AIRCRAFT VIBRATION AND FLUTTER

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ABSTRACT

The paper outlines the theoretical and experimental procedure one has to adopt for flutter prevention during the various stages (project, design and prototype) of the development of a modern aircraft. With the advent of high speed, the aerodynamic coefficients have to be calculated with due regard to the effects of compressibility, finite aspect ratio of the lifting surfaces, sweep back and other peculiar shapes of the wings. The use of thin, small aspect ratio wings with external masses, necessitates the computation of higher frequency modes of vibration. Single degree of freedom flutter and the effect of control surface non-linearities has also become very important.

Thus, it is shown how the availability of high speed computing machines, improved experimental technique for model and full scale testing has not kept pace with the uncertainties associated with the transonic speeds, low aspect ratio and the high frequency modes. Cross-checking of theoretical and experimental results at every stage seems to be the only answer.

Introduction

The problem of aircraft flutter arising as an interaction of inertial, elastic and aerodynamic forces is solved by three groups of workers. One group evaluates the aerodynamic forces which act on an oscillating aerofoil, the second solves the vibration problem due to inertial and elastic forces alone. The third group is to solve the differential equations which express the interaction of all the three forces.

The increase in the aircraft speed and the use of thin swept back or delta shaped aerofoils with small aspect ratios and external masses have complicated the whole problem, because the simplifying assumptions which one could previously make are no longer valid. The theoretical aerodynamic derivatives become quite uncertain in the transonic range, because of the presence of mixed flow and shock waves. One has to consider a greater number of vibration modes, body degrees of freedom and higher overtones. Thus only high speed computing machines, capable of handling six or more degrees of freedom can solve the flutter equations.

Due to the above mentioned complications the flutter problem is solved in at least three different stages. In the project stage, one uses simple semi-empirica formulae to get the flutter speed and to know the mass-balancing required to avoid control surface flutter. In the design stage, one carries out flutter calculations with a limited number of arbitrary or normal vibration modes and using two-dimensional incompressible aerodynamic derivatives with empirical compressibility corrections. When the prototype stage is reached, more elaborate flutter calculations are carried out. The normal vibration modes are checked experimentally. The supersonic and compressible subsonic aerodynamic derivatives (depending upon the aircraft design speed) are used after due regard to sweep and aspect ratio effects. For the transonic range only the aerodynamic derivatives which have been checked experimentally are made use of. The flutter equations include all the important vibration modes. Finally, flutter models are tested in the wind tunnels and in free flight with the help of ground launched rockets.

If the flight tests and the abovementioned elaborate flutter calculations predict marginal stabilities, full scale flight flutter testing has to be carried out.

The paper attempts at presenting the present stage of progress in the different branches of flutter, especially in the aerodynamic field. The flutter procedure adopted these days is also outlined.

Oscillating Aerofoil Theory

The aerodynamic forces are expressed in terms of non-dimensional aerodynamic derivatives, which are given in the form of tables for different values of the reduced frequency parameter $k=\omega c/2V$, where ω is the oscillation frequency, c is the chord length and V is the aircraft speed.

The two-dimensional incompressible aerodynamic derivatives include the effect of control surface and tab deflections, their aerodynamic balance and the effect of air gap between the different moving surfaces. On each span-wise aerofoil section the aerodynamic force refers to the corresponding chord length, thus making use of the *strip theory*, under which the particular section is assumed to be a part of an infinitely long rectangular aerofoil. Extension of Multhopp's lifting surface theory to the case of oscillating aerofoil has not found much favour because the incompressible aerodynamic derivatives are now-a-days used for getting only an approximate answer to the flutter problem.

For supersonic flow, the linearised differential equations of the hyperbolic type satisfied by Prandtl's acceleration potential function have been solved by several methods, namely, superposition of sources and sinks, Laplace transformation and Riemann's method of characteristics. The two-dimensional derivatives have been tabulated for the various values of the Mach number M.

Theoretically it was observed and experimentally confirmed that the pitching damping coefficient became negative at low supersonic speeds ($1 < M^2 < 2 \cdot 5$) for small values of the reduced frequency parameter. This would lead to single degree of freedom flutter and the phenomenon has since been studied theoretically as well as experimentally.

Extension of the two-dimensional theory to three dimensions has been confined to particular aerofoil shapes, namely, rectangular and triangular, the later referring to delta shaped wings. One has to consider purely supersonic, purely sub-sonic or mixed flow conditions according to the delineation of the lifting surface by the Mach lines from the leading tips. The effect of the Mach lines from the control surface leading tips is also being considered.

Calculation of the aerodynamic derivatives in the case of subsonic compressible flow is more difficult than the supersonic case, because the airflows above and below the aerofoil and in the wake are no longer independent. This region is a very important one requiring further investigation because all high speed aircraft have to pass through it and will therefore be discussed in greater detail.

The linearised partial differential equation satisfied by the acceleration potential function is of the elliptic type. Possio obtained the solution of this equation in the form of an integral and expressed the relationship between the aerofoil vertical velocity w(x) (or down-wash) and the lift distribution L () as under

$$w(x) = \frac{\omega}{\rho_o V^2} \int_{-1}^{+1} L(\xi)\, K(M, x-\xi)\, d\xi \qquad ..(1).$$

The time factor *exp* (*iwt*) is omitted from both sides and ρ_o represents the undisturbed fluid density. x and ξ are non-dimensional chordwise coordinates. The leading and trailing edges correspond to $x = -1$ and $x = +1$ respectively. The *kernel* K (M, $x - \xi$) has a singular point at $x = \xi$ which has been isolated in several ways by different authors. Possio[1] and Schwarz[2] put

$$K(M, x-\xi) = K_{Sing}(M, x-\xi) + K_1(M, x-\xi),$$

where K_{sing} (M, $x-\xi$) represents the singular part and K_1 (M, $x - \xi$) stands for the non-singular portion. Tables of both K (M, $x-\xi$) and K_1 (M, $x-\xi$) have been published by Possio[1], Schwarz[2], Dietz[3] and Schade[4]. Frazer[5] isolated the complete expansion of K (M, $x-\xi$) in the vicinity of $x = \xi$. Dietze[3] expresses K (M, $x - \xi$) as the sum of three expressions, one expressing its counterpart in the case M = O. Second is a power series in ($x-\xi$) with indices higher than unity and the third contains the singular terms, constant terms and terms having ($x-\xi$) as a factor.

Function L (ξ) also has a singularity at the point corresponding to the leading edge of the aerofoil. Possio[1] expressed it as

$$\frac{L(\xi)}{\rho_o V^2} = A_o \operatorname{Cot}\frac{\theta}{2} + \operatorname{Sin}^2\theta \sum_{n=1}^{\infty} A_n \operatorname{Cos}^{n-1}\theta \qquad ..(2).$$

where $\xi = -\operatorname{Cos}\theta$. Thus the first term contains the singularity. The aerodynamic forces obtained by using this expression involve a number of co-efficients A_O, A_1,.................... To avoid this Frazer[5] adopted the common expression used in the steady aerofoil theory, namely,

$$\frac{L(\xi)}{\rho_o V^2} = A_o \operatorname{Cot}\frac{\theta}{2} + \sum_{n=1}^{\infty} A_n \operatorname{Sin} n\theta \qquad ..(3).$$

The normal force (measured downwards) is given by the expression

$$-\int_{o}^{\pi} L(\xi) \operatorname{Sin}\theta\, d\theta = -\pi\rho_o V^2 \left(A_o + \frac{1}{2} A_1\right) \qquad ..(4).$$

The pitching moment about the quarter-chord is given by expression

$$\int_{0}^{\pi} L(\xi)\left(\text{Cos}\theta - \frac{1}{2}\right)\text{Sin}\,\theta\,d\theta = -\frac{\pi}{4}\rho_o V^2 (A_1 - A_2) \quad .. \quad .. \quad (5)$$

Using the tabulated values of the kernel K (M, x—ξ) and substituting expression (3) for L (ξ) in equation (1), one obtains the values of the coefficients A_0 , A_1,.............in terms of the aerofoil oscillation. Possio[1] has used only three collocation points (leading edge, trailing edge and the mid-chord point) whereas Frazer[5] made use of 5-point collocation and even seven-point collocation for higher values of the reduced frequency. Dietz[3] has solved equation (1) by an iterative method, whereas Eichler[6] and Schade[4] reduce the problem to the solution of a set of linear algebraic equations, the latter using a series in Legender's polynomial for the non-singular portion of L (ξ).

With a view to checking theoretical results Jones[7] obtained the solution in terms of velocity potential and arrived at the results

$$2\pi(W + I) = \int_{-1}^{\infty} \frac{1}{x-\xi}\frac{\partial k}{\partial \xi}\,d\xi \quad .. \quad .. \quad (6)$$

where $$2\pi I = \int_{-1}^{\infty} \frac{1}{x-\xi}\frac{d}{d\xi}\left[K(\xi)\,\psi\,(x-\xi)\right]d\xi \quad .. \quad .. \quad (7)$$

and $$W = \frac{w}{\sqrt{1-M^2}}\exp\left[-iM^2k/(1-M^2)\right]$$

Equation (6) is similar to that obtained in the incompressible flow. The equations (6) and (7) may be solved either by the method of iteration [7] or by expressing[8] K as a series in the well known functions K_1, K_2, ... which occur in incompressible flow theory. By applying the method of successive approximation, any order of desired accuracy may be attained.

The other approach to the two-dimensional compressible flow equations suggested by several workers, but pursued by workers at National Luchtvaartlaboratorium[9], is to solve it directly in terms of series of Mathieu functions by using elliptic co-ordinates.

The three dimensional theory for the unsteady compressible subsonic flow has been attacked in two different ways. One approach involves the transformation of the governing differential equation by a suitable choice of orthogonal coordinates followed by the use of classical method of separation of variables. This method was used by Schade and Krienes[10] for the oscillating circular plate and has been recently generalised by Kuessner[11]. The cases of infinitely long ribbon, circular and elliptic aerofoils have been studied. In the first case the solution is expressible in terms of Mathieu functions, in the second case the solution involves Lame's functions. The third case involves still less known Wave functions.

The unsteady lifting surface theory is also built up by generalising Possio's integral equation as

$$w(x, y) = \frac{1}{\rho_0 V^2} \iint_S L(\xi, \eta)\, K(M, x-\xi, y-\eta)\, d\xi d\eta \quad .. \quad (8)$$

where S denotes the aerofoil surface. The two dimensional kernel $K(M, x-\xi, y-\eta)$ has been extensively treated by Watkins, Runyan and Woolston[12]. It has been expressed in terms of Bessel and Struve functions and non-singular integrals. The singularities at $y=\eta$ and $x-\xi \leqslant 0$ have been isolated. Finally the kernel has been expressed as a series in powers of reduced frequency. Using these results Runyan and Woolston[13] have shown how three-dimensional derivatives can be calculated for any plan form in the subsonic and sonic cases. Specific results for rectangular and triangular wings compare favourably with the experimental values.

All these theories are based on the assumption of linearised flow, inviscid fluid, small aerofoil thickness, etc. The boundary layer effect and the presence of shock waves and mixed flow are overlooked. To meet the above objection, experiments have been conducted to measure the aerodynamic derivatives in the subsonic, sonic and supersonic regions. These experiments also establish the effects of finite aspect ratio, sweepback and other non-conventional plan forms. For measuring the aerodynamic derivatives one employs rigid models on which only selected degrees of freedom are permitted. Both overall wing derivatives or control surface derivatives can be measured. The experiments can either be conducted in a wind tunnel or in free flight with the help of ground launched rockets, in which case three models pitching about different axes are needed in order to get complete set of derivatives. As experiments on models cannot be completely representative of the actual situation, these derivatives cannot be directly employed for exact flutter calculations. Instead they provide a good check to the theoretical values.

The Vibration Problem

The vibration problem is solved on the basis of the principle of 'semi-rigidity' where the structure is permitted only a certain number of degrees of freedom. The selection of these degrees of freedom is very important. In the beginning one assumed parabolic deflection mode for flexure and linear deflection law for the torsion of fixed root wings. Later simple engineering formulae were used to get the wing deflection mode, and for the torsion mode one assumed the wing to be a pure torque box. The effects of cut out were included by assuming that certain members carried only compressive stresses whereas the others carried shear stresses only. These developments led to what are called uncoupled modes. In general, however, flexure and torsion exist simultaneously in any vibration mode and hence normal modes (coupled) of vibration are being used for more exact flutter calculations. Special methods are applied to treat the case of swept back and delta wings where the various stress-carrying elements are not mutually perpendicular. The effect of including body freedoms and localised masses has also been investigated. This usually necessitates the inclusion of higher modes of vibration.

Even the advanced procedure for calculating the vibration modes is based on simplifying assumptions. Experimental verification at each stage is desirable. Firstly, one can obtain the flexural and torsional influence coefficients by simply measuring the deflections corresponding to applied loads and torques. Secondly for getting the un-coupled resonance modes one excites the structure under investigation by one or more exciters suitably phased and positioned. The deflections at the various points are measured through an arrangement of pick-ups, amplifiers and recorders. Lastly when the first prototype is ready, experiments are conducted to get the normal modes of vibrations.

The results obtained experimentally should not be used for flutter calculations directly, as they have their own limitations. They provide a very good check for the theoretical calculations. Any discrepancies can be investigated and the data used in flutter calculations can be modified accordingly.

Flutter Equations

The interaction of inertial, elastic and aerodynamic forces is expressed as Lagrangian equations of motion for a non-conservative system. For n degrees of freedom one gets n equations of the form

$$\sum_{r=1}^{n} [\, A_{nr}\, \ddot{x}_r + (B_{nr}\, V + D_{nr})\, \dot{x}_r + (C_{nr}\, V^2 + E_{nr})\, x_r \,] = 0 \qquad (9)$$

The coefficients A, B, C, D,........etc., represent the structural and aerodynamic characteristics and depend upon the reduced frequency parameter. At the critical flutter speed where the motion may be assumed to be simple harmonic, the above equations reduce to simultaneous algebraic equations with complex coefficients. Such equations have been solved with the help of direct analogue computors, simultaneous equations solvers or even desk calculators.

It is desirable to solve equation (9) in a general manner. Firstly, because one would like to know the vibration characteristics as the flutter speed is approached and secondly, such a solution may suggest means to avoid flutter. Flutter simulators and differential analysers have been used for this purpose. A flutter simulator capable of handling six degrees of freedom is in operation at the Royal Aircraft Establishment[14], U.K. On the electronic differential analyser (PREDA) available at the Indian Institute of Science, Bangalore, only a simple two degree of freedom problem could be solved[15].

The basic method of solving the flutter equations on flutter simulators or differential analysers is as follows. Assuming a certain value of the reduced frequency $k = wc/2V$ one calculates the coefficients A, B, C........etc. The problem is then set on the machine, with V as variable. Corresponding to the different V values, the machine provides the oscillation frequencies ω. The aircraft speed V and the corresponding ω, satisfying the chosen reduced frequency provides the answer. For this aircraft speed, the oscillation mode is also obtained from the machine.

The calculations are repeated for several values of the reduced frequency, each giving the oscillation mode corresponding to a certain aircraft speed.

This ensemble provides the ranges of aircraft speeds for which the oscillations are either of divergent or damping nature.

For checking the theoretical flutter calculations, flutter models are designed to represent the full scale structure as far as possible. They are tested in wind tunnels, thus providing the flutter speeds and the modes of flutter, for various stiffnesses and mass positions. For high speed work, free flight models are also used with the help of ground launched rockets.

Flutter prediction in practice

The full scale flutter calculations are very complicated, therefore, one improves the accuracy of these calculations as the design proceeds as indicated below.

Design Criteria

In the project stage simple formulæ based on experimental and theoretical considerations are used to get preliminary weight estimates. To avoid wing flexure torsion flutter, a torsional stiffness criterion giving the relationship between the flutter speed and the torsional stiffness is considered sufficient[16]. The criterion includes the effect of Mach number and sweepback and is considered valid for M less than 0·95 and Aspect ratios greater than 3. To avoid control surface (unpowered) flutter the design criteria requires static and dynamic mass balancing of the control surfaces. In addition, the aileron should also satisfy a torsional stiffness criterion, and the powered controls should be operated through either a truly irreversible unit or damper unit of adequate power, having minimum flexibility and blacklash.

Preliminary flutter calculations

Once the design is chosen, simple flutter calculations are carried out. Only a limited number of degrees of freedom are chosen. Arbitrary or uncoupled modes of vibrations are used. The two-dimensional aerodynamic derivatives corrected for the effects of compressibility, aspect ratio (empirical correction) and sweepback are employed, and the problem is solved to get the critical flutter speed. Such a procedure for a conventional unswept wing has been given in a report prepared by the author[17].

Final flutter calculations

As soon as a prototype is ready, normal vibration modes are obtained experimentally and checked with the calculated normal modes. Aerodynamic derivatives appropriate to the plan form and the speed range under investigation and possibly checked by experiments are used, in conjunction with the normal modes for flutter calculations. Higher modes of vibrations and body degrees of freedom are also included. The flutter equations are solved on differential analysers or flutter simulators to get not only the flutter speed, but also the oscillation trend as the flutter speed is approached.

Flight testing

During the course of flight testing a multichannel vibration equipment is installed in the aircraft with a view to finding out if there are any poorly damped vibration modes because such modes generally lead to single degree of freedom flutter. In successive flights the aircraft speed is increased and the vibration modes are recorded. If such tests indicate marginal stabilities, comprehensive Flight Flutter testing is undertaken. The aircraft structure or controls are set into continuous sinusoidal oscillations, and the vibration response is recorded as the oscillation frequency is increased. Successive flights are made at increasing speeds, the approach to the critical flutter condition being indicated by an increase in the response amplitude with air speed. The other alternative is to cut off the exciter suddenly and record the decaying oscillations. As the critical flutter speed is approached, the oscillations take more time to damp out.

Conclusions

The compressible oscillating aerofoil theory in the subsonic range needs further examination and should be extended to three dimensions. The theoretical flutter calculations should invariably be checked by the experimental measurements and *vice versa.*

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