BALLISTIC EFFECTS OF BORE RESISTANCE FOR GUNS

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ABSTRACT

Ballistic effects of bore resistance for guns have been dealt with in this paper. The equations have been integrated for the most general case by taking both the resistances which occur during the band engraving and during the motion of the shell in the gun barrel. Detailed solution has been obtained when covolume terms are neglected.

Introduction

An important contribution to the problem of bore resistance was made by Corner [1,2] when he gave his 'weighing factor' method for calculating the effects of bore resistance on shot travel, peak pressure and muzzle velocity. He considered the case when $\eta=1/\delta$ i.e. the covolume of the propellant gases is equivalent to the reciprocal of densty of the solid propellant. Recently Tawakley [3] extended the results by taking into account the covolume terms. Results have also been obtained due to bore resistance by Kapur[4] for the general form $z = \phi(f)$. In all the cases only the resistance that occurs after the band engraving has been taken into account, and the resistance that occurs in the initial stages is not considered.

In the present paper the author deals both with the resistance that occurs during the engraving of the band and that which occurs after the engraving of the band. The latter persists for larger part of the shot travel along the barrel of the gun. The initial resistance is so important in ballistics that it is desirable to investigate every promising representation of it, *therefore* it must be taken into account. Here the author has integrated the most general equations of interna ballistics taking both types of resistances into account and also assuming $\eta\neq 1/\delta$ i.e. considering the covolume terms. A detailed solution has been obtained for the case when covolume terms are neglected. The effects of bore resistance have been obtained for the following important quantities;

(*a*) Shot travel at all burnt;

(*b*) Pressure at all burnt;

(*c*) Velocity at all burnt;

(*d*) Maximum pressure;

and (*e*) Muzzle velocity.

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Let A be the area of cross-section of the bore. The standard bore resistance will be taken as a constant, AP_o, over a short travel $\triangle x$ near the point x_o and zero at all other shot positions. It is assumed, that $\triangle x$ is infinitesimal, and $P_o \triangle x$ is finite. The effects on ballistics to the first order in $P_o \triangle x$ is calculated by subtracting an energy $AP_o \triangle x$ from the kinetic energy of the projectile, as it passes through the point x_o. The ballistic equations from the point x_o onwards are exactly the same as before except for a perturbing term linear in $P_o \triangle x$. Approximations are the same as those of Corner [1,2] and Tawakely[3].

The Fundamental Equations

With the usual notations as that of Corner [2], the fundamental equations of the internal ballistics before the occurrence of standard bore-resistance are as follows:—

The energy equation is

$$z(T_o - T) = \frac{\bar{\gamma}-1}{2}\left(W_1 + \frac{C}{3}\right)\frac{V^2}{CR}, \quad \ldots \quad \ldots \qquad (1)$$

where $\bar{\gamma}$ is the value of γ suitably increased to allow for heat loss to the barrel.

The equation of state of the gas is

$$P\left[U + Ax - \frac{C(1-z)}{\delta} - cz\eta\right] = \frac{Cz\,RT\left(1 + \dfrac{C}{2W_1}\right)}{1 + \left(\dfrac{C}{3W_1}\right)} \quad ..(2)$$

the form function is

$$z = (1-f)(1+\theta f), \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (3)$$

the rate of burning equation is

$$D\frac{df}{dt} = -\beta P^{\alpha}, \quad \text{where } \alpha = 1 \quad \ldots \quad \ldots \quad \ldots \qquad (4)$$

and the dynamical equation of motion of the shot is

$$\left(W_1 + \frac{C}{2}\right)\frac{dv}{dt} = AP \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (5)$$

where W_1 is the effective mass of the projectile accounting for its rotational inertia.

Solution with Standard Bore Resistance

The energy equation (1) with bore resistance becomes,

$$z(T_o - T) = \frac{\bar{\gamma}-1}{CR}\left[\left(W_1 + \frac{C}{3}\right)\frac{V^2}{2} + AP_o\triangle x\right] \quad \ldots \qquad (6)$$

and the equations (2) and (5) remain the same after the projectile passes the point x_o. Just before meeting the resistance the kinetic energy of the charge and projectile is $\frac{1}{2}\left(W_1 + \frac{C}{3}\right)V_o^2$, and immediately afterwards the kinetic energy is $\frac{1}{2}\left(W_1 + \frac{C}{3}\right)V_o^2 - AP^o\triangle x$. The instantaneous drop of the velocity of the projectile is transmitted through the gas with a finite velocity, namely a_o the velocity of sound in the propellant gases and the time taken for the waves to settle down is of order $2\,(U+Ax_o)/Aa_o$ which is comparable with the time taken for the shot to reach the muzzle. If the velocity of the projectile immediately after the resistance is written as V_1, then assuming this delayed redistribution of energy as the immediate process, so that the kinetic energy is at all times $\frac{1}{2}\left(W_1+\frac{C}{3}\right)V^2$, where V is the velocity, we get

$$\frac{1}{2}\left(W_1+\frac{C}{3}\right)V_1^2=\frac{1}{2}\left(W_1+\frac{C}{3}\right)V_o^2-AP_o\Delta x \quad .. \qquad (7)$$

Also eliminating P from (4) and (5) and integrating, we get finally,

$$V=V_1+\frac{AD\,(f_o-f)}{\beta\left(W_1+\frac{C}{2}\right)} \quad .. \quad .. \quad .. \quad .. \qquad (8)$$

Therefore the equation (6) may be written as

$$\frac{CRz\,(T_o-T)}{\gamma-1}=\frac{1}{2}\left(W_1+\frac{C}{3}\right)\times\left[V_1^2+\left\{\frac{AD\,(f_o-f)}{\beta\left(W_1+\frac{C}{2}\right)}\right\}^2+\frac{2ADV_1\,(f_o-f)}{\beta\left(W_1+\frac{C}{2}\right)}\right]+AP_o\Delta x.$$

Using equation (7), we have

$$\frac{CRz\,(T_o-T)}{\gamma-1}=\frac{1}{2}\left(W_1+\frac{C}{3}\right)\times\left[V^2_o+\left\{\frac{AD\,(f_o-f)}{\beta\left(W_1+\frac{C}{2}\right)}\right\}^2+\frac{2ADV_1\,(f_o-f)}{\beta\left(W_1+\frac{C}{2}\right)}\right] \qquad (9)$$

Combining equations (2) an (9) we find

$$CRT_oz=\frac{\gamma-1}{2}\left(W_1+\frac{C}{3}\right)\times\left[V_o^2+\left\{\frac{AD\,(\gamma-f)}{\beta\left(W_1+\frac{C}{2}\right)}\right\}^2+\frac{2ADV_1\,(f_o-f)}{\beta\left(W_1+\frac{C}{2}\right)}\right]$$

$$+\frac{AP\,[\,x+l\,(1-bz)\,]\left(W_1+\frac{C}{3}\right)}{\left(W_1+\frac{C}{2}\right)} \quad .. \quad .. \qquad (10)$$

when $Al=U-\frac{C}{\eta}$ (initial free space)

and $b=\frac{C\left(\eta-\frac{1}{\delta}\right)}{Al}$

So far the equations have been exact, but now keeping only first order terms in (7), (8) and (10), we have,

$$V_1 = V_\circ - \frac{AP_\circ \triangle x}{\left(W_1 + \frac{C}{3}\right)V_\circ} \quad .. \quad .. \quad .. \quad .. \qquad (11)$$

$$V = \frac{AD\,(f_s - f)}{\beta\left(W_1 + \frac{C}{2}\right)} - \frac{AP_\circ \triangle x}{\left(W_1 + \frac{C}{3}\right)V_\circ} \quad .. \quad .. \quad .. \qquad (12)$$

$$CRT_\circ z = \frac{\gamma - 1}{2}\left(W_1 + \frac{C}{3}\right)$$

$$\left[\left\{\frac{AD\,(f_s - f)}{\beta\left(W_1 + \frac{C}{2}\right)}\right\}^2 - \frac{2AP_\circ \triangle x\,(f_\circ - f)}{\left(W_1 + \frac{C}{3}\right)(f_s - f_\circ)}\right]$$

$$+ \quad \frac{AP\,[x + l\,(1 - bz)]\left(W_1 + \frac{C}{3}\right)}{\left(W_1 + \frac{C}{2}\right)} \quad .. \quad .. \qquad (13)$$

where f is the fraction of D, websize remaining at the instant shot begins to start.

Combining (3) and (13), we have,

$$(1 - f)\,(1 + \theta f) = \frac{\bar{\gamma} - 1}{2} M\,(f_s - f)^2 - \frac{(\bar{\gamma} - 1)\,AP_\circ \triangle x\,(f_\circ - f)}{CRT_\circ\,(f_s - f_\circ)}$$

$$- \frac{df}{dx}\left[M\,(f_s - f) - \frac{AP_\circ \triangle x}{CRT_\circ\,(f_s - f_\circ)}\right][x + l\,(1 - bz)] \qquad (14)$$

by conveniently introducing the 'Central ballistic parameter'

$$M = \frac{A^2 D^2\left(1 + \frac{C}{3W_1}\right)}{\beta^2\,CRT_\circ\left(1 + \frac{C}{2W_1}\right)^2}$$

leading to

$$Z + M\,[x + l\,(1 - bz)]\,\frac{df}{dx} = \frac{AP_\circ \triangle x}{CRT_\circ\,(f_s - f_\circ)\,(f_s - f)}$$

$$\left[[x + l\,(1 - bz)]\,\frac{df}{dx} - (\bar{\gamma} - 1)\,(f_\circ - f)\right] \quad .. \qquad (15)$$

$$\text{when } Z = \frac{(1 - f)\,(1 + \theta f)}{(f_s - f)} - \frac{\bar{\gamma} - 1}{2} M\,(f_s - f) \quad .. \quad .. \qquad (16)$$

Equation (15) shows the first order effects of the resistance on the equation of unresisted motion,

$$Z + M\,[x + l\,(1 - bz)]\,\frac{df}{dx} = 0 \quad .. \quad .. \qquad (17)$$

$$\text{or } [x + l(1 - bz)] \frac{df}{dx} = -\frac{Z}{M} \qquad \ldots \qquad \ldots \qquad (18)$$

This gives

$$Z + M[x + l(1 - bz)]\frac{df}{dx} = -\frac{AP_\circ \triangle x}{CRT_\circ (f_s - f_\circ)(f_s - f)} \times \left[\frac{Z}{M} + (\bar{\gamma} - 1)(f_\circ - f)\right] \qquad \ldots \qquad (19)$$

$$\text{or } \frac{dx}{df} + \left[\frac{M}{Z} - \frac{AP_\circ \triangle x}{CRT_\circ (f_s - f_\circ)(f_s - f)}\left\{\frac{1}{Z} + \frac{M(\bar{\gamma}-1)(f_\circ - f)}{Z^2}\right\}\right](x + l)$$

$$= blz\left[\frac{M}{Z} - \frac{AP_\circ \triangle x}{CRT_\circ (f_s - f_\circ)(f_s - f)}\left\{\frac{1}{Z} + \frac{M(\bar{\gamma}-1)(f_\circ - f)}{Z^2}\right\}\right] \qquad (20)$$

This is a linear differential equation, which could be easily integrated and whose integrating factor is given by

$$I.F. = e^{\int \left[\frac{M}{Z} - \frac{AP_\circ \triangle x}{CRT_\circ (f_s - f_\circ)(f_s - f)}\left\{\frac{1}{Z} + \frac{M(\quad - 1)(f_\circ - f)}{Z^2}\right\}\right] df} \qquad \ldots \qquad \ldots \qquad (21)$$

Hence, we get

$$I.F. = exp\left\{\phi(f) + \frac{AP_\circ \triangle x}{2CRT_\circ (f_s - f_\circ)\theta_1}\psi(f)\right\} \qquad \ldots \qquad (22)$$

where

$$\phi(f) = \frac{M}{2\theta_1}\left[\log(f^2 + pf - q) - \frac{f_s + p/_2}{\sqrt{q + p^2/_4}}\log\frac{(f + p/_2) - \sqrt{q + p^2/_4}}{(f + p/_2) + \sqrt{q + p^2/_4}}\right] \quad (23)$$

$$\psi(f) = \left[\frac{1}{\sqrt{q + p^2/_4}}\log\frac{(f + p/_2) - \sqrt{q + p^2/_4}}{(f + p/_2) + \sqrt{q + p^2/_4}}\left\{1 - \frac{M(\gamma - 1)}{\theta_1}\left(1 - \frac{p(p + f_\circ + f_s) + 2(f_\circ f_s + q)}{4(q + p^2/_4)}\right)\right\} - \frac{(p + f_\circ + f_s)M(\quad - 1)}{\theta_1(f^2 + pf - q)}\right.$$

$$\left. + \frac{M(\bar{\gamma} - 1)[p(p + f_\circ + f_s) + 2(f_\circ f_s + q)]}{2\theta_1(q + p^2/_4)} \cdot \frac{f + p/_2}{f^2 + pf - q}\right] \qquad (24)$$

$$p = \frac{[1 - \theta - (\bar{\gamma} - 1)Mf_s]}{\theta_1} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (25)$$

$$q = \frac{1 - \frac{\bar{\gamma} - 1}{2}Mf_s^2}{\theta_1} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (26)$$

$$\text{and } \theta_1 = \theta + \frac{\bar{\gamma} - 1}{2}M \qquad \ldots \qquad \ldots \qquad \ldots \qquad (27)$$

Therefore integrating equation (20) and retaining first order terms of $AP_o\Delta x$, we get

$$[x+l(1-bz)] = l(1-bz)e^{\phi(f_s)-\phi(f)} + ble^{-\phi(f)}[\chi(f)-\chi(f_s)]$$
$$+\frac{AP_o\Delta x}{2CRT_o(f_s-f_o)\theta_1}\Big[l(1-bz_s)e^{\phi(f_s)-\phi(f)}[\psi(f_o)-\psi(f)]$$
$$+ble^{-\phi(f)}[\tau(f)-\tau(f_o)]\Big] \quad .. \quad .. \quad (28)$$

$$\text{where} \int(1-\theta+2\theta f)\,e^{\phi(f)}df = \chi(f) \quad .. \quad .. \quad (29)$$

$$\text{and} \int(1-\theta+2\theta f)\,e^{\phi(f)}\psi(f)\,df = \tau(f) \quad .. \quad .. \quad (30)$$

As the Covolume term is very small and contributes negligible difference in the shot travel, we consider the case in more details when $b = o$, as a particular case of more practical importance.

We have

$$\frac{x+l}{l} = e^{\phi(f_s)-\phi(f)} + \frac{AP_o\Delta x}{2CRT_o(f_s-f_o)\theta_1}e^{\phi(f_s)-\phi(f)}[\psi(f_o)-\psi(f)] \quad (31)$$

$$\text{or}\log\left(1+\frac{x}{l}\right) = [\phi(f_s)-\phi(f)] + \frac{AP_o\Delta x}{2CRT_o(f_s-f_o)\theta_1}[\psi(f_o)-\psi(f)] \quad (32)$$

Solution with "unburnt" at muzzle is not of practical importance. Hence we consider the solution for "burnt" inside gun. Here we need the change in $\log(x_B + l)$, where suffix B refers to conditions at "burnt", due to bore resistance which is given as,

$$\Delta\log(x_B+l) = \frac{AP_o\Delta x}{2CRT_o(f_s-f_o)\theta_1}[\psi(f_o)-\psi(f_B)] \quad .. \quad (33)$$

But, $f_B = 0$, and therefore, we have

$$\Delta\log(x_B+l) = \frac{AP_o\Delta x}{2CRT_o(f_s-f_o)\theta_1}$$
$$\Bigg\{\frac{1}{\sqrt{q+p^2/4}}\left[1-\frac{M(\bar\gamma-1)}{\theta_1}\left(1-\frac{p(p+f_o+f_s)+2(f_of_s+q)}{4(q+p^2/4)}\right)\right]$$
$$\times\log\frac{[(f_o+p/2)-\sqrt{q+p^2/4}]\,[p/2+\sqrt{q+p^2/4}]}{[(f_o+p/2)+\sqrt{q+p^2/4}]\,[p/2-\sqrt{q+p^2/4}]}$$
$$-\frac{p(p+f_o+f_s)M(\bar\gamma-1)}{\theta_1}\left[\frac{1}{f_o^2+pf_o-q}+\frac{1}{q}\right]$$
$$+\frac{[p(p+f_o+f_s)+2(f_of_s+q)]M(\bar\gamma-1)}{2\theta_1(q+p^2/4)}$$
$$\left[\frac{f_o+p/2}{f_o^2+pf_o-q}+\frac{p/2}{q}\right]\Bigg\} \quad .. \quad .. \quad (34)$$

Maximum Pressure: From equation (13), we have

$$P\,[x+l\,(1-bz)] = \frac{CRT_\circ\left(1+\dfrac{C}{2W_1}\right)}{A\left(1+\dfrac{C}{3W_1}\right)}\times$$

$$\left[(f_s-f)\,Z+\frac{(\bar\gamma-1)\,AP_\circ\Delta x\,(f_\circ-f)}{CRT_\circ\,(f_s-f_\circ)}\right] \quad .. \quad (35)$$

Also from equation (13) and (20), we have

$$\frac{dx}{df} = -\frac{CRT_\circ\left(1+\dfrac{C}{2W_1}\right)}{AP\left(1+\dfrac{C}{3W_1}\right)}\left[M\,(f_s-f)-\frac{AP_\circ\Delta x}{CRT_\circ\,(f_s-f_\circ)}\right] \quad (36)$$

We here introduce a dimensionless quantity,

$$k = \frac{\left(\eta-\dfrac{1}{\delta}\right)P_m\left(1+\dfrac{C}{3W_1}\right)}{RT_\circ\left(1+\dfrac{C}{2W_1}\right)} \quad .. \quad . \quad (37)$$

The equation (35) now becomes

$$P(x+l) = \frac{CRT_\circ\left(1+\dfrac{C}{2W_1}\right)(f_s-f)}{A\left(1+\dfrac{C}{3W_1}\right)}\times$$

$$\left[Z+\frac{k\,(1-f)\,(1+\theta f)}{(f_s-f)}+\frac{(\bar\gamma-1)\,AP_\circ\Delta x\,(f_\circ-f)}{CRT_\circ\,(f_s-f_\circ)\,(f_s-f)}\right] \quad (38)$$

For maximum pressure $\dfrac{dP}{df} = 0$

and hence, we have, the value of f at maximum pressure as,

$$f_m = \frac{\gamma'\,M f_s+\theta-1}{\gamma' M+2\theta} + \text{ a term proportional to } AP_\circ\Delta x, \quad (39)$$

where $\gamma' = \dfrac{\bar\gamma}{1+k}$

For first order effects of bore resistance, we need only substitute

$f_m = \dfrac{\bar\gamma' M f_s+\theta-1}{\bar\gamma' M+2\theta}$, in equations (28) and (35).

For pressures upto 25 tons/sq in; $RT_\circ$ greater than 60 (tons/sq in) $\times$ (c.c./g), η less than $1{\cdot}02$ cc/g, and δ less than $1{\cdot}67$ g/c.c., the maximum value of k is $0{\cdot}175$. For maximum pressure using suffix m, we have from equation (28),

$$[\,x_m+l\,(1-bz_m)\,] = l\,(\,1-bz_s\,)\;e^{\phi(f_s)-\phi(f_m)}+ble^{-\phi(f_m)}\,[\,\chi(f_m)-\chi(f_s)\,]$$

$$+ \frac{AP_{\circ}\Delta x}{2CRT_{\circ}(f_s - f_{\circ})\theta_1}\Big[l(1 - bz_s)\, e^{\phi(f_s) - \phi(f_m)}$$

$$[\psi(f_{\circ}) - \psi(f_m)] + ble^{-\phi(f_m)}[\tau(f_m) - \tau(f_{\circ})]\Big] \qquad (40)$$

Also from equation (35), we have,

$$[x_m + l(1 - bz_m)] = \frac{CRT_{\circ}\left(1 + \frac{C}{2W_1}\right)}{AP_m\left(1 + \frac{C}{3W_1}\right)} \times$$

$$\left[(f_s - f_m) Z_m + \frac{(\gamma - 1) AP_{\circ}\Delta x (f_{\circ} - f_m)}{CRT_{\circ}(f_s - f_{\circ})}\right] \quad .. \qquad (41)$$

Therefore we find from equations (40) and (41)

$$\frac{CRT_{\circ}\left(1 + \frac{C}{2W_1}\right)}{AP_m\left(1 + \frac{C}{3W_1}\right)}\left[(f_s - f_m) Z_m + \frac{(\gamma - 1) AP_{\circ}\Delta x (f_{\circ} - f_m)}{CRT_{\circ}(f_s - f_{\circ})}\right]$$

$$= l(1 - bz_s)\, e^{\phi(f_s) - \phi(f_m)} + ble^{-\phi(f_m)}[\chi(f_m) - \chi(f_s)]$$

$$+ \frac{AP_{\circ}\Delta x}{2CRT_{\circ}(f_s - f_{\circ})\theta_1}\Big\{ l(1 - bz_s)\, e^{\phi(f_s) - \phi(f_m)}[\psi(f_{\circ}) - \psi(f_m)]$$

$$+ ble^{-\phi(f_m)}[\tau(f_m) - \tau(f_{\circ})]\Big\} \quad .. \quad .. \quad .. \qquad (42)$$

Hence we find,

$$\frac{CRT_{\circ}\left(1 + \frac{C}{2W_1}\right)}{AP_m\left(1 + \frac{C}{3W_1}\right)} = \frac{l(1 - bz_s)\, e^{\phi(f_s) - \phi(f_m)}}{(f_s - f_m) Z_m} + \frac{ble^{-\phi(f_m)}}{(f_s - f_m) Z_m}$$

$$[\chi(f_m) - \chi(f_s)] + \frac{AP_{\circ}\Delta x}{2CRT_{\circ}(f_s - f_{\circ})\theta_1 (f_s - f_m) Z_m} \times$$

$$\Big[l(1 - bz_s)\, e^{\phi(f_s) - \phi(f_m)}[\psi(f_{\circ}) - \psi(f_m)] + ble^{-\phi(f_m)}[\tau(f_m) - \tau(f_{\circ})]$$

$$- \frac{2(\gamma - 1)\theta_1(f_{\circ} - f_m)}{(f_s - f_m) Z_m}\Big\{ l(1 - bz_s)\, e^{\phi(f_s) - \phi(f_m)} + ble^{-\phi(f_m)} \times$$

$$[\chi(f_m) - \chi(f_s)]\Big\}\Big] \quad .. \quad .. \quad .. \qquad (43)$$

Therefore the change in the maximum pressure due to bore resistance $AP_o\triangle x$ occurring at a certain point, during the period of burning is given by,

$$\frac{CRT_o\left(1+\frac{C}{2W_1}\right)}{A\left(1+\frac{C}{3W_1}\right)}\triangle\left(\frac{1}{P_m}\right)=\frac{AP_o\triangle x}{2CRT_o(f_s-f_o)\theta_1(f_s-f_m)Z_m}$$

$$\Big[\,l(1-bz_s)\,e^{\phi(f_s)-\phi(f_m)}[\psi(f_o)-\psi(f_m)]+ble^{-\phi(f_m)}$$

$$[\tau(f_m)-\tau(f_o)]-\frac{2(\;-1)\,\theta_1(f_o-f_m)}{(f_s-f_m)Z_m}$$

$$\Big\{l(1-bz_s)\,e^{\phi(f_s)-\phi(f_m)}$$

$$+ble^{-\phi(f_m)}[\chi(f_m)-\chi(f_s)]\Big\}\Big] \qquad (44)$$

Again neglecting covolume terms *i.e.* $b=o$ we find the change in the maximum pressure as

$$\frac{CRT_o\left(1+\frac{C}{2W_1}\right)}{A\left(1+\frac{C}{3W_1}\right)}\triangle\left(\frac{1}{P_m}\right)=\frac{AP_o\triangle x}{2CRT_o(f_s-f_o)\theta_1(f_s-f_m)Z_m}\Big[\,le^{\phi(f_s)-\phi(f_m)}$$

$$\times\left\{[\psi(f_o)-\psi(f_m)]-\frac{2(\;-1)\theta_1(f_o-f_m)}{(f_s-f_m)Z_m}\right\}\Big] \quad .. \quad (45)$$

Now when $\eta=\frac{1}{\delta}$ *i.e.* when $b=o$, K, the dimensionless quantity is also zero. Hence $f_m=f'_m$ and $Z_m=Z'_m$ as from (39)

$$f_m=\frac{\bar{\gamma}Mf_s+\theta-1}{\bar{\gamma}M+2\theta}$$

$$=\frac{(\theta-1)+Mf_s\,\bar{\gamma}(1-k)}{2\theta+M\bar{\gamma}(1-k)}$$

$$=\frac{(\theta-1)+Mf_s\,\bar{\gamma}-M\,f_s\,k}{2\theta+M\bar{\gamma}}\left[1+\frac{M\bar{\gamma}k}{2\theta+M}\right]$$

$$=\frac{\theta-1+M\bar{\gamma}f_s}{2\theta+M\bar{\gamma}}+k\,\frac{(\theta-1+M\,f_s)M_r-M\bar{\gamma}f_s(2\theta+M\bar{\gamma})}{(2\theta+M\bar{\gamma})^2}$$

$$=f_m+k\omega \text{ where } \omega \text{ is the constant} \quad .. \quad .. \quad .. \quad (39a)$$

Similarly from (16)

$$Z_m=\frac{(1-f_m)(1+\theta f_m)}{(f_s-f_m)}-\frac{\bar{\gamma}-1}{2}M(f_s-f_m)$$

putting $f_m=f'_m+k\omega$, we find

$$Z_m=\frac{(1-f'_m)(1+\theta f'_m)}{(f_s-f'_m)}-\frac{\gamma-1}{2}M(f_s-f'_m)+k\omega\left[\frac{\theta-1-2\theta f'_m}{f_s-f'_m}+\frac{\bar{\gamma}-1}{2}M\right.$$

$$\left.+\frac{(1-f'_m)(1+\theta f'\;)}{(f_s-f'_m)^2}\right]$$

$$=Z'_m+k\omega' \text{ where again } \omega' \text{ is the constant} \quad .. \quad .. \quad (16a)$$

Hence we get the change in maximum pressure for $b=o$ as

$$\frac{CRT_0\left(1+\frac{C}{2W_1}\right)}{A\left(1+\frac{C}{3W_1}\right)}\triangle\left(\frac{1}{P_m}\right)=\frac{AP_o\triangle x}{2CRT_o(f_s-f_o)\theta_1(f_s-f'_m)Z'_m}$$

$$\times\left[le^{\phi(f_s)-\psi(f'_m)}\left\{[\psi(f_0)-\psi(f'_m)]-\frac{2(\gamma-1)\theta_1(f_0-f'_m)}{(f_s-f'_m)Z'_m}\right\}\right] \quad .. \quad (46)$$

By substituting the values of (f) and ψ(f), the change in maximum pressure is given by

$$\frac{CRT_0\left(1+\frac{C}{2W_1}\right)}{A\left(1+\frac{C}{3W_1}\right)}\triangle\left(\frac{1}{P_m}\right)=\frac{AP_o\triangle x}{2CRT_o(f-f_o)\theta_1(f_s-f'_m)Z'_m}$$

$$\left\{l\left[\frac{f^2_s+pf_s-q}{f'^2_u+pf'_m-q}\right]^{\frac{M}{2\theta_1}}\right.$$

$$\times\left[\frac{[(f+p/_2)+\sqrt{q+p^2/_4}][(f'_m+p/_2)-\sqrt{q+p^2/_4}]}{[(f_s+p/_2)-\sqrt{q+p^2/_4}][(f'_m+p/_2)+\sqrt{q+p^2/_4}]}\right]^{\frac{M(f_s+p/_2)}{2\theta_1\sqrt{q+p^2/_4}}}$$

$$\times\left[\left\{\frac{1}{\sqrt{q+p^2/_4}}\left(1-\frac{M(\gamma-1)}{\theta_1}\left[1-\frac{p(p+f_o+f_s)+2(f_of_s+q)}{4(q+p^2/_4)}\right]\right)\right.\right.$$

$$\times\log\frac{[f_o+p/_2)-\sqrt{q+p^2/_4}][(f'_m+p/_2)+\sqrt{q+p^2/_4}]}{[(f_o+p/_2)+\sqrt{q+p^2/_4}](f'_m+p/_2)-\sqrt{q+p^2/_4}]}-\frac{(p+f_o+f_s)M(\gamma-1)}{\theta_1}$$

$$\left[\frac{1}{f_o^2+pf_o-q}-\frac{1}{f^{2\prime}_m+pf'_m-q}\right]$$

$$+\frac{[p(p+f_o+f_s)+2(f_of_s+q)]M(\gamma-1)}{2\theta_1(q+p^2/_4)}\left[\frac{f_o+p/_2}{f_o^2+pf_o-q}-\frac{f'_m+p/_2}{f'^2_m+pf'_m-q}\right]\Bigg)$$

$$\left.\left.-\frac{2(\gamma-1)\theta_1(f_o-f'_m)}{(f_s-f'_m)Z'_m}\right]\right\} \quad .. \quad .. \quad (47)$$

This holds if,

$$f_o\geqslant\frac{\gamma' Mf_s+\theta-1}{\gamma' M+2\theta}\geqslant 0$$

If,

$$f_o<\frac{\gamma' Mf_s+\theta-1}{\gamma' M+2\theta}\geqslant 0$$

then there is no change in the maximum pressure.
The only other possible case is

$$f_o\geqslant 0>\frac{\gamma' Mf+\theta-1}{\gamma' M+2\theta}$$

in which the peak pressure always occurs at 'burnt'. Therefore we now consider the change in peak pressure when $f=0$, We have from equations (35) and (28),

$$\frac{CRT_0\left(1+\frac{c}{2w_1}\right)f_sZ_B}{AP_B\left(1+\frac{c}{3w_1}\right)}=l(1-bz_s)e^{\phi(f_s)-\phi(0)}+ble^{-\phi(0)}[\chi(0)-\chi(f_s)]$$

$$+\frac{AP_0\triangle x}{2CRT_0(f_s-f_0)\theta_1}\left\{l(1-bz_s)\,e^{\phi(f)-\phi(0)}\left[\{\psi(f_0)-\psi(0)\}-\frac{2\theta_1(\bar{\gamma}-1)f_0}{f_sZ_B}\right]\right.$$

$$\left.+\,ble^{-\phi(0)}\left[[\tau(0)-\tau(f_0)]-\frac{2\theta_1\bar{\gamma}(\bar{\gamma}-1)f_0}{f_sZ_B}[\chi(0)-\chi(f_s)]\right]\right\} \qquad (48)$$

Hence the change in the peak pressure when $f = o$ is given by

$$\frac{CRT_0\left(1+\frac{c}{2w_1}\right)f_sZ_B}{A\left(1+\frac{c}{3w_1}\right)}\triangle\left(\frac{1}{P_B}\right)=\frac{AP_0\triangle x}{2CRT_0(f_s-f_0)\theta_1}$$

$$\left\{l(1-bz_s)e^{\phi(f)-\phi(0)}\left[[\psi(f_0)-\psi(0)]-\frac{2\theta_1(\bar{\gamma}-1)f_0}{f_sZ_B}\right]\right.$$

$$\left.+ble^{-\phi(0)}\left[[\tau(0)-\tau(f_0)]-\frac{2\theta_1(\bar{\gamma}-1)f_0}{f_sZ_B}[\chi(0)-\chi(f_s)]\right]\right\} \qquad (49)$$

Considering the particular case when $\eta=\frac{1}{\delta}$ *i.e.* $b = o$ we get the equation (49) as,

$$\frac{CRT_0\left(1+\frac{c}{2w_1}\right)f_sZ_B}{A\left(1+\frac{c}{3w_1}\right)}\triangle\left(\frac{1}{P_B}\right)=\frac{AP_0\triangle x}{2CRT_0(f_s-f_0)\theta_1}\left\{le^{\phi(f_s)-\phi(0)}\right.$$

$$\left.\times\left[[\psi(f_0)-\psi(0)]-\frac{2\theta_1(\bar{\gamma}-1)f_0}{f_sZ_B}\right]\right\} \qquad (50)$$

Therefore the change in peak pressure due to bore resistance is given by

$$\frac{CRT_0\left(1+\frac{c}{2w_1}\right)f_sZ_B}{A\left(1+\frac{c}{3w_1}\right)}\triangle\left(\frac{1}{P_B}\right)=\frac{AP_0\triangle x}{2CRT_0(f_s-f_0)\theta_1}\left\{l\left[\frac{q-pf_s-f_s^2}{q}\right]^{\frac{M}{2\theta_1}}\right.$$

$$\times\left[\frac{[(f+p/_2)+\sqrt{q+p^2/_4}]\,[p/_2-\sqrt{q+p^2/_4}]}{[(f+p/_2)-\sqrt{q+p^2/_4}]\,[p/_2+\sqrt{q+p^2/_4}]}\right]^{\frac{M(f_s+p/_2)}{2\theta_1\sqrt{q+p^2/_4}}}$$

$$\times\left[\left[\frac{1}{\sqrt{q+p^2/_4}}\left(1-\frac{M(\bar{\gamma}-1)}{\theta_1}\left[1-\frac{p(p+f_0+f_s)+2(f_0f_s+q)}{4(q+p^2/_4)}\right]\right)\right.\right.$$

$$\times\log\frac{[(f_0+p/_2)-\sqrt{q+p^2/_4}]\,[p/_2+\sqrt{q+p^2/_4}]}{[(f_0+p/_2)+\sqrt{q+p^2/_4}]\,[p/_2-\sqrt{q+p^2/_4}]}-\frac{(p+f_0+f_s)M(\bar{\gamma}-1)}{\theta_1}\times$$

$$\left[\frac{1}{f_0^2+pf_0-q}+\frac{1}{q}\right]+\frac{[p(p+f_0+)+2(f_0f_s+qf_s)]M(\bar{\gamma}-1)}{2\theta_1(q+p^2/_4)}\left[\frac{f_0+p/_2}{f_0^2+pf_0-q}\right.$$

$$\left.\left.+\frac{p/_2}{q}\right]\right]-\frac{2\theta_1(\bar{\gamma}-1)f_0}{f_sZ_B}\Bigg]\Bigg\} \quad \ldots \quad \ldots \quad (51)$$

Muzzle velocity: After all burnt position the expansion of gases is adiabiatc, and therefore the pressure at any travel x greater than x_B is

$$P=P_B\, r^{-\bar{\gamma}} \quad \ldots \quad (52)$$

$$\text{Where } r=\frac{x+l(1-b)}{x_B+l(1-b)} \quad \ldots \quad (53)$$

Equation of motion of the shot is

$$\left(W_1+\frac{c}{2}\right)\frac{dv}{dt}=AP \quad \ldots \quad (54)$$

Therefore from equations (52), (53) and (54) we get

$$V^2=V^2_B+\frac{AP_B\,[x_B+l(1-b)]\,\dot{\phi}}{\left(W_1+\frac{c}{2}\right)} \quad \ldots \quad (55)$$

$$\text{Where} \quad \Phi=\frac{2}{\bar{\gamma}-1}\left(1-r^{1-\bar{\gamma}}\right) \quad \ldots \quad (56)$$

Let suffix E refer to values at shot ejection, we have

$$V^2_E=V^2_B+\frac{AP_B\,[x_B+l(1-b)]\Phi_E}{\left(W_1+\frac{c}{2}\right)} \quad \ldots \quad (57)$$

$$\text{Where} \quad \Phi_E=\frac{2}{\gamma-1}\left[1-\left\{\frac{x_E+l(1-b)}{x_B+l(1-b)}\right\}^{1-\gamma}\right] \quad \ldots \quad (58)$$

Also from (35) we have,

$$\frac{AP_B\,[x_B+l(1-b)]}{\left(W_1+\frac{c}{2}\right)}=\frac{CRT_\circ f_s\,Z_B}{\left(W_1+\frac{c}{3}\right)}+\frac{(\bar{\gamma}-1)f_\circ AP_\circ\triangle x}{\left(f_s-f_\circ\right)\left(W_1+\frac{c}{3}\right)} \quad \ldots \quad (59)$$

and from (58)

$$\triangle\Phi_E=2\bar{r}_E^{-\bar{\gamma}}\triangle r_E=-2r_E^{1-\bar{\gamma}}\frac{\triangle[x_B+l(1-b)]}{[x_B+l(1-b)]} \quad \ldots \quad (60)$$

From equation (12), we have

$$V_B=\frac{ADf_s}{\beta\left(W_1+\frac{c}{2}\right)}-\frac{AP_\circ\triangle x}{\left(W_1+\frac{c}{3}\right)V_\circ} \quad \ldots \quad (61)$$

therefore, we get

$$\triangle(V^2_B)=-\frac{2AP_\circ\triangle x\cdot f_s}{\left(W_1+\frac{c}{3}\right)(f_s-f_\circ)} \quad \ldots \quad (62)$$

Hence, we have

$$\Delta(V^2_E) = -\frac{2AP_\circ \Delta x(f_s - f_\circ + f_\circ r_E^{\,1-\bar{\gamma}})}{\left(W_1 + \frac{c}{3}\right)(f_s - f_\circ)}$$

$$-\frac{f_s Z_B r_E^{\,1-\bar{\gamma}} AP_\circ \Delta x}{\left(W_1 + \frac{c}{3}\right)(f_s - f_\circ)\theta_1[x_B + l(1-b)]}$$

$$\times \left[l(1-bz_s)e^{\phi(f_s)-\phi(o)}[\psi(f_0) - \psi(0)] + ble^{-\phi(o)}[\tau(o) - \tau(f_\circ)] \right] \quad \ldots (63)$$

and finally

$$\frac{\left(W_1 + \frac{c}{3}\right)V_E(f_s - f_0)\Delta V_E}{AP_0 \Delta x} = -f_s + f_0 - f_0 r_E^{\,1-\bar{\gamma}} - \frac{f_s Z_B r_E^{\,1-\bar{\gamma}}}{2\theta_1[x_B + l(1-b)]}$$

$$\times \left[l(1-bz_s)e^{\phi(f_s)-\phi(o)}[\psi(f_0) - \psi(0)] + ble^{-\phi(0)}[\tau(o) - \tau(f_0)] \right] \quad \ldots (64)$$

Considering the particular when $\eta = \frac{1}{\delta}$ we have

$$\frac{\left(W_1 + \frac{c}{3}\right)V_E(f_s - f_0)\Delta V_E}{AP_\circ \Delta x} = -f_s + f_0 - f_\circ r_E^{\,1-\bar{\gamma}}$$

$$-\frac{f_s Z_B r_E^{\,1-\bar{\gamma}}}{2\theta_1[x_B + l]}\left[le^{\phi(f_s)-\phi(o)}[\psi(f_0) - \psi(o)] \right] \quad \ldots \quad \ldots (65)$$

and hence, we get

$$\frac{\left(W_1 + \frac{c}{3}\right)V_E(f_s - f_0)\Delta V_E}{AP_0 \Delta x} = -f_s + f_0 - f_\circ r_E^{\,1-\bar{\gamma}} - \frac{f_s Z_B r_E^{\,1-\bar{\gamma}}}{2\theta_1[x_B + l]} \left\{ l\left[\frac{q - pf_s - f_s^2}{q}\right]^{\frac{M}{2\theta_1}}, \right.$$

$$\times \left[\frac{[(f + p/_2) + \sqrt{q + p^2/_4}]\,[p/_2 - \sqrt{q + p^2/_4}]}{[(f_s + p/_2) - \sqrt{q + p^2/_4}]\,[p/_2 + \sqrt{q + p^2/_4}]}\right]^{\frac{M(f_s + p/_2)}{2\theta_1\sqrt{q + p^2/_4}}}$$

$$\times \left[\frac{1}{\sqrt{q + p^2/_4}}\left(1 - \frac{M(\gamma - 1)}{\theta_1}\left[1 - \frac{p(p + f_0 + f_s) + 2(f_0 f_s + q)}{4(q + p^2/_4)}\right]\right)\right.$$

$$\times \log \frac{[(f_0 + p/_2) - \sqrt{q + p^2/_4}]\,[p/_2 + \sqrt{q + p^2/_4}]}{[(f_0 + p/_2) + \sqrt{q + p^2/_4}]\,[p/_2 - \sqrt{q + p^2/_4}]} - \frac{(p + f_0 + f_s)M(\gamma - 1)}{\theta_1}$$

$$\left[\frac{1}{f_0^2 + pf_0 - q} + \frac{1}{q}\right]$$

$$\left. + \frac{[p(p + f_0 + f_s) + 2(f_0 f_s + q)]M(\gamma - 1)}{2\theta_1(q + p^2/_4)}\left[\frac{f_0 + p/_2}{f_0^2 + pf_0 - q} + \frac{p/_2}{q}\right]\right]\right\} \quad \ldots (66)$$

This gives the change of muzzle velocity $\triangle V_E$, due to a bore resistance $AP_\circ \triangle x$ occurring at the point (f_0, Z_0), in terms of the characteristics of the solution without resistance. For resistance at "burnt", $f_0 = o$ and

$$\frac{\left(W_1+\frac{c}{3}\right)V_E \triangle V_E}{AP_\circ \triangle x}=-1$$

and the muzzle velocity falls. When the resistance occurs during the adiabatic expansion the kinetic energy of the shot and gases at the muzzle are reduced by the work done on the resistance, without other effects on the ballistics (except on the time to traverse the bore). Hence

$$\frac{\left(W_1+\frac{c}{3}\right)V_E \triangle V_E}{AP_\circ \triangle x}=-1$$

if the resistance occurs after "all burnt".

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