FRAGMENTATION AND LETHALITY

by

V. R. Thiruvenkatachar

Defence Science Laboratory, New Delhi

ABSTRACT

The lethality of a H.E. shell or bomb depends on its ability to produce high velocity fragments and blast. The relative importance of these two damaging agents depends on the nature of the targets it is proposed to destroy. Small, high-velocity fragments are effective for the attack of personnel in the open, but aircraft targets require larger fragments. The blast effect from shell-burst inside aircraft wings does considerable damage, but blast is of relatively little importance against heavily armoured targets such as tanks. Fragment effect ceases to be of primary importance here and if the HE shell is to be lethal to such targets it must carry a very large charge of explosive, which will either "scab" the armour or do extensive structural damage by blast and shock.

For assessing the effectiveness of a fragmenting shell or bomb against a given type of target, we have to take into account different characteristics of ammunition and target. The solution of the problem of lethality of ammunition will involve a determination of fragmentation in regard to total number of fragments, their mass distribution, their initial velocities etc. In order to achieve a design with a specific level of lethality in a given situation, it will be necessary to predict the performance for given design data, a process which demands a theoretical treatment if possible, or at least a suffiicent quantity of experimental data which can yield reliable empirical formulae. In this paper an account is given of the various theoretical and empirical aspects and a discussion of these with reference to certain special cases.

Introduction

The lethality of a HE shell or bomb depends on its ability to produce high-velocity fragments and blast. High velocity fragments are effective for the attack of personnel in the open or of aircraft targets. Since fragments produce damage by penetration or perforation, it is necessary to establish quantitative relations which determine the depth to which a fragment of given characteristics such as mass, velocity etc. can penetrate into a target of given quality. This depth of penetration depends on the mass and area of the fragment as well as its velocity at the instant of impact which in turn depends on its initial velocity and the drag due to air-resistance experienced by it during flight. The resistance to a fragment in flight or after penetration into target material depends on the presented area of the fragment. It is therefore important to know this presented area at any instant and the manner in which it varies. Thus prediction of the penetration performance by fragments from a given shell involves a knowledge of the following :—

(1) the mean coefficient of area and the frequency distribution function,

(2) the air resistance to fragments,

(3) mass-distribution of fragments,
(4) initial velocity of the fragments,
(5) the angular distribution of the fragments.

When a shell (or bomb) bursts, it produces irregular fragments of different masses and areas but with a common initial velocity, *viz.* the velocity of the shell casing at the moment of burst.

Coefficient of area and frequency distribution function

If A_i is the frontal area that a fragment presents at any instant when viewed from a point in its line of flight, we define a dimensionless coefficient of area a_i by $A_i = a_i v^{2/3}$ where v is the volume of the fragment.

On account of the instability of their motion, fragments rotate during flight, resulting in a considerable variation of the presented area. The simplest case is that in which the fragment rotates randomly so that all possible orientations are presented in turn with equal frequency. The average presented area over all orientations, assuming random rotation, is called the *mean projected area* 'A' (MPA). Correspondingly we can define a mean coefficient of area 'a' by

$$A = av^{2/3}$$

The coefficients of area ai for natural fragments are determined experimentally by measuring the projected area in different directions and taking the average. In addition to the coefficients of area and their standard deviations it is required to know the frequency distribution of the projected area about the mean value *i.e.* the probability $G(A_i)dA_i$ that A_i lies between A_i and $A_i + dA_i$ Then

$$A = \int A_i \; G(A_i) dA_i$$

and the standard deviation

$$\sigma_{A_i} = \int (A_i - A)^2 \, G(A_i) dA = \overline{A_i^2} - A^2$$

It is found that for natural fragments the distribution function $G(a_i)$ can be represented by

$$G(a_i) = \frac{1}{\sqrt{2\pi}\,\sigma_{a_i}} \exp\left\{- (a_i - a)^2/2\,\sigma_{a_i}^{\,2}\right\}$$

From this we can calculate the probability p (x) that a_i is less than x :

$$p(x) = \tfrac{1}{2}\left[1 + \exp\left(\frac{x - a}{\sqrt{2}\,\sigma_{a_i}}\right)\right]$$

Air resistance to fragments

This is given by $R = \frac{1}{2} C_D \rho A_i V^2$, V=velocity of fragment.

The best estimates of C_D for sharp edged fragments are

$C_D = 1 \cdot 308$ at supersonic speeds

$= 0 \cdot 887$ at subsonic speeds

For smooth bodies such as ellipsoids

$$C_D = \frac{1.038}{1 + 2 \times 10^{-5} V_m}$$

where V_m is the mean velocity over the trajectory. Although C_D in this case varies with the velocity, the variation is small over the range of velocities used and the assumption of constant C_D is sufficiently accurate. Taking C_D as constant and assuming that the fragment rotates randomly so as to present all orientations with sufficient frequency, the equation of motion of the fragment

$$mV \frac{dV}{dr} = -\frac{1}{18} C_D \rho A_i V^2 \quad (\text{'m' in ounces})$$

can be integrated to give the remaining velocity at distance r:

$$V = V_o \exp.\left(-0.002039\, r a_r\, m^{-\frac{1}{3}} \frac{\rho_h}{\rho_o}\right) \quad , V > V_s$$

$$V = V_o \exp.\left(-0.001382\, r a_r\, m^{-\frac{1}{3}} \frac{\rho_h}{\rho_o}\right) \quad , V < V_s$$

where ρ_o is the air density at sea-level, ρ_h is the corresponding density at height h and V_s is the velocity of sound in air at height h. We may without appreciable error, take ρ_h to be the density at the height h of the shell burst.

Mass Distribution of fragments

(*a*) *Mott's formula*: The number of fragments dN whose masses lie between m and m + dm (m in ounces) is given by

$$dN = C \exp.(-M/M_A)\, dM.$$

where $M = m^{1/2}$ and C, M_A are constants depending on the shell and the explosive under consideration. This formula is based on the assumption of random fracture with cracks distributed according to the laws of chance. From an analysis of the results of strawboard trials, allowance being made for secondary breakup, Mott has suggested (for TNT fillings) the formula

$$M_A = 0{\cdot}35\, t^{5/6} d_2^{\,1/3} \left(1 + \frac{t}{d_2}\right)$$

where t is the wall thickness of the shell casing and d_2 its external diameter (both in inches). For other fillings the values of M_A obtained from the formula must be multiplied by an appropriate factor.

Payman's formula—Payman observed that a more realistic representation of mass distribution can be attained by considering a mass frequency rather than a number frequency, i.e. by considering the total mass of the fragments, and not the number of fragments, in each mass interval. If dW is the total mass of all the fragments of individual mass between m and m+dm, Payman's mass-distribution function is defined by

$$dW = w(m)\, dm$$

so that the total mass of all the fragments of individual mass greater than m is given by

$$W = \int_m^\infty w(m)\, dm$$

Payman suggests the mass-distribution formula

$$\log_{10}\left(W/W_o\right) = -cm$$

where W_o =mass of the shell casing and c is a constant for a given shell. If p is the percentage mass of metal in the fragments of mass greater than m, so that

$$p = 100\ W/W_o$$

then Payman's formula can be written as

$$2 - \log_{10}p = cm$$

The value of c can thus be determined for any given shell by finding experimentally the values of p for various values of m and plotting $\log_{10}p$ against m.

Payman's formula gives the best fit of experimental results, although neither Payman's nor Mott's formula is satisfactory for fragments of mass $<0 \cdot 125$ oz.

(*b*) *Effect of various factors on mass distribution*—The mass distribution for natural fragmentation may be altered by varying

(*i*) the strength of the HE filling

(*ii*) the thickness of the shell casing

(*iii*) the nature of the steel of the casing (viz. its carbon content) and

(*iv*) by insertion of a cardboard liner in the shell casing. The results of trials on these lines are summarised here:—

(*i*) Obviously the fragmentation of a shell depends on its filling. Except for shells filled RDX/TNT there was little variation in Payman's constant c with different explosives.

(*ii*) Trials with casings of various thicknesses showd a decrease in the value of c with increasing thickness.

(*iii*) In general a decrease in carbon content of the steel casing gives coarser fragmentation.

(*iv*) The insertion of a cardboard liner also results in coarser fragmentation.

(*c*) *Controlled fragmentation*—The effectiveness of a fragmenting bomb or shell is a maximum when all the fragments omitted have approximately the same mass. The optimum fragment mass can be determined from effectiveness calculations. Almost always the optimum fragment shape is that which gives the minimum coefficient of area and thus ideal fragments would be spherical in shape. Manufacturing and technical difficulties however rule this out. For fragments in the shape of rectangular parallelopipeds, one dimension of the fragment is fixed by the specified wall thickness of the shell casing and hence the minimum coefficient of area is obtained by making the other two dimensions equal. There are several methods of achieving this condition in practice:—

(*i*) Preformed fragments

(*ii*) Grooved ring bomb

(*iii*) Grooved charges and fluted liners.

Initial velocities of fragments

The theoretical derivation of a formula for fragment velocities will require the representation of the detonation process by an idealised model and the various treatments of the problem differ according to the fidelity with which the model represents the actual detonation process. In the simplest of such theories (Gurney) it is assumed that the detonation velocity is infinite and that the detonation products have a constant density throughout the volume at any instant. Considering the shell as a long cylindrical bomb and equating the energy of the charge converted into kinetic energy to the sum of the kinetic energies of the casing and of the gaseous products we get the formula

$$V = (2\Phi)^{1/2}\left\{\frac{E/C}{1+\frac{1}{2}E/C}\right\}^{1/2}$$

where V=initial velocity of the fragments
E=weight of explosive filling
C=weight of shell casing
Φ=energy of charge per unit mass which is converted into kinetic energy.

Extensive trials with model shell casings of various diameters and thicknesses have shown that for shells filled CE/TNT 30/70 or TNT, the relation fitted the results remarkably well. Velocities for other charges can be obtained

$$V = 7850\left\{\frac{E/C}{1+\frac{1}{2}E/C}\right\}^{1/2} - 160$$

by multiplying the velocity given by the above formula by a suitable conversion factor.

Various improvements in the theoretical treatment have been carried out. Thus Thomas and Sterne took into account the variation in the density of the gaseous products while still retaining the assumption of infinite detonation velocity. In G. I. Taylor's theory of the detonation of a long cylindrical bomb, the detonation wave is taken to move with finite velocity. Further the casing is assumed to be heavy, so that the outward velocity of detonation product is small compared to the detonation velocity. The theory is thus applicable for small E/C only. Wilkinson has developed a corresponding theory for light thin casings. Wilkinson has also modified Taylor's theory to take account of the resistance to expansion of the steel casing; like Taylor's theory this modified theory is applicable for low E/C.

Before leaving this topic mention should be made of a formula which has been proposed by Mott:

$$V^2 = K f \times 10^7 . \frac{E}{C}$$

where K is a constant depending on the filling and f depends on E/C. This formula gives a better estimate of the mean fragment velocities from Service shell, which differ from the idealised long cylindrical shell and in general give lower fragment velocities for given values of E/C.

Penetration and perforation by fragments

Experimental results have shown that to a reasonably good approximation, the resistance R of a target to a projectile is proportional to the projected area A of the projectile on the target. The resistance also obviously depends on the projectile velocity V, the density ρ and the strength B of the target material. The resistance per unit area R/A is thus some function of V, B and ρ:

$$R/A_i = f\,(V_1\,B_1\,\rho)$$

One form of this relation which has been studied in some detail is

$$R/A_i = (K_o B + K_2\,\rho V^2)$$

which amounts to assuming that the resistance may be regarded as composed of two parts : a static part which is simply proportional to the strength of the target material, and a dynamical part which arises by movement of the target material

. The equation of motion of a fragment

$$mV\ \frac{dV}{dx} = -R = -A_i\left(K_o B + K_2 \rho V^2\right)$$

may now be integrated to give the relation:

$$\log_e\left(1 + \rho\ \frac{V^2_{ip}}{B}\right) = 2\,\rho\,A_i\ t/m$$

where t=thickness of the plate

V=striking velocity just sufficient to ensure penetration.

and the constant K_0, K_2 have been given suitable values by comparison with results of experiments. In this form the equation has been found to be in remarkably good agreement with results of experiments. In actual fragment calculat ons, for the sake of ease and simplicity in computation, the following formula is used:

$$t = \frac{137.5\ m^{1/3}}{\rho a_i\ (B/\rho)^{1/2}}\left[V_{ip} - 0\cdot 259\left(\frac{B}{\rho}\right)^{1/2}\right]$$

Here t is in inches, V_{ip} in feet/sec, m in ounces, ρ in pounds per cubic foot and B in poundals per square foot. In general this equation holds up to velocities of 4000 ft/sec., For velocities below 1000 ft/sec. the results of experiments can be represented closely by the formula

$$\frac{m^{1/3}\ V_p}{ta_i} = \frac{K}{2.726}$$

where K is a constant, which is chosen so that the two formulae agree at a specified velocity V_e. If the impact velocity V_i is greater than that to just ensure penetration viz. V_{ip}, the fragment perforates the target. Let

$$a_s = \frac{137\cdot 5\ m^{1/3}}{\rho t\ (B/\rho)^{1/2}}\left[V_i - 0.259\left(\frac{B}{\rho}\right)^{1/2}\right] \text{ if } V_i > V_e$$

$$= \frac{2.726\ m^{1/3}\ V_i}{Kt} \text{ if } V_i < V_e$$

then the fragment perforates the target if a_i at the moment of stirke is less than a_s . We have already seen that this probability p is given by

$$p = \tfrac{1}{2}\left[1 + exp\ \frac{a_s - a}{\sqrt{2}\quad \sigma_{a_i}}\right]$$

If N is the total number of fragments which hit the plate then the number of fragments which perforate is Np.

The angular distribution of fragments

In order to estimate the effectiveness of a shell or bomb we require, in addition to a knowledge of the mass distribution and initial velocities of the fragments, information on the direction in which the fragments are projected by the explosion. According to Taylor's theory mentioned in a previous paragraph, the direction of emission of the fragments from a nose-initiated shell is given by

$$\theta = \pi/2 + \mathrm{Sin}^{-1}\,(\mathrm{V}/2\,\mathrm{U})$$

Here θ is measured from the forward direction of the axis of the shell, V is the radial velocity of the expanding case at rupture and U is the velocity of the detonation wave in the explosive. In general this formula gives a reasonably good estimate of the average angle of emission. In actual practice there is an angular spread which may be due to several factors including the constraint existed by the ends of the shell, the variable fragmentation along the length due to the non-planar form of the detonation wave etc. Moreover the distribution for an ogival shell is less concentrated about the peak than for a parallel sided shell. For a shell detonated in flight, the forward velocity of the shell at detonation affects both the velocity and the direction of emission of the fragments. If U is the forward velocity of the shell, the fragments will be emitted in the direction θ_1, with velocity V, where

$$\cot\theta_1 = (u + \mathrm{V}\cos\theta)/\mathrm{V}\,\mathrm{Sin}\theta$$
$$\mathrm{V}_1^{\,2} = u^2 + \mathrm{V}^2 + 2u\mathrm{V}\cos\theta.$$

As the forward velocity of the shell increases, θ_1, at the peak of the angular distribution moves towards the rear of the distribution, since the fragment velocity at the peak is near the highest fragment velocity. Further the spread of the angular distribution is reduced for a shell in flight. On account of the high velocity of the fragments, the spin of a shell has only a minor effect on the angular distribution.

General information of the fragment zones of shell suggests that there is a marked peak zone of fragments which may vary from 10° to 25° in width in which 70—90 per cent of the fragments are projected.

Lethality

The effectiveness of a shell is usually expressed in terms of the "Area of effect" ("vulnerable area" "lethal area") which represents the number of casualties to be expected from a single shell burst among targets uniformly distributed over a large area. Assuming that there is no shielding of targets by each other in an infinite array of targets with a density of distribution of *1* per unit area, the area of effect can be defined mathematically as

$$\mathrm{A.\,E.} = \iint \left(1 - e^{-p} \right) d\mathrm{S}$$

where p is the probable number of hits on the target at any point on the ground and dS is an element of area surrouding that point. In the simplified case where the expected number of hits $p(x)$ at a distance x is the same in all directions the integral takes the form

$$\theta \int_{0}^{x} \left[1 - exp \left\{ -p(x) \right\} \right] x\,dx$$

where θ is the angle swept by the fragments (in radians). The value of $p(x)$ is

$$p(x) = g.\frac{N(x).A}{x^2}$$

where N (x) is the number of incapacitating fragments, A is the effective presented area of the target and g is a factor governing the distribution of fragments. If the projectile bursts just above the ground the only factor governing the distribution of fragments is the burst itself. Fragments are normally concentrated in a fairly well defined zone with subsidiary zones at nose and tail which do not affect the area of effect. Areas of effect calculated on the assumption that all fragments are concentrated in a sharp fragment zone of uniform density are found to agree approximately with the true area of effect. In this case the value of g is $1/\Omega$ where Ω is the solid angle covered by the assumed fragment zone. If the projectile bursts in contact with the ground, some of the fragments will be cut off by small irregularities in the surface, causing a reduction in the density of the fragments. The pressure on the ground at the moment of break-up may directly affect the fragmentation. The combined effect of these factors produces the factor g. Its value is best determined from trials.

For a projectile with a marked peak zone of fragment emission the area of effect is practically independent of Ω on the other hand for projectiles with a steep angle of descent (e.g. bombs) a slight change in the width of the fragment zone makes a big difference in the angle swept by the fragments.

The number of incapacitating fragments at a distance x is given by

$$N(x) = N_o f$$

where N is the number of fragments of mass m and f is the chance of incapacitation. This latter depends on the adopted criterion of incapacitation and is a function of the velocity V_x at distance x from the burst which, in turn, is related to the initial velocity V_o of the fragments by the retardation laws discussed earlier.

The area of effect of a given type of shell against a given type of target is determined by a lethality trial in which a number of rounds are fired against a target layout. If the number of targets is N and the number of effective rounds is n then the number surviving n rounds is

$$N_s = N\left(1 - \frac{A}{NB}\right)^n$$

where A is the area of effect and 1/B is the density of targets. This formula does not take into account all the factors influencing the result. Some of these factors, the general tendency of which is to give a value of A less than the true value are —

(*a*) If the shell bursts are not spread uniformly over the target area considerable overhitting may occur in the region where they are concentrated while other areas will have few casualties.

(*b*) Shell bursting near the edge of the target area will cause fewer casualties than those bursting near the centre, so that the equivalent number of effective rounds is less than the nominal figure.

Discussion

Mr. B. N. Sen (Kirkee) wanted to know the effect on fragmentation of the flight of the shell in fragmentation. Dr. Thiruvenkatachar replied that if a shell was in motion when the fragmentation took place one had to consider both the linear and angular velocities of the projectile. The spin in general had only a small effect. The velocity of the shell altered the direction of emission of the fragments and also their initial velocity.

Mr. B. N. Sen (Kirkee) then enquired whether the formulae derived took account of the secondary fragmentation and whether they were applicable to all sizes of fragments. Dr. Thiruvenkatachar replied that neither Mott's nor Payman's formulae are applicable for very small fragments i.e. less than 1/8 oz. They also took no account of the secondary fragmentation as it is a phenomenon incidental to the process of collection of the fragments and has nothing to do with the mechanism of fragmentation.

In reply to a question from Mr. Sivaramakrishnan (Kirkee) as to the extent to which the theory predicted the influence of the finite velocity of the detonation wave on the size of fragments, Dr. Thiruvenkatachar poind out that Taylor's theory did take into account the finiteness of the velocity of the detonation wave in determining the initial velocity of the fragments. He further said that since the initial velocity of the fragments entered into the calculations of fragment size in Mott's theory there was a definite relation between the two, finite velocity of the detonation wave and the fragment size. However there were no quantitative results available immediately.