AN EVALUATION OF ROCKET PARAMETERS

by

J. N. Beri

Defence Science Laboratory, Delhi

ABSTRACT

The dependence of conventional parameters of internal ballistics of Solid Propellant Rockets using external burning cruciform charge, on the geometry of charge and rocket motor is discussed and results applied in a special case.

Introduction

The primary function of a rocket motor is to deliver a certain impulse $\int F dt$, to control mass flux and pressure and that the shape of the force-time curve is of secondary importance. However, the design of propellant grains in solid propellant rockets places rather definite limitations on the thrust programming. In order to obtain the desired programming of gas evolution charges are designed with many different shapes. Considering internal burning cylindrical charge Price[1] has deduced expressions showing the dependence of four important ballistic parameters of solid propellant rockets viz. the ratio of burning surface area to nozzle throat area, burning surface area to internal channel area, nozzle throat area to internal channel area and charge mass, on charge and motor geometry and has tried to approximate all the charges by certain programmes of burning surface variations during operation. But in actual practice the scope of that paper is rather limited, and in many cases, *e.g.*, cruciform charge which has external burning surface, the results arrived at by him are not at all applicable. Moreover, for many applications, particularly forward firing aircraft rockets it is necessary to utilize a greater weight of propellant in a specified motor than is possible with a tubular grain. Hence, the design of propellant charge comes into play. Though there are no definite rules for the choice of a particular grain for a given application, the comparative study of tubular and external burning grain indicated that a cross section which can be used with considerable success is the cruciform shape. A cruciform grain definitely permits the use of a greater weight of a propellant in a motor of given inside diameter than a tubular grain and hence the more effective is ballistics.

In the present paper the dependence of conventional parameters on the charge geometry of internal ballistics of solid propellant rockets using cruciform charge is studied. The method is the same as that followed by Price[1].

Notations

a = fractional change in surface area

A_c = cross-sectional area of the propellant charge

q = perimeter of the cross section of the burning charge

r = burning rate

A_m = cross-sectional area of the interior of the motor tube.

A_p = port area, $A_m - A_e$

A_t = throat area of the sonic nozzle

C_d = discharge coefficient

d = web size

D = inside diameter of motor tube

G = internal area ratio S_c / A_p

J = channel area ratio A_t / A_p

K = nozzle area ratio, S_c / A_t

L = length of the propellant charge

M = propellant mass

p = chamber pressure

r_i = defined in fig. (1)

S_c = burning surface area of charge.

T = time from start of burning

V = volume of propellant chagre

x = distance burnt into charge

α, β = constants in the burning rate rule $r = \alpha + \beta p$

ρ_p = density of propellant charge

Subscripts

i = initial conditions

f = final conditions

Charge Geometry and Ballistic Parameters

The assumptions made are:

(1) The propellant charge is cruciform in the general sense and remains so during burning. All the four peripherial ends are inhibited with non-inflammable material to prevent end burning.

(2) The area of the burning surface is a linear function of the distance burned into the web from the initial surface.

Assumptions (1) and (2) help us to define the geometry of the charge at any instant during burning and the following geometrical properties can be inferred from Fig. (1).

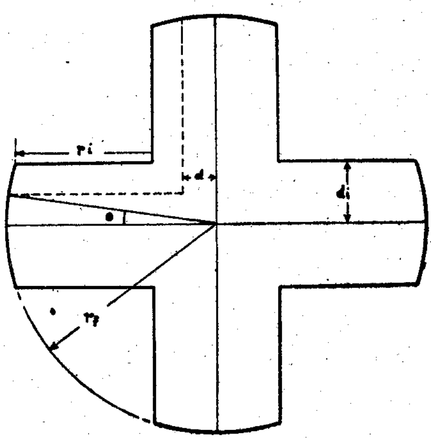

Fig. 1 Charge geometry at any time during burning

$$a = \frac{q_f - q_i}{q_i} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1.1)$$

$$\frac{d_i}{r_i} = \frac{a^2 + 2a}{1 + \sqrt{2a^2 + 4a + 1}} = f(a) \quad \ldots \quad \ldots \quad (1.2)$$

$$q = q_i\ (a+1)\left[\sqrt{1 - \left(\frac{f(a)}{a+1}\ \frac{d}{d_i}\right)^2} - \frac{f(a)}{a+1}\ \frac{d}{d_i}\right] \quad (2)$$

$$V_e = A_e L \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

$$S = qL \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

$$A_e = \left[\ q + \frac{a+1}{f(a)}\ \frac{di}{d}\ q_f\ Sin^{-1}\left(\frac{f(a)}{a+1}\ \frac{d}{d_i}\right)\right]\frac{d}{2} \quad \ldots \quad (5)$$

For geometrical interpretations of the results, dimension-less variables are most suitable and the analysis can be generalised to a greater extent. Following Price[1] we then express the charge geometry in terms of the following dimensionless variables:

$b = \frac{q}{\pi D}$ —burning perimeter of cross-section shape

$l = \frac{L}{D}$ —length of charge

$R = A_e\ / A_m$ —loading density

$w = \frac{d}{D}$ —web thickness

The variations during burning in these variables are of considerable interest from design as well as stability point of view, and in particular the following relations, which are used often, can be deduced.

$$w_i = \frac{Ri\ (1+a)}{2b_f} \Big/ \left[1 + \frac{(a+1)^2}{f(a)}\ S_{in}{}^{-1}\ \frac{f(a)}{a+1}\right] \quad \ldots \quad \ldots \quad (6)$$

$$b_i = \frac{b_f}{1+a} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

$$b = \frac{b_f}{R_i\ (1+a)^2}\left[\sqrt{\left\{R_i\ (1+a)^2\right\}^2 - \left\{2b_f\ f(a)\left[1 + \frac{(a+1)^2}{f(a)}\ \ {}^{-1}\ {}_{a+1}\right]\omega\right\}^2}\right.$$
$$\left. - 2\ b_f\ f(a)\left[1 + \frac{(a+1)^2}{f(a)}\ S_{in}{}^{-1}\ \frac{f(a)}{a+1}\right]\omega\ \right] \quad \ldots \quad (8)$$

$$R = 2\omega\ b_f\ \left\{\sqrt{1 - \left[\frac{f(a)}{a+1}\ \omega \frac{2b_f}{R_i\ (a+1)}\left(1 + \frac{(a+1)^2}{f(a)}\ S_{in}{}^{-1}\frac{f(a)}{a+1}\right)\right]^2}\right.$$
$$- \frac{f(a)}{a+1}\ \omega\ \frac{2b_f}{R_i\ (a+1)}\left(1 + \frac{(a+1)^2}{f(a)}\ S_{in}{}^{-1}\ \frac{f(a)}{a+1}\right)$$
$$+ \frac{a+1}{\omega . f(a)}\ \frac{R_i\ (a+1)}{2\ b_f\left[1 + \frac{(a+1)^2}{f(a)}\ S_{in}{}^{-1}\frac{f(a)}{a+1}\right]}$$
$$\left.\times\ S_{in}{}^{-1}\left[\omega\frac{f(a)}{a+1}\ \frac{2b_f}{R_i\ (a+1)}\left(1 + \frac{(a+1)^2}{f(a)}S_{in}{}^{-1}\frac{f(a)}{a+1}\right)\right]\right\} \quad (9)$$

The choice of the variables, w, R_i and b_f though arbitrary is justified by their utility in terms of rocket problems in general.

The form of equations (8) and (9) can be considerably simplified if the variable w is transformed by the following relation:

$$w = \frac{w}{w_i} w_i = \frac{W R_i (1 + a)}{2b_f \left[1 + \frac{(a+1)^2}{f(a)} S_{in}^{-1} \frac{f(a)}{a+1} \right]} \quad .. \quad .. \quad (10)$$

where the variable $W\left(= \frac{w}{w_i} = \frac{d}{d_i} \right)$ is the fractional web-thickness at any time during burning, and then using the modified fractional web-thickness defined by the relation

$$W_1 = \frac{f(a)}{a+1} W \quad .. \quad .. \quad .. \quad .. \quad .. \quad (11)$$

Hence equations (8) and (9) can be written as

$$b = b_f \left[\sqrt{1 - W_1^2} - W_1 \right] \quad .. \quad .. \quad .. \quad .. \quad .. \quad (12)$$

$$R = \frac{R_i W_1 \frac{(a+1)^2}{f(a)}}{1 + \frac{(a+1)^2}{f(a)} S_{in}^{-1} \frac{f(a)}{a+1}} \left[\sqrt{1 - W_1^2} - W_1 + \frac{S_{in}^{-1} W_1}{W_1} \right] \quad .. \quad (13)$$

Ratio of burning surface Area to Nozzle Throat Area

Gases are evolved by the burning of the charge which is assumed to be burning by parallel layers and are being discharged through the nozzle throat. Hence the ratio K of the burning surface of the charge and nozzle throat area is of utmost importance especially in rocket motors where the gas velocity is not very high.

Under the assumptions laid down

$$K = \frac{S_c}{A_t} = \frac{S_c}{A_m} \Big/ \frac{A_t}{A_m} = \frac{qL}{\frac{\pi^2 D^2}{4}} \Big/ \frac{A_t}{A_m} = 4\, lb \Big/ \frac{A_t}{A_m} \quad .. \quad (14)$$

From the relations (14) and (15) we have

$$\frac{k}{lb_f} \frac{A_t}{A_m} = 4 \left(\sqrt{1 - W_1^2} - W_1 \right) \quad .. \quad .. \quad .. \quad (14{\cdot}1)$$

and at the end of burning (and also for $a = 0$)

$$K_f = \frac{4\, lb_f\, A_m}{A_t}$$

Ratio of Burning Surface Area to Internal Channel Area

The ratio $G = S_e / A_p$ is another parameter which plays an important role in the performance of solid propellant Rockets.

Under the assumptions of present paper we have

$$G = \frac{4lb_f\,(\sqrt{1-W_1^2}-W_1)}{1-\dfrac{Ri\,W_1\,(a+1)^2/f(a)}{1+\dfrac{(a+1)^2}{f(a)}\,Sin^{-1}\,\dfrac{f(a)}{a+1}}\left(\sqrt{1-W_1^2}-W_1+\dfrac{Sin^{-1}\,W_1}{W_1}\right)} \quad (15)$$

and for special cases of finish and start of burning

$$\left(\frac{G}{l}\right)_f = 4b_f \quad \ldots \quad (15\cdot1)$$

$$\left(\frac{G}{l}\right)_i = \frac{4b_f\left(\sqrt{1-\left(\dfrac{f(a)}{a+1}\right)^2}-\dfrac{f(a)}{a+1}\right)}{1-\dfrac{Ri\,(a+1)}{1+\dfrac{(a+1)^2}{f(a)}\,Sin^{-1}\,\dfrac{f(a)}{a+1}}\left(\sqrt{1-\left(\dfrac{f(a)}{a+1}\right)^2}-\dfrac{f(a)}{a+1}\times+\dfrac{Sin^{-1}\dfrac{f(a)}{a+1}}{\dfrac{f(a)}{a+1}}\right)} \quad \ldots \quad (15\cdot2)$$

Ratio of Nozzle Throat Area to Internal Channel Area

The third ballistic parameter J—the ratio of nozzle throat area—A_t to Internal Channel Area—A_p of the flow channel in the rocket motor is of more importance than the parameter K when the ratio J approaches unity. In one dimensional isentropic flow the equilibrium conditions in the rocket motor are sufficiently sensitive to A_p when J is large. But as it is customary to describe conditions in terms of K and G, even when J is large, J is determined in terms of K and G ard hence in terms of $\frac{A_m}{A_t}$ and W_1, a and R_i.

$$J = J\frac{A_t}{A_p} = \frac{A_t}{Am} \Big/ \frac{A_p}{A_m} = \frac{A_t}{A_m} \Big/ (1-R)$$

$$\therefore \frac{A_m}{A_t} = \frac{1}{1-\dfrac{Ri\,W_1(a+1)^2/f(a)}{1+\dfrac{(a+1)^2}{f(a)}\,Sin^{-1}\,\dfrac{f(a)}{a+1}}\left(\sqrt{1-W_1^2}-W_1+\dfrac{Sin^{-1}\,W_1}{W_1}\right)}$$

$$= \frac{G}{4lb_f\,(\sqrt{(1-W_1^2}-W_1)} \quad \ldots \quad (16)$$

and for initial and final conditions

$$\left(\frac{A_m}{A_t} J\right)_i = \frac{1}{1 - \dfrac{R_i\,(a+1)}{1+\dfrac{(a+1)^2}{f(a)} Sin^{-1}\dfrac{f(a)}{a+1}}\left[\sqrt{1-\left(\dfrac{f(a)}{a+1}\right)^2} - \dfrac{f(a)}{a+1} \times + \dfrac{Sin^{-1}\dfrac{f(a)}{a+1}}{\dfrac{f(a)}{a+1}}\right]} \qquad \ldots \quad \ldots \quad 16{\cdot}1$$

$$\left(\frac{A_m}{A_t} J\right)_f = 1 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 16{\cdot}2$$

Charge Mass

The importance of Charge Mass M, can be seen from the fact that in all cases to determine acceleration we require nozzle geometry, pressure in and outside the venturi and the rocket mass—including unburnt propellant at any time during burning.

Hence if ρ_p is density of solid propellant, M is mass and v the volume, then

$$M = \rho_p V = \rho_p A_e L = \rho_p \frac{A_e}{A_m} A_m L$$

$$= \rho_p R \frac{\pi D^2}{4} lD = \rho_p \frac{\pi D^3 l}{4} R \qquad \ldots \quad \ldots \quad \ldots \quad (17)$$

$$M = \frac{\rho_p \pi D^3 l}{4} \quad \frac{R_i\, W_1\,(a+1)^2/f(a)}{1+\dfrac{(a+1)^2}{f(a)} S_{in}^{-1}\dfrac{f(a)}{a+1}}\left[\sqrt{1-W_1^2} - W_1 + \frac{S_{in}^{-1} W_1}{W_1}\right] \qquad (18)$$

But in order to interpret the results in terms of loading conditions we express M in terms of R the loading density,

$$\frac{M}{M_i} = \frac{R}{R_i} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18{\cdot}1)$$

i.e., the variations of charge mass to initial charge mass are equivalent to the variations of loading density to initial loading density

Application of Results in Elementary Theory of Solid Propellant Rocket Motor

In a simple case, neglecting the variations in conditions in the different parts of rocket motor, the equilibrium between mass burning rate of the charge and mass discharge rate through the rocket nozzle is given by the relation

$$\rho_p S_e r = C_D A_t p \qquad \ldots \quad \ldots \quad \ldots \quad (19)$$

Assuming that the burning rate of the propellant is given by the relation $r = \alpha + \beta p$, the above equation with the help of equation (14·1) gives

$$p = \frac{\alpha \rho_p \, 4lb_f \, \frac{A_m}{A_t}\left(\sqrt{1-W_1^2} - W_1\right)}{C_D - \beta \rho_p \, 4lb_f \, \frac{A_m}{A_t}\left(\sqrt{1-W_1^2} - W_1\right)} \quad .. \quad .. \quad (20)$$

If dx is the distance burnt into the web, the time during burning is given by the relation $dt = \frac{dx}{r}$

$$\text{and} \because dx = -d\,(Dw) = -d\,(Dw_i \; W)$$

$$= - \frac{DR_i \;(a+1)^2/f(a)}{2b_f \left[1 + \frac{(a+1)^2}{f(a)} S_{in}^{-1} \frac{f(a)}{a+1}\right]} dW_1$$

then

$$dt = - \frac{DR_i \;(a+1)^2/f(a)}{2b_f \left[1 + \frac{(a+1)^2}{f(a)} S_{in}^{-1} \frac{f(a)}{a+1}\right]} \frac{dW_1}{\alpha + \beta_p} \quad .. \quad 21$$

and using equation (20) we have

$$dt = - \frac{DR_i \;(a+1)^2/f(a)}{2b_f \left[1 + \frac{(a+1)^2}{f(a)} S_{in}^{-1} \frac{f(a)}{a+1}\right]} \times$$

$$\frac{C_D - \beta \rho_p \, 4lb_f \, \frac{Am}{At}\left(\sqrt{1-W_1^2} - W_1\right) dW_1}{2C_D}$$

This relation is to be integrated between the limits

$$t = o,\; W_1 = \frac{f(a)}{a+1} \text{ to } t = T,\; W = W_1$$

Then we have

$$T = \frac{DR_i \;(a+1)^2/f(a)}{2b_f \left[1 + \frac{(a+1)^2}{f(a)} S_{in}^{-1} \frac{f(a)}{a+1}\right] \alpha C_D} \Bigg\{ - \Bigg[C_D W' - 2lb_f \frac{Am}{At} \beta \rho_p W_1$$

$$\left(\sqrt{1-W_1^2} - W_1 + \frac{S_{in}^{-1} W_1}{W_1}\right)\Bigg] + \frac{f(a)}{a+1}\Bigg[C_D - 2lb_f \frac{Am}{At} \beta \rho_p$$

$$\left(\sqrt{1-\left(\frac{f(a)}{a+1}\right)^2} - \right.$$

$$\left. \frac{f(a)}{a+1} + \frac{S_{in}^{-1} \frac{f(a)}{a+1}}{\frac{f(a)}{a+1}} \right)\Bigg]\Bigg\} \quad .. \quad .. \quad .. \quad (23)$$

The dependence of p on T, through the parameter W_1 can easily be compiled from the above results and in a particular case the results are represented in Fig. (2).

$a = \cdot 72$

$b_f = 1 \cdot 275$

$C_D = \cdot 007/\text{sec}$

$D = 3 \cdot 09$ in

$l = 14$

$\rho_p = \cdot 0571$ lbs/cu. in.

$\alpha = \cdot 13$ in/sec

$\beta = \cdot 0002284$

$A_t = 1 \cdot 651$ sq. in.

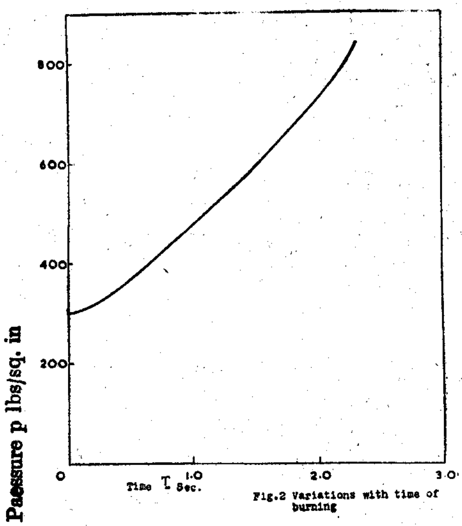

Fig.2 Variations with time of burning

In a simpler case when the peripherial ends are straight lines and not circular arc, the procedure is a simple one and results are just the same as those deduced by Price[1] for tubular charges.

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