ON HEYDENREICH METHOD OF INTERNAL BALLISTICS

by

J. P. Sirpal

Institute of Armament Studies, Kirkee

ABSTRACT

The pressure space curve standardized by Heydenreich has been analysed. It has been found that the index of expansion of the gases after burnt is not a constant and the standardized curve is probably only a mean curve obtained from a number of experimental firing results. A cruve of the form pV^n with a good fit to the Heydenreich values of pressure decay after all-burnt shows that the index of expansion is 1·116 and the chamber volume is 1·2 times the volume of the bore swept up to p_{max}. No defined value of the shot-start pressure is revealed.

Introduction

The Heydenreich method of solution of Internal Ballistics is often used by designers both for development and design work because of the ease and quickness with which the method gives the various ballistics quantities. The method aims at fixing a standardized pressure space curve during the motion of the projectile in the bore, based on a number of experimental firing results such that the energy deduced from the p-s curve equals that of the shot and the gases i.e.

$$E = \frac{w + c/3}{2g} . v_e^2 = p_{mean} . A . s_e . k.$$

where E equals the product of the bore area and the area under the p-s curve, w and c the weights of the projectile and the charge, v_e the muzzle velocity, s_e the projectile travel, A the effective cross-sectional area of the bore, k a factor (usually between 0·9 to 0·96) to take into account losses due to friction, band engraving etc. and p_{mean} the mean pressure of the gases over the length of travel of the projectile. If the ratios p_{mean} to p_{max} and travel of projectile at any instant (s) to its travel up to p_{max} (s_1) are represented by η and λ, then all internal ballistic quantities are represented in this method by some functions of η or λ which are as follows:—

$$s_1 = s_e \, \Sigma(\eta) \qquad t_e = \frac{2s_e}{v_e} \tau(\eta)$$

$$v_1 = v_e \, \phi(\eta) \qquad p_s = p_{max} \, \psi(\lambda)$$

$$t_1 = \frac{2s_e}{v_e} \theta(\delta) \qquad t_s = t_1 \, \xi(\lambda)$$

$$p_e = p_{mean} \, \pi(\eta) \qquad v_e = v_1 \, \Omega(\lambda)$$

where suffix 1 refers to conditions at maximum pressure, suffix e refers to exit conditions and t, v and p represent the time, velocity and pressure at any instant when the travel is s. Two sets of tables are available, one for progressive and the other for degressive burning powders. Little is, however, known about the standardized p-s curve or any other assumptions made in the compilation of these tables. This paper, therefore, aims at the study of the standardized p-s curve and the assumptions, if any, made in the derivation of other Heydenreich functions.

Analysis

It is found that the values of the function $p_s/p_{max} = \psi(\lambda)$ are the same both for progressive and degressive burning powders beyond $\lambda = 1{\cdot}5$. From this it follows that the all burnt occurs for $1 \leqslant \lambda \leqslant 1{\cdot}5$. The nature of the curve showing the pressure growth to its maximum value is however, different in the two cases as expected.

Since after burnt a relation of the form pV^n where p and V are the pressure and the volume of the gases at any instant and n the ratio of specific heats of the product gases, is valid, a curve of this form which would give the best fit to the tabulated values of $p_s/p_{max} = \psi(\lambda)$ for $\lambda \geqslant 1{\cdot}5$ was analysed. If δ be defined as $\dfrac{c_v}{1{\cdot}5\, As_1}$ where c_v is the chamber volume and A the effective cross-sectional area of bore, then

$$p_s\,(c_v + As)^n = p_{1\cdot5} \cdot s_1 \cdot (c_v + 1{\cdot}5As_1)^n$$

whence $$\frac{ps}{p_{max}} = \psi(\lambda)_{1\cdot5} \left[\frac{\delta + 1}{(\delta + \lambda/1{\cdot}5)}\right]^n \text{ for } \lambda \geqslant 1{\cdot}5$$

From the analysis it is found that the values of n and δ vary with the points chosen for their calculations. This is expected because the temperature of the gases changes as the shot moves, consequently affecting the composition and the specific heats of the product gases. However, a fit within $\pm$ 2 per cent is obtained to the Heydenreich points with a value of $n = 1{\cdot}116$ and $\delta = 1{\cdot}2$. Earlier than all burnt a relation of the form PV^n would not be valid. But assuming that such a relation were valid it is found that the above values of n and δ give an error of + 6 per cent at the point of p_{max}.

The points for the determination of an equation representing p_s/p_{max} as a function of λ before the instant of p_{max} are few and a curve of the form $p_s/p_{max} = \dfrac{a\lambda}{(b + \lambda)}$ *with $a = 1{\cdot}2169$ and $b = 0{\cdot}1909$ gives a fit which is within $\pm$ 2 per cent of the tabulated values in the case of degressive powders while for the progressive powders $a = 1{\cdot}1574$ and $b = 0{\cdot}1358$ for the same accuracy of fit. This, however, suggests that the shot start pressure in either case is zero.

As a matter of comparison it may be remarked here that the ratio of the specific heats of the propellant gases is nearly, 1·25 for British propellants

* Fitting of a curve of the form $p_s/p_{max} = a\lambda/b+\lambda)^s$ as is done in the Le Duc's method was tried but the best fit obtained for such a curve gave errors which were about + 9%.

while in the Le Duc's method (American) it is taken to be 7/6. Comparative values of δ as calculated for some of the existing equipments are given below:—

1. B.L. 5.5″. Gun/How	1·70
2. Q.F. 3.7″ How	1·80
3. Q.F. 25 pr	1·10
4. Q.F. 6 pr 7 cwt	1·10

This, therefore, suggests that the Heydenreich tables might have been derived by assuming a mean value of δ for various equipments and a value of n which is even lower than that used in the Le-Duc's method. This may be so with the cooler propellants used in Germany.

Assuming that the Heydenreich p-s curve can be represented by the function $p_s/p_{max} = \psi(\lambda)$, the other Heydenreich functions can be calculated from the following equations which can be easily derived.

(*a*) For the function s_1/s_e :—

P_{mean} for the p-s curve is given by:

$$\frac{p_{mean}}{p_{max}} = \eta = s_1/s_e \int_0^{\lambda_e} \psi(\lambda)\, d\lambda = 1/\lambda_e \int_0^{\lambda_e} \int \psi(\lambda)\, d\lambda$$

where $\lambda_e = s_e/s_1$ from which it follows that s_1/s_e can be represented as a function of η.

(*b*) For exit perssure:

$$p_e \left(\delta + \frac{s_e}{1{\cdot}5 s_1}\right)^n = {}^{p.}1{\cdot}5{\cdot}(\delta + 1)^n \text{ whence}$$

$$p_e = p_{mean} \frac{\psi(\lambda)_{1\cdot5}}{\eta} \left[\frac{\delta + 1}{\delta + \lambda_e/1.5}\right]^n$$

and since λ_e a function of η as determined in (a), p_e can be tabulated as a function of η and hence.

$$p_e = p_{mean}\, \pi(\eta)$$

(*c*) For velocity of projectile at p_{max} (v_1):—

By equating the work under the p-s curve for λ varying between 1 and λ_e to the energy gained by the projectile during this interval we have :

$$\left(\frac{v_e}{v_1}\right)^2 = 1 + \frac{\int_0^{\lambda_e} \psi(\lambda)\, d\lambda}{\int_0^1 \psi(\lambda)\, d\lambda}$$

and since λ_e is a function of η, the relation between v_1 and v_e can be put in the form $v_1 = v_e\, \phi(\eta)$.

(*d*) Velocity of projectile at any instant:—

Again from the work considerations under the p-s curve up to any travel λ_s, the projectile velocity can be shown to be given by

$$\left(\frac{v_s}{v_1}\right)^2 = 1 + \frac{\int_0^{\lambda_s} \psi(\lambda)\, d\lambda}{\int_0^1 \psi(\lambda)\, d\lambda}$$

and hence the function $v_s = v_1\, \Omega(\lambda)$.

(*e*) Time of travel to p_{max} (t_1) :—

$$t_1 = \int_0^1 \frac{s_1}{v}\, d\lambda = \int_0^1 \frac{\Sigma(\eta)}{\varphi(\eta)} \cdot \frac{s_e}{v_e} \cdot \frac{d\lambda}{\Omega(\lambda)}$$

$$= \frac{2\, s_e}{v_e} \cdot \frac{\Sigma(\eta)}{2\varphi(\eta)} \int_0^1 \frac{d\lambda}{\Omega(\lambda)}$$

which in the Heydenreich tables is given as

$$t_1 = \frac{2\, s_e}{v_e}\, \theta(\eta)$$

since $\int_0^1 \frac{d\lambda}{\Omega(\lambda)}$ is a definite integral it follows that $\frac{\Sigma(\eta)}{\varphi(\eta)\,\theta(\eta)}$ is a constant for any value of η. From the Heydenreich tables, it is however, found that this constant varies from 1·74 to 1·78 for progressive burning powders and from 1·83 to 1·91 for degressive burning powders which also indicates that the Heydenreich tables are more likely to have been constructed on the basis of fired results rather than by a theoretical approach.

(f) Time of travel (t_s) :—

As above $$t_s = \int_0^{\lambda} \frac{s_1\, d\lambda}{v} = \frac{2s_e}{v_e} \cdot \frac{\Sigma(\eta)}{2\phi(\eta)} \int_0^{\lambda} \frac{d\lambda}{\Omega(\lambda)}$$

whence $$\frac{t_s}{t_1} = \frac{\int_0^{\lambda} \frac{d\lambda}{\Omega(\lambda)}}{\int_0^1 \frac{d\lambda}{\Omega(\lambda)}}$$

and hence the function $t_s = t_1 \xi(\lambda)$

(*g*) Time to exit (t_e) :—

$$\text{similarly } t_e = s_1 \int_0^{\lambda_e} \frac{d\lambda}{v} = \frac{s_e}{v_e} \frac{\Sigma(\eta)}{\Omega(\eta)} \int_0^{\lambda_e} \frac{d\lambda}{\Omega(\lambda)}$$

and since λ_e is a function of η as determined in (a), t_e can be tabulated as a function of η.

From the above it therefore follows that having fixed p-s curve all other Heydenreich functions can be derived. The method actually followed in constructing the tables is not, however, purely theoretical as mentioned earlier.

Conclusions

(1) This method of solution of internal ballistics is completely empirical and can therefore be used with accuracy only with guns having ballistic similarity to the one which gives the standardized p-s curve of Heydenreich.

(2) It has been shown that all the Heydenreich functions can be generated when once $\psi(\lambda)$ is known as a function of λ.

(3) A mean value of η after burnt which gives the best fit to the tabulated values of $\psi(\lambda)$ is 1·116 which is rather low in comparison with the values used in the LeDuc's method ϑ ($\simeq$7/6) and the British systems δ($\simeq$1·25). The value of δ however in actual designs varies and is of the order of 1·5 but in the Heydenreich method it is assumed to be 1·2.

(4) From the analysis no definite value of shot-start pressure for the standardized p-s curve is revealed.

Acknowledgement

The author is thankful to Dr B. Patnaik for the helpful suggestions given by him in writing this paper.