THE COVOLUME FUNCTION IN GOLDIE'S METHOD OF INTERNAL BALLISTICS

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ABSTRACT

In the present paper, tables of Goldie's covolume function G(f), for four values of θ, eleven values of M, three values of ϕ_o , and five values of f, have been given and these have been used to examine further the validity of Goldie's approximation formula for G(f).

Introduction

An approximation formula was given by Goldie[1] to evaulate the covolume function in his method of solving the equations of Internal Ballistics. About the order of approximation, he made the statement [Corner].[2] that the error would be greatest at all burnt, where it may be as high as $\pm 10\%$. Later Kapur[3] was able to express this covolume function G (f) in terms of incomplete Beta and Gamma functions, and by actually evaluating this function in one particular case, with the help of Pearsons[4]. tables of these functions, he showed that the error in certain cases could be as high as 30%. This led Kapur[5,6] to investigate the validity of Goldie's approximation fourmula, but since use of Pearsons tables require extensive interpolations, he had to use an approximation in his investigation viz he neglected squares and higher powers of ϕ_o, corresponding to the shot—start pressure. His results could therefore be valid only for small values of ϕ_o. To investigate the validity for all ϕ_o, it was necessary to have accurate tables of G (f). Dr. Corner, at the request of the author, agreed to get these tables prepared on an electronic computor in the U.K., to resolve unambiguously, the qestion of the range of validity of Goldie's approximation formula, apart from the necessity of these tables for the accurate computation of pressure-spare curves. These tables have now been prepared and are being reproduced below with Dr. Corner's permission. We have also discussed here the validity of Goldies approximation formula and compared the results obtained here with these obtained earlier for small values of ϕ_o.

Tables of Goldie's Covolume Function

We give below the tables of G (f) for three values of ϕ_o, [0, 0·1, 0·2], four of θ [0, 0·2, 0·5, 1·0], eleven of M [0, 0·4, 0·8, 1·2, 1·6, 2·0, 2·4, 2·8, 3·2, 3·6, 4·0] and five of f (·8, ·6, ·4, ·2, 0)

TABLE I

Goldie's Covolume function $G(f)$ for $\phi_0 = 0$

0	f	M=0	0·4	0·8	1·2	1·6	2·0	2·4	2·8	3·2	3·6	4
0	·8	·200	·193	·185	·178	·171	·165	·159	·153	·147	·142	·137
	·6	·400	·370	·342	·317	·294	·273	·254	·236	·220	·206	·193
	·4	·600	·533	·475	·424	·380	·342	·309	·280	·255	·233	·214
	·2	·800	·684	·587	·507	·440	·385	·340	·301	·270	·243	·220
	0	1	·822	·681	·570	·481	·412	·356	·311	·275	·246	·222
0·2	·8	·232	·225	·218	·211	·204	·197	·191	·185	·179	·174	·169
	·6	·448	·420	·393	·369	·346	·325	·306	·288	·272	·257	·243
	·4	·648	·588	·534	·486	·443	·405	·372	·342	·316	·292	·271
	·2	·832	·731	·644	·570	·507	·454	·408	·368	·335	·306	·281
	0	1	·851	·729	·630	·548	·481	·426	·380	·342	·310	·284
0·5	·8	·280	·273	·266	·259	·252	·246	·240	·234	·228	·222	·217
	·6	·520	·494	·469	·445	·423	·403	·383	·365	·348	·332	·317
	·4	·720	·667	·618	·573	·533	·496	·463	·433	·406	·381	·358
	·2	·880	·796	·721	·656	·598	·548	·503	·464	·429	·399	·372
	0	1	·886	·788	·704	·633	·573	·521	·476	·437	·404	·375
1	·8	·360	·353	·346	·340	·333	·327	·320	·314	·308	·302	·297
	·6	·640	·616	·593	·570	·549	·529	·510	·491	·474	·457	·441
	·4	·840	·794	·751	·712	·674	·640	·608	·578	·550	·524	·500
	·2	·960	·895	·836	·782	·733	·689	·648	·611	·577	·546	·518
	0	1	·927	·861	·802	·748	·700	·657	·617	·582	·550	·520

TABLE II

Goldie's Covolume Function $G(f)$ for $\phi_0 = 0{\cdot}1$

0	·8	·200	·200	·199	·199	·198	·198	·197	·197	·196	·196	·196
	·6	·400	·392	·385	·378	·371	·364	·357	·351	·345	·339	·334
	·4	·600	·574	·550	·527	·506	·486	·467	·450	·434	·419	·405
	·2	·800	·744	·693	·647	·606	·570	·537	·507	·481	·457	·436
	0	1	·901	·816	·743	·680	·626	·579	·538	·504	·474	·448

TABLE II—*contd.*

0	f	M=0	0·4	0·8	1·2	1·6	2·0	2·4	2·8	3·2	3·6	4·0
0·2	·8	·232	·231	·231	·230	·229	·228	·228	·227	·226	·226	·225
	·6	·448	·439	·431	·423	·415	·407	·400	·392	·386	·379	·372
	·4	·648	·621	·596	·573	·551	·530	·511	·493	·476	·460	·446
	·2	·832	·778	·730	·686	·646	·610	·578	·548	·522	·498	·476
	1	1	·912	·835	·768	·710	·659	·614	·576	·542	·513	·487
0·5	·8	·280	·279	·278	·277	·276	·274	·273	·272	·271	·270	·269
	·6	·520	·510	·500	·491	·482	·473	·465	·457	·449	·441	·433
	·4	·720	·693	·668	·644	·622	·600	·581	·562	·544	·527	·512
	·2	·880	·831	·786	·746	·708	·674	·663	·614	·588	·564	·542
	0	1	·928	·864	·807	·756	·711	·672	·636	·605	·576	·551
1	·8	·360	·358	·357	·355	·353	·352	·350	·348	·347	·345	·344
	·6	·640	·629	·618	·607	·597	·587	·577	·568	·558	·549	·541
	·4	·840	·814	·789	·765	·743	·721	·701	·681	·663	·645	·629
	·2	·960	·919	·881	·846	·813	·782	·754	·727	·702	–679	·657
	0	1	·952	·909	·868	·831	·797	·765	·736	·710	·685	·662

TABLE III

Goldie's Covolume Function G (f) For $\varphi_0 = 0\cdot 2$

9	·8	·200	·200	·200	·200	·200	·200	·200	·200	·200	·200	·200
	·6	·400	·398	·396	·395	·393	·391	·389	·388	·386	·384	·383
	·4	·600	·589	·578	·568	·558	·548	·539	·530	·522	·514	·506
	·2	·800	·770	·741	·715	·690	·667	·646	·625	·607	·589	·573
	0	1	·940	·885	·836	·792	·752	·716	·683	·654	·628	·604
0·2	·8	·232	·232	·232	·232	·232	·232	·232	·232	·232	·232	·232
	·6	·448	·445	·443	·440	·438	·435	·433	·430	·428	·425	·423
	·4	·648	·635	·623	·611	·600	·589	·578	·568	·558	·549	·540
	·2	·832	·801	·772	·745	·719	·696	·673	·653	·634	·616	·599
	0	1	·943	·892	·845	·803	·765	·731	·700	·672	·646	·623

Conclusions from the Tables

1. The formula is exact for $\phi_o = 0$, and the error increases as ϕ_o increases.

2. The overall error is larger for tubular charges than for cord charges, and decreases as θ increases.

3. As M increases, the positive error decreases, but the negative error increases in magnitude.

4. The percentage error at shot-start is always positive and goes on decreasing as f decreases, till at all-burnt.

(*i*) If M is small and θ is large, it will be positive.

(*ii*) If M is large and θ is small, it vanishes somewhere before all-burnt and is negative at all burnt.

5. Except for large M and small θ, the error at shot-start would be larger than the error at all burnt.

6. For every value of θ and ϕ_o, there will be some value of M for which the error would be zero—and thus minimum at all-burnt. For $\theta=0$, this value has between 0 and 1·2, for $\theta=0 \cdot 5$, it lies between 1·2 and 2·4; and for $\theta=1 \cdot 0$, it lies between 1·2 and 2·4 for $\phi_o = 0 \cdot 1$ and between 2·4 and 3·6 p for $\phi_o = 0 \cdot 2$.

7. For the ranges of θ, M, ϕ_o considered, the error lies between—29% and 100%.

These conclusions are consistent with those deduced in Kapur[5] by approximate methods. Qualitatively the trend of the error function is similar, though quantitatively, the results here are more exact.

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References

1. Goldie, A. W., A method of calculation of pressure-space curves and its application to a problem in gun design. A.R.D. Bal. 18/46 (1946).

2. Corner J., Theory of the Interior Ballistics of Guns. John Wiley & Sons, New York (1950).

3. Kapur, J. N., Use of incomplete Bela and Ganeno functions in Internal Ballistics Mem. Art. Fran. Paris (1958).

4. Pearson K., Tables of Incomplete Beta and Gamma Functions. Biometrika Office, London (1922, 1934).

5. Kapur, J. N. The evaluation of the covolume Functions in Goldies method of Internal Ballistics (1957).

6. ———— Appendix Note to above paper. Proc. Sec. Cong. Th. & App. Mech. 219 (1957).