are the same.

In this paper an attempt has been made to find the conditions for the maximum pressure to occur at different positions in the case of composite hepta-tubular powders used in an orthodox gun under certain conditions, viz., the ratio of specific heats has been assumed to be the same for both the charges and the covolume corrections have not been neglected.

Principal Notations

The following principal notations have been used here:—

μ =The fictitious mass of the projectile, taking into consideration the different passive resistance as well the differences of pressure between the base and the breech.

v =Velocity of the projectile.

σ =Cross section of the arm taking into consideration the rifling.

P =Mean-pressure under which the powder burns.

n_i =Ratio of the two specific heats of the gases for the ith powder.

$\nu_i = \dfrac{n_i - 1}{2}$.

z_i =Fraction of the ith powder burnt at any instant 't'.

y_i =Fraction of the thickness (web size) of the powder grains remaining at time 't'.

η_i = Covolume of the gases.

$\frac{1}{\delta_i}$ =Specific volume of the gases.

f_i =Force constant of the powder.

$r_i = \frac{\nu_i \mu v^2}{f_i \bar{\omega}_i}$

A_i =Vivacity of the powder.

D'_i =Web-size.

β'_i =Linear velocity of the combustion of the powder under the unit pressure.

S_{io} =Surface of the powder initially exposed to the combustion.

S_i =Surface of the combustion at the instant 't'.

W_{io} =Initial volume of the powder.

$\varphi_i (z_i) = \frac{S_i}{S_{io}}$ =Function of the progressivity of the powder.

x =Shot-travel at the instant 't'.

C^*=Internal volume of the gun upto the base of the projectile at the instant 't', (i.e. equal to $C' + \sigma x$).

C'=Internal volume at the instant when t=0.

Basic Equations

The equations of Internal Ballistics using composite charges can be written as[7]:—

(1) Resals' Equation of Energy:—

$$z_1 + Lz_2 - r_1 = \frac{P\left(C' - \frac{\bar{\omega}_1}{\delta_1} - \frac{\bar{\omega}_2}{\delta_2}\right)}{f_1 \bar{\omega}_1} [Y - B_1 z_1 - B_2 z_2] \qquad (1)$$

(2) Equation of Inertia:—

$$\mu \frac{dv}{dt} = \sigma P \qquad (2)$$

(3) Equations of combustion:—

$$\frac{dz_i}{dt} = A_i P \varphi_i (z_i) \qquad (3)$$

The equations of combustion can also be taken in the form

$$D'_i \frac{dy_i}{dt} = -\beta'_i P \qquad (4)$$

Where

$$L = \frac{f_2 \bar{\omega}_2}{f_1 \bar{\omega}_1} \qquad (5)$$

$$r_1 = \frac{\nu_1 \mu v^2}{f_1 \bar{\omega}_1} \quad \ldots \quad (6)$$

$$B_1 = \frac{\bar{\omega}_1 \left(\eta_1 - \dfrac{1}{\delta_1} \right)}{C' - \dfrac{\bar{\omega}_1}{\delta_1} - \dfrac{\bar{\omega}_2}{\delta_2}} \quad \ldots \quad (7)$$

$$B_2 = \frac{\bar{\omega}_2 \left(\eta_2 - \dfrac{1}{\delta_2} \right)}{C' - \dfrac{\bar{\omega}_1}{\delta_1} - \dfrac{\bar{\omega}_2}{\delta_2}} \quad \ldots \quad (8)$$

$$Y = \frac{C^* - \dfrac{\bar{\omega}_1}{\delta_1} - \dfrac{\bar{\omega}_2}{\delta_2}}{C' - \dfrac{\bar{\omega}_1}{\delta_1} - \dfrac{\bar{\omega}_2}{\delta_2}} \quad \ldots \quad (9)$$

$$\beta''_i = \frac{\beta'_i}{D'_i} \quad \ldots \quad (10)$$

Now let us put

$$P_c = \frac{f_1 \bar{\omega}_1 + f_2 \bar{\omega}_2}{C' - \dfrac{\bar{\omega}_1}{\delta_1} - \dfrac{\bar{\omega}_2}{\delta_2}} \quad \ldots \quad (11)$$

$$V_i (z_i) = \int_{z_{io}}^{z_i} \frac{dz_i}{\varphi_i (z_i)} \quad \ldots \quad (12)$$

$$\xi_i = \frac{\nu_i \sigma^2}{f_1 \bar{\omega}_1 A_1^2 \mu} \quad \ldots \quad (13)$$

As the equation of energy (1) can be written with the help of (11) and (5) as

$$z_1 + L z_2 - \xi_1 V_1^2 = \frac{P(1+L)}{P_c} \left[Y - B_1 z_1 - B_2 z_2 \right] \quad \ldots \quad (14)$$

whence

$$P = \frac{P_c}{1+L} \left[\frac{z_1 + L z_2 - \xi_1 V_1^2}{Y - B_1 z_1 - B_2 z_2} \right] \quad \ldots \quad (15)$$

where

$$V_i = \xi_i V_i^2 (z_i) \quad \ldots \quad (16)$$

$$V_1 = V_1 (z_i) \quad \ldots \quad (17)$$

Also from (4), (3), (12), and (2) after simplification and satisfying the initial conditions, we have [Gupta (1959-b)]

$$y_2 = 1 - \frac{\beta''_2}{\beta''_1}(1-y_1) \quad \ldots \quad \ldots \quad (18)$$

$$V_i(z_i) = \frac{A_i}{\beta''_i}(y_{io} - y_i) \quad \ldots \quad \ldots \quad (19)$$

$$v = \frac{\sigma}{A_i\,\mu} V_i(z_i) \quad \ldots \quad \ldots \quad (20)$$

$$v = \frac{\sigma}{\mu\,\beta''_i}(y_{io} - y_i) \quad \ldots \quad \ldots \quad (21)$$

$$y_i = y_{io} - \frac{\mu\,\beta''_i}{\sigma} v \quad \ldots \quad \ldots \quad (22)$$

Maximum Pressure

From (15) we have

$$P = \frac{P_c}{1+L}\left[\frac{\zeta(y_1) + L\,\zeta(y_2) - \xi_1 \frac{A_1^2\,\mu^2}{\sigma^2} v^2}{Y - B_1\,\zeta(y_1) - B_2\,\zeta(y_2)}\right] \quad \ldots \quad (23)$$

Differentiating (23) with respect to v, and simplifying with the help of (20), (9) and (22), we have

$$\Big[Y - B_1\,\zeta(y_1) - B_2\,\zeta(y_2)\Big]^2 \frac{dp}{dv}$$

$$= \frac{P_c}{1+L}\,\frac{\mu}{\sigma}\Big[\xi_1\Big[E_1^2 v^2\Big\{E_3\,\zeta'(y_1) + E_4\,\zeta'(y_2)\Big\} - E_5 v$$

$$\Big\{Y - B_1\,\zeta(y_1) - B_2\,\zeta(y_2)\Big\}\Big] - \Big[\beta''_1\,\zeta'(y_1)\Big\{Y + E_2\,\zeta(y_2)\Big\}$$

$$+ \beta''_2\,\zeta'(y_2)\Big\{LY - E_2\,\zeta(y_1)\Big\}\Big]\Big] \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

with

$$E_1 = \frac{A_1\,\mu}{\sigma} \quad \ldots \quad (24A)$$

$$E_2 = B_1 L - B_2 \quad \ldots \quad (24B)$$

$$E_3 = B_1\,\beta''_1 \quad \ldots \quad (24C)$$

$$E_4 = B_2\,\beta''_2 \quad \ldots \quad (24D)$$

$$E_5 = \frac{A_1^2\,\mu}{\sigma}\left(2 + \frac{1}{\nu_1}\right) \quad \ldots \quad (24E)$$

Now initially for hepta-tubular powders,

$\zeta(y_1) = o, \quad \zeta'(y_1) = b_1 + c_1 - a_1,$

$\zeta(y_2) = o, \quad \zeta'(y_2) = b_2 + c_2 - a_2,$

$v = o, \quad Y = 1,$

$$\therefore \left(\frac{dp}{dv}\right)_{v=0} = \frac{P_c}{1+L}\,\frac{\mu}{\sigma}\Big[(a_1 - b_1 - c_1)\,\beta''_1 + (a_2 - b_2 - c_2)\,\beta''_2\Big] \quad (25)$$

= a positive quantity,

At the all-burnt position of both the propellants, when the propellants are burnt out at different times, we have

$$\zeta(y_1) = \zeta(y_2) = 1; \ \zeta'(y_1) = \zeta'(y_2) = o$$

$$v = v_{B1} = \frac{\sigma}{\mu\beta''_1}(y_{10} - y_{1\,min})$$

$$Y = Y_{B1}, \text{ say*}.$$

$\therefore$ from (24), we have

$$\left[Y_{B1} - B_1 - B_2\right]^2\left(\frac{dp}{dv}\right)_{v\,=\,v_{B1}}$$

$$= \frac{P_c}{1+L}\ \frac{\mu}{\sigma}\left[-E_5\, v_{B1}\,(Y_{B1} - B_1 - B_2)\right] \quad \ldots \quad \ldots \quad (26)$$

= a negative quantity.

In the case of simultaneous all-burnt position, we have

$$\zeta(y_1) = \zeta(y_2) = 1,$$

$$\zeta'(y_1) = \zeta'(y_2) = 0,$$

$$v = v_{BB}, \text{ say*}.$$

$$Y = Y_{BB}, \text{ say*}.$$

$\therefore$ from (24), we have

$$\left[Y_{BB} - B_1 - B_2\right]^2\left(\frac{dp}{dv}\right)_{v\,=\,v_{BB}}$$

$$= \frac{P_c}{1+L}\ \frac{\mu}{\sigma}\left[-A^2{}_1\,\frac{\mu}{\sigma}\,v_{BB}\left(2+\frac{1}{\nu_1}\right)(Y_{BB} - B_1 - B_2)\right] \quad (27)$$

=a negative quantity.

$\therefore$ from (25), (26) and (27) it is evident that the maximum pressure must occur before the all-burnt position, whether the propellants burn simultaneously or at different times.

Now again in the case when both the propellants burn at different times i.e. when they do not burn simultaneously, we shall have:—

(*i*) At the rupture of second charge, while the first is burning before its rupture, we have

$$v = v_{r_2} = \frac{y_{20}\,\sigma}{\mu\,\beta''_2},$$

$$y_{1r_2} = y_{10} - \frac{\beta''_1}{\beta''_2}\,y_{20},$$

$$y_2 = 0,$$

$$Y = Y_{r_2}, \text{ say†}.$$

$$\zeta(y_1) = \zeta(y_{1r_2})$$

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*Ref.—Equations (71), (84) and (85) respectively of Gupta (1959-b).

†Ref.—Equation (45) of Gupta (1959-b).

where $\zeta(y_{1r_2})$ is given by

$$\zeta(y_{1r_2}) = (1 - y_{1r_2})(a_1 - b_1 y_{1r_2} - c_1 y^2_{1r_2}),$$

$$\zeta'(y_1) = \zeta'(y_{1r_2})$$

where $\zeta'(y_{1r_2})$ is given by

$$\zeta'(y_{1r_2}) = -(a_1 + b_1) + 2(b_1 - c_1) y_{1r_2} + 3c_1 y^2_{1r_2},$$

$$\zeta(y_2) = a_2 \text{ and } \zeta'(y_2) = -(a_2 + b_2).$$

∴ from (24), the maximum pressure will occur at the rupture of the second charge, while the first is burning before its rupture,

$$\text{if } \xi_1 = \frac{\zeta'(y_{1r_2})\beta''_1[y_{r_2} + E_2a_2] - (a_2 + b_2)\beta''_2[Ly_{r_2} - E_2\zeta(y_{1r_2})]}{[E_1v^2_{r_2}\{E_3\zeta'(y_{1r_2}) - E_4(a_2 + b_2)\} - E_5v_{r_2}\{y_{r_2} - B_1\zeta(y_{1r_2}) - B_2a_2\}]} \quad \ldots \quad \ldots \quad (28)$$

The maximum pressure will occur before the rupture of second charge, while the first is burning before its rupture, *i.e.* when both the charges are burning before their respective ruptures

$$\text{if } \xi_1 > \frac{\beta''_1\zeta'(y_{1r_2})[y_{r_2} + E_2a_2] - (a_2 + b_2)\beta''_2[Ly_{r_2} - E_2\zeta(y_{1r_2})]}{[E^2_1v^2_{r_2}\{E_3\zeta'(y_{1r_2}) - E_4(a_2+b_2)\} - E_5v_{r_2}\{y_{r_2} - B_1\zeta(y_{1r_2}) - B_2a_2\}]} \quad \ldots \quad \ldots \quad (29)$$

The maximum pressure will occur after the rupture of second charge, while the first is burning before its rupture,

$$\text{if } \xi_1 < \frac{\beta''_1\zeta'(y_{1r_2})[y_{r_2} + E_2a_2] - (a_2 + b_2)\beta''_2[Ly_{r_2} - E_2\zeta(y_{1r_2})]}{[E^2_1v^2_{r_2}\{E_3\zeta'(y_{1r_2}) - E_4(a_2 + b_2)\} - E_5v_{r_2}\{y_{r_2} - B_1\zeta(y_{1r_2}) - B_2a_2\}]} \quad \ldots \quad \ldots \quad (30)$$

(*ii*) Now at the rupture of the first charge, while the second is burning after its rupture, we have

$$v = v_{r_1} = \frac{y_{10}\sigma}{\mu\beta''_1}, \text{ say*}.$$

$$y_2 = y_{2r_1} = y_{20} - \frac{\mu\beta''_2}{\sigma} v_{r_1} = y_{20} - \frac{\beta''_2}{\beta''_1} y_{10},$$

$$y_1 = 0,$$

$$y = y_{r_1}, \text{ say*}.$$

$$\zeta(y_1) = a_1, \qquad \zeta'(y_1) = (a_1 + b_1),$$

$$\zeta(y_2) = \zeta(y_{2r_1}), \qquad \zeta'(y_2) = \zeta'(y_{2r_1}),$$

where

$$\zeta(y_{2r_1}) = 1 - \frac{3(m_2 + 1)}{8\pi(m_2^2 - 7)} \left[1 + \frac{1}{\rho_2 m_2} - \frac{m_2 + 1}{4\rho_2 m_2 \cos\omega_{2r_1}}\right] H_2(\omega_{2r_1})$$

$$\sec(\omega_{2r_1}) = \left[1 - \frac{m_2 - 3}{m_2 + 1} y_{2r_1}\right]$$

†Ref.—Equations (51) and (53) respectively of Gupta (1959-b).

From (24), we easily get that the maximum pressure will occur at the rupture of first charge, while the second is burning after its rupture,

$$\text{if } \xi_1 = \frac{[-(a_1+b_1)\beta_1''\{y_{r_1} + E_2\zeta(y^{2r_1})\} + \zeta'(y_{2r_1})\beta_2''\{Ly_{r_1} - E_2a_1\}]}{[E_1^2v^2_{r_1}\{-E_3(a_1+b_1)+E_4\zeta'(y_{2r_1})\} - E_5v_{r_1}\{y_{r_1} - B_1a_1 - B_2\zeta(y_{2r_1})\}]} \quad \ldots \quad (31)$$

The maximum pressure will occur after the rupture of the first charge, while the second is burning after its rupture (*i. e.* when both the charges are burning after their respective ruptures),

$$\text{if } \xi_1 < \frac{[-(a_1+b_1)\beta_1''\{y_{r_1} + E_2\zeta(y_{2r_1})\} + \zeta'(y_{2r_1})\beta_2''\{Ly_{r_1} - E_2a_1\}]}{[E_1^2v^2_{r_1}\{-E_3(a_1+b_1)+E_4\zeta'(y_{2r_1})\} - E_5v_{r_1}\{y_{r_1} - B_1a_1 - B_2\zeta(y_{2r_1})\}]} \quad \ldots \quad \ldots \quad (32)$$

The maximum pressure will occur before the rupture of the first charge, while the second is burning after its rupture,

$$\text{if } \xi_1 > \frac{[-(a_1+b_1)\beta_1''\{y_{r_1} + E_2\zeta(y_{2r_1})\} + \zeta'(y_{2r_1})\beta_2''\{Ly_{r_1} - E_2a_1\}]}{[E_1^2v^2_{r_1}\{-E_3(a_1+b_1)+E_4\zeta'(y_{2r_1})\} - E_5v_{r_1}\{y_{r_1} - B_1a_1 - B_2\zeta(y_{2r_1})\}]} \quad \ldots \quad \ldots \quad (33)$$

(*iii*) again when the second charge is just completely burnt out, while the first is burning before its rupture, we have

$$v = v_{B_{21}} = \frac{\sigma}{\mu\beta_2''}(y_{20} - y_{2\,min}),$$

$$y_2 = y_{2\,min}, \qquad y_1 = y_{1B_{21}},$$

$$\zeta'(y_2) = \zeta'(y_{2min}) = 0,$$

$$\zeta'(y_1) = \zeta'(y_{1B_{21}})$$

where

$$\zeta'(y_{1B_{21}}) = -(a_1+b_1) + 2(b_1 - c_1)y_{1B_{21}} + 3c_1y^2_{1B_{21}}$$

$$\zeta(y_1) = \zeta(y_{1B_{21}}) = (1 - y_{1B_{21}})(a_1 - b_1y_{1B_{21}} - c_1y^2_{1B_{21}})$$

$$\zeta(y_2) = \zeta(y_{2min}) = 1$$

$$y = y_{B_{21}}, \text{ say.*}$$

$\therefore$ from (24) the maximum pressure will occur at the all-burnt position of the second charge, while the first is burning before its rupture

$$\text{if } \xi_1 = \frac{\zeta'(y_{1B_{21}})\beta_1''[y_{B_{21}} + E_2]}{E_1^2v^2_{B_{21}}\{E_3\zeta'(y_{1B_{21}})\} - E_5v_{B_{21}}\{y_{B_{21}} - B_1\zeta(y_{1B_{21}}) - B_2\}} \quad \ldots \quad \ldots \quad (34)$$

The maximum pressure will occur after the all-burnt position of the second charge while the first is just at the point of its rupture,

$$\text{if } \xi_1 = \frac{(a_1+b_1)\beta_1''\{y_{r_1} + E_2\}}{E_1^2v^2_{r_1}\{E_3(a_1+b_1)\} + E_5v_{r_1}\{y_{r_1} - B_1a_1 - B_2\}} \quad \ldots \quad \ldots \quad (35)$$

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* Ref.—Equation (59) of Gupta (1959-b).

The maximum pressure will occur after the all-burnt position of the second charge while the first is burning after its rupture

$$\text{if } \xi_1 < \frac{(a_1+b_1)\beta_1''\{y_{r_1}+E_2\}}{E_1^2 v^2_{r_1}\{E_3(a_1+b_1)\} + E_5 v_{r_1}\{y_{r_1} - B_1 a_1 - B_2\}} \quad \cdots \quad \cdots \quad (36)$$

(*iv*) When the propellants are burning simultaneously, we have at their simultaneous ruptures,

$v = v_{rr}$, say*

$y = y_{rr}$, say*

$\zeta(y_1) = a_1, \quad \zeta(y_2) = a_2,$

$\zeta'(y_1) = -(a_1+b_1), \quad \zeta'(y_2) = -(a_2+b_2)$

∴ from (24) the maximum pressure will occur at their simultaneous ruptures

$$\text{if } \xi_1 = \frac{(a_1+b_1)\beta_1''\{y_{rr}+E_2 a_2\} + (a_2+b_2)\beta_2''\{L y_{rr} - E_2 a_1\}}{E_1^2 v^2_{rr}\{E_3(a_1+b_1) + E_4(a_2+b_2)\} + \{E_5 v_{rr}(y_{rr} - B_1 a_1 - B_2 a_2)\}} \quad \cdots \quad \cdots \quad (37)$$

The maximum pressure will occur before their simultaneous ruptures

$$\text{if } \xi_1 > \frac{(a_1+b_1)\beta_1''\{y_{rr}+E_2 a_2\} + (a_2+b_2)\beta''_2\{L y_{rr} - E_2 a_1\}}{E_1^2 v^2_{rr}\{E_3(a_1+b_1) + E_4(a_2+b_2)\} + \{E_5 v_{rr}(y_{rr} - B_1 a_1 - B_2 a_2)\}} \quad \cdots \quad \cdots \quad (38)$$

The maximum pressure will occur after their simultaneous ruptures,

$$\text{if } \xi_1 < \frac{(a_1+b_1)\beta_1''\{y_{rr}+E_2 a_2\} + (a_2+b_2)\beta_2''\{L y_{rr} - E_2 a_1\}}{E_1^2 v^2_{rr}\{E_3(a_1+b_1) + E_4(a_2+b_2)\} + \{E_5 v_{rr}(y_{rr} - B_1 a_1 - B_2 a_2)\}} \quad \cdots \quad \cdots \quad (39)$$

The three conditions (37), (38) and (39) reduce to (65), (63) and (64) respectively of Gupta (1959-a) the present author when only one charge is taken into consideration *i.e.* when the values of all the terms for the second charge and L are put equal to zero. The conditions for the maximum pressure in the case of a single charge as deduced by the author can also be obtained from (31), (33) and (32) easily.

The maximum pressure at any instant is given by (15) as :—

$$P = \frac{P_c}{1+L}\left[\frac{z_1 + L z_2 - \xi_1 V_1^2}{y - B_1 z_1 - B_2 z_2}\right] \quad \cdots \quad \cdots \quad (15)$$

where at the point of maximum pressure $dP=0$.

∴ differentiating (15) logarithmically and putting $dP = 0$, we, get

Ref.—Equations (79) and (80) respectively of Gupta (1959-b).

$$\frac{dz_1 + Ldz_2 - \xi_1 2V_1(z_1)\dfrac{1}{\varphi_1(z_1)}dz_1}{z_1 + Lz_2 - \xi_1 V_1^2(z_1)} + \frac{B_1 dz_1 + B_2 \dfrac{dz_2}{dz_1}dz_1}{y - B_1 z_1 - B_2 z_2} = \frac{dy}{y - B_1 z_1 - B_2 z_2} \quad .. \quad .. \quad (40)$$

But from (25) of Gupta (1959-b)

$$\frac{dy}{dz_1} - \left(\frac{\xi_1}{\nu_1}\frac{V_1}{\varphi_1}\cdot\frac{1}{z_1 + Lz_2 - \xi_1 V_1^2}\right)y = -\frac{\xi_1}{\nu_1}\frac{V_1}{\varphi_1}\cdot\frac{B_1 z_1 + B_2 z_2}{z_1 + Lz_2 - \xi_1 V_1^2} \quad .. \quad .. \quad (40A)$$

$\therefore$ from (3), (40) and (40A), we get

$$\frac{1 + L\dfrac{A_2\varphi_2(z_2)}{A_1\varphi_1(z_1)} - 2\xi_1\dfrac{V_1}{\varphi_1}}{z_1 + Lz_2 - \xi_1 V_1^2} + \frac{B_1 + B_2\dfrac{A_2\varphi_2(z_2)}{A_1\varphi_1(z_1)}}{Y - B_1 z_1 - B_2 z_2} = \frac{\xi_1 V_1}{\nu_1\varphi_1[z_1 + Lz_2 - \xi_1 V_1^2]} \quad .. \quad (40B)$$

$\therefore$ at the point of maximum pressure, we get (denoting the values at maximum pressure by the suffix m)

$$\frac{\dfrac{\xi_1 V_{1m}}{\varphi_{1m}}\left(2 + \dfrac{1}{\nu_1}\right) - L\dfrac{A_2\varphi_{2m}}{A_1\varphi_{1m}} - 1}{z_{1m} + Lz_{2m} - \xi_1 V^2_{1m}} = \frac{B_1 + B_2[A_2\varphi_{2m}(z_{2m})/A_1\varphi_{1m}(z_{1m})]}{y_m - B_1 z_{1m} - B_2 z_{2m}} \quad .. \quad .. \quad (41)$$

Here the quantities φ_{1m}, φ_{2m}, z_{1m}, z_{2m}, all can be expressed as functions of y_{im} ($i = 1, 2$) which in turn is a function of v_m, (v_m being the value of v at the point of maximum pressure) given by (22) as :—

$$y_{im} = y_{io} - \frac{\mu\beta''_i}{\sigma}v_m \quad .. \quad .. \quad .. \quad (42)$$

Also from (20), V_{1m} is function of v_m given as

$$V_{1m} = \frac{A_1\mu}{\sigma}v_m \quad .. \quad .. \quad .. \quad .. \quad (43)$$

Thus (41) is a relation between V_m and Y_m.

This equation combined with (29) of Gupta (1959-b) *viz.*,

$$\Big[Y.H\Big]_{z_{10}}^{z_1} = -\frac{\xi_1}{\nu_1}\int_{z_{10}}^{z_1} H\cdot\frac{V_1}{\varphi_1}\;\frac{B_1 z_1 + B_2 z_2}{z_1 + Lz_2 - \xi_1 V_1^2}dz_1 \quad .. \quad (44A)$$

gives the value of V_m and Y_m at the point of maximum pressure.

Hence the maximum pressure P_m is given from (15) as:—

$$P_m = \frac{P_c}{1+L}\left[\frac{z_{1m} + Lz_{2m} - \xi_1 V^2_{1m}}{y_m - B_1 z_{1m} - B_2 z_{2m}}\right] \quad \ldots \quad (44B)$$

Or from (41), we have

$$P_m = \frac{P_c}{1+L}\left[\frac{\xi_1 \dfrac{V_{1m}}{\varphi_{1m}}\left(2+\dfrac{1}{\nu_1}\right) - L\,\dfrac{A_2\varphi_{2m}}{A_1\varphi_{1m}} - 1}{A_1B_1\varphi_{1m} + A_2B_2\varphi_{2m}}\right] A_1\varphi_{1m} \quad (45)$$

Now the maximum pressure may occur when,

(*a*) both the charges are burning before their respective ruptures,

(*b*) the second charge is burning after its rupture and, the first charge before its rupture,

(*c*) the second charge is burning after its rupture and the first is just at the point of its rupture,

(*d*) both the charges are burning after their respective ruptures,

(*e*) the second charge has been burnt out and the first is burning before its rupture,

(*f*) the second charge has been burnt out and the first is just at the point of its rupture,

(*g*) the second charge has been burnt out and the first charge is burning after its rupture.

Case (*a*):—In this case P_m is given by equation (44*B*) which with the help of (35) and (36) of Gupta (1959-*b*) reduces to

$$P_m = \frac{P_c}{1+L}\left[\frac{(K_1 - v_m)\ (K_2 + v_m)\ (K_3 - K\,v_m)}{Y_m - (N_o + N_1 v_m + N_2 v^2_m - N_3 v^3_m)}\right] \quad \ldots \quad (46)$$

where Y_m and v_m are given from (44*A*) and (41).

The equation (44*A*) in this case reduces to (40) of Gupta (1959-b), where $v = v_m$ and $Y = Y_m$.

Also by substituting the values of z_{1m}, z_{2m}, φ_{1m} φ_{2m} and V_{1m} in terms of v_m, the equation (41) after simplification becomes,

$$\frac{\xi_1\left(2+\dfrac{1}{\nu_1}\right) Z(v_m) - L\,\dfrac{A_2}{A_1}\,Z^*(v_m) - 1}{(K_1 - v_m)\ (K_2 + v_m)\ (K_3 - K\,v_m)}$$

$$= \frac{B_1 + B_2\,\dfrac{A_2}{A_1}\,Z^*(v_m)}{[\,Y_m - (N_o + N_1 v_m + N_2 v^2_m - N_3 v^3_m)\,]} \quad \ldots \quad \ldots \quad (47)$$

where

$$Z(v_m) = \frac{V_{1m}}{\varphi_{1m}} = \frac{\dfrac{A_1\mu_1}{\sigma}\,v_m}{\alpha_1 - \beta_1 y_{1m} - \gamma_1 y^2_{1m}}$$

$$Z^*(v_m) = \frac{\varphi_{2m}}{\varphi_{1m}} = \frac{\alpha_2 - \beta_2 y_{2m} - \gamma_2 y^2_{2m}}{\alpha_1 - \beta_1 y_{1m} - \gamma_1 y^2_{1m}} \quad \ldots \quad \ldots \quad \ldots \quad (47A)$$

with

$$y_{im} = y_{io} - \frac{\mu \beta''_i}{\sigma} v_m .$$

Conditions:—The conditions for occurrence for maximum pressure in this stage are:—

(i) $\beta''_2 > \beta''_1$

(ii) $z_{1m} < a_1$ and $z_{2m} < a_2$

i.e. $y_{1m} > o$ and $y_{2m} > o$

i.e. $v_m \leqslant \frac{y_{2o}\ \sigma}{\mu \beta''_2}$

and (iii) condition given by (29).

Case (b) :—In this case P_m is given by equation (44), which with the help of (47) and (48) of Gupta (1959-b) reduces to

$$P_m = \frac{P_c}{1 + L} \left[\frac{F_1 (v_m) + LF_2^* (v_m) - M_1 \nu_1 v^2_m}{Y_m - B_1 F_1 (v_m) - B_2 F_2^* (v_m)} \right] \quad .. \quad .. \quad (48)$$

where v_m is given by (41), while Y_m is given by (49) of Gupta (1959-b).

The quantities involved in (41) in this case also being functions of v_m given as

$$\frac{V_{1m}}{\varphi_{1m}} = Z (v_m) \quad \text{from} \quad (45).$$

$$\varphi_{2m} = \text{a function of } v_m .$$

$$= \theta^*_2 (v_m).$$

$\therefore$ from $\varphi_i (z_i) = \alpha_i - \beta_i y_i - \gamma_i y^2_i$, we have

$$\frac{\varphi_{2m}}{\varphi_{1m}} = \frac{\theta^*_2 (v_m)}{\alpha_1 - \beta_1 y_{1m} - \gamma_1 y^2_{1m}}$$

$$= Z_2^* (v_m)$$

$$z_{1m} + Lz_{2m} - \xi_1 V^2_{1m} = F_1 (v_m) + L F_2^* (v_m) - M_1 \nu_1 v^2_m$$

$$Y_m - B_1 z_{1m} - B_2 z_{2m} = Y_m - B_1 F_1 (v_m) - B_2 F_2^* (v_m)$$

Conditions :—The conditions for the occurrence of maximum pressure during this stage are:—

(i) $\beta''_2 > \beta''_1$

(ii) $z_{2m} > a_2$ and $z_{1m} < a_1$

i.e. $y_{2m} < o$ and $y_{1m} > o$

i.e. $\frac{\sigma}{\mu \beta_2''} y_{2o} < v_m < \frac{\sigma}{\mu \beta''_1} y_1$

and (iii) conditions given by (30) and (33).

Case (*c*) :—In this case P_m is given by equation (44B) which with the help of (47) and (48) of Gupta (1959-b) reduces to

$$P_m = \frac{P_c}{1+L}\left[\frac{F_1(v_{r_1}) + L\,F_2^*(v_{r_1}) - M_1\,\nu_1\,v^2_{r_1}}{Y_{r_1} - B_1\,F_1(v_{r_1}) - B_2\,F_2^*(v_{r_1})}\right] \quad .. \quad (49)$$

$$= P_{r_1} \quad \text{say*}.$$

Here Y_{r_1}, and v_{r_1}, are given by (53) and (51) of Gupta (1959-b).

Conditions :—The conditions for the occurrence of maximum pressure at this point are

(*i*) $\beta''_2 > \beta''_1$

(*ii*) $z_{1m} = a_1$ and $a_2 < z_{2m} < 1$

i.e. $y_{1m} = o$ and $y_{2m} < o$

i.e. $v_m = v_{r_1} = \dfrac{y_{10}\,\sigma}{\mu\,\beta''_1}$.

and (*iii*) that given by (31).

Case (*d*) :—In this case P_m is given by equation (44B) which with the help of (55) and (56) of Gupta (1959-b) reduces to

$$P_m = \frac{P_c}{1+L}\left[\frac{F_1^*(v_m) + L\,F_2^*(v_m) - M_1\,\nu_1\,v^2_m}{Y_m - B_1\,F_1^*(v_m) - B_2\,F_2^*(v_m)}\right] \quad .. \quad (50)$$

Here Y_m and v_m are given by (57) of Gupta (1959-b) and (41) respectively as explained above.

Conditions :—The conditions for the occurrence of maximum pressure during this stage are

(*i*) $\beta''_2 > \beta''_1$

(*ii*) $a_1 < z_{1m} < 1$ and $a_2 < z_{2m} < 1$

i.e. $y_{1m} < o$ and $y_{2m} < o$

i.e. $v_m > \dfrac{y_{10}\,\sigma}{\mu\,\beta''_1}$.

and (*iii*) that given by (32).

Case (*e*) :—In this case P_m is given by equation (44B) which with the help of (61) and (62) of Gupta (1959-b) reduces to

$$P_m = \frac{P_c}{1+L}\left[\frac{F_1(v_m) + L - M_1\,\nu_1\,v^2_m}{Y_m - B_1\,F_1(v_m) - B_2}\right] \quad .. \quad .. \quad (51)$$

or from (37), we have

$$P_m = \frac{P_c}{1+L}\left[\frac{(L + T_{10}) + T_{11}\,v_m + (T_{12} - M_1\,\nu_1)\,v^2_m - T_{13}\,v^3_m}{Y_m - B_1\,(T_{10} + T_{11}\,v_m + T_{12}\,v^2_m - T_{13}\,v^3_m) - B_2}\right] \quad (52)$$

* Ref:—Equation (54) of Gupta (1959-b).

where Y_m and v_m are given by (63) of Gupta (1959-b) and (41) respectively, which with the help of (31) of Gupta (1959-b) in this case reduce to

$$[Y \cdot H]_{v_{B_{21}}}^{v_m} = -M_1 \int_{v_{B_{21}}}^{v_m} \frac{H \cdot v\,[(B_1 T_{10} + B_2) + T_{11}\, v + T_{12}\, v^2 - T_{13}\, v^3]\,dv}{(L + T_{10}) + T_{11}\, v + (T_{12} - M_1 \nu_1)\, v^2 - T_{13}\, v^3} \quad .. \quad .. \quad .. \quad .. \quad (53)$$

and

$$\frac{\xi_1 \left(2 + \frac{1}{v_1}\right) Z(v_m) - 1}{(L + T_{10}) + T_{11}\, v_m + (T_{12} - M_1\, v_1)\, v^2_m - T_{13}\, v^3_m} = \frac{B_1}{Y_m - B_1 (T_{10} + T_{11}\, v_m + T_{12}\, v^2_m - T_{13}\, v^3_m) - B_2} \quad .. \quad (54)$$

Conditions :—In this case the conditions for occurrence of maximum pressure are

(i) $\beta''_2 > \beta''_1$

(ii) $z_{1m} < a_1$ aud $z_{2m} = 1$

i.e. $y_{1m} > o$ and $y_{2m} = y_{2\ min.}$

i.e. $v_m = \dfrac{\sigma}{\mu\, \beta''_2}\, [y_{20} - y_{2\ min.}] < \dfrac{y_{10}\, \sigma}{\mu \beta''_1}$

and (*iii*) that given by (34).

Case (*f*) :—In this case when the second charge is completely burnt out and the first is just at the point of its rupture, the pressure P_m is given by (49)

Conditions :—The conditions for the occurrence of maximum pressure at this stage are

(i) $\beta''_2 > \beta''_1$

(ii) $z_{1m} = a_1$ and $z_{2m} = 1$

i.e. $y_{1m} = o$ and $y_{2m} = y_{2min}$

i.e. $v_m = v_{r_1} = \dfrac{y_{10}\sigma}{\mu \beta_1''}$

and (*iii*) that given by (35).

Case (*g*) :—In this case P_m is given by (44B). The quantities y_m and v_m are given by (69) of Gupta (1959-b) and (41) respectively as explained above.

Conditions :—The conditions for occurrence of maximum pressure during this stage are

(i) $\beta''_2 > \beta''_1$

(ii) $1 > z_{1m} > a_1$ and $z_{2m} = 1$

i.e. $y_{1m} < o$ and $y_{2m} = y_{2min}$

i.e. $v_m = \dfrac{\sigma}{\mu \beta''_2}\, [y_{20} - y_{2\min}] > \dfrac{y_{10}\sigma}{\mu \beta''_1}$

and (*iii*) that given by (36).

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