OPERATIONAL RESEARCH IN WEAPON SYSTEM

by

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ABSTRACT

The paper is divided into three parts:—

(*a*) The first part deals with what operational research is.

(*b*) The second part gives what we mean by Weapon Systems and discusses considerations that determine the choice of a particular weapon system from a class of weapon systems.

(*c*) The third part deals with some aspects of weapon replacement policy.

The effectiveness of a weapon system is defined as

$$E = D/C$$

where E is weapon effectiveness (a comparative figure of merit); D is total damage inflicted or prevented and C is total cost, D and C being reduced to common dimensions.

During the course of the investigations, criteria regarding the choice of weapon or weapons from a set of weapon systems are established through production function and military effect curves. A procedure is described which maximises the expectation of military utility in order to select a weapon system from the class of weapon systems. This is done under the following simplifying assumptions:

(*a*) Non-decreasing utility function;

(*b*) Constant average cost for each kind of weapons; and

(*c*) Independence of the performance of each unit of weapon.

Some of the difficulties which arise when any of these restrictions is relaxed are briefly mentioned.

Finally, the policy of weapon replacement and the factors governing the same are described.

Introduction

Operational Research since the World War II has developed quite a lot and has tackled problems not only of direct military importance but of industrial and social nature as well. One of the useful techniques in Operational Research is that of linear and non-linear programming. The choice of an optimum weapon-system has been solved by applying this technique. It is also interesting to find that weapon-system is a topic in which we can apply some of the methods of economics and human rational behaviour. In this paper a brief account of the work so far done on Operational Research in Weapon-System is given.

Section II of this paper deals with what Operational Research is. Section III gives what we mean by Weapon-System and how to make a choice of a particular weapon-system or systems from a class of weapon systems. For the sake of clarity this Section is divided into three parts. In the first of these, weapon system concept is explained. In the second part, for a given combat situation, it is discussed (*i*) how to choose a weapon, (*ii*) in what proportion the various weapons be used, and (*iii*) given proportion of the weapon system how many of them should be employed. In the last part of this Section a mathematical procedure is developed which consists, keeping the cost fixed all through, in maximizing the expectation of military utility in order to select the optimum weapon or class of weapon-systems. Certain assumptions are initially made with a view to getting definite results and then an attempt is made to present the difficulties in relaxing the restrictions. In the last Section the Weapon Replacement Policy is discussed.

Operational research

It is difficult to define Operational Research. Experts in Operatinoal Research have explained in their own ways what Operational Research is. I will, however, give two definitions of Operational Research. The first is due to Morse and Kimball. According to them Operational Research is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control. Professor R.L. Ackoff in the recent book on Operational Research written in collaboration with Mr. Arnoff and Mr. Churchman says that Operational Research is the application of scientific methods, techniques and tools to problems involving the operations of a system so as to provide those in control of the system with optimum solution for the problem. However, whatever be the definition, it should be clear that Operational Research is not concerned with the development of science such as Physics, Chemistry or Mathematics, but it is concerned with the development of techniques of applying scientific knowledge. This is a very important point to note.

I think it will be better if I just explain what we mean by tools, techniques and methods, the words which occur in the definition given in the recent book just mentioned. Calculus, as most of us know, is a scientific tool. Employing calculus to find an optimum value of a variable in a mathematical model of a system is a scientific technique. The plan of utilizing a mathematical model to optimize a system is a scientific method. The different phases of Operational Research may be put down as—

(*i*) formulating the problem;
(*ii*) constructing a mathematical model to represent the system under study;
(*iii*) deriving a solution from the model;
(*iv*) testing the model and the solution derived from it;
(*v*) establishing controls over the solution;
(*vi*) implementation of the solution or solutions.

Weapon system

As we all know, weapon is an instrument of combat for offence or defence. Formerly its assessment was made by its performance alone. The concept of weapon has now been extended to what we call a weapon system which includes not only the main combat equipment but also the related equipment, installations, personnel, maintenance, supply and training, etc. We may, therefore, say that a weapon system is "a composite of equipment, skill and techniques that form an instrument of combat". The main point about the weapon system concept is the definition of what constitutes a weapon, the manner in which it is conceived, researched, developed and produced.

Performance of weapon system is judged by its effectiveness. Under the extended concept of a weapon system, weapon effectiveness, however, is a complex quantity and is usually defined as

$$E = \frac{D}{C}$$

where E is weapon effectiveness (a comparative figure of merit), D is total damage inflicted (or prevented) and C is total cost, D and C being both reduced to common dimensions such as rupees. The effectiveness of a weapon system must always be taken with respect to a particular combat situation. If the mission changes, the effectiveness of weapon system will also change. A rigorous study of the appropriate weapon system components is often necessary for the determination of the effectiveness of military weapon system. I will explain this point by considering the situation where a regiment of entrenched troops has to be defended against attack by enemy tanks. We shall have, therefore, to consider two weapon-systems—Bazooka rocket versus surface-to-surface anti-tank missile. We must, therefore, consider the efficiency or effectiveness of these two weapons in this particular situation.

An example of different nature is given by the fact that infantry is used for patrol and reconnaisance but is also used to destroy bunkers. Tanks are not only used to destroy the enemy bunkers but are also used for patrolling and reconnaissance. Here we shall have to consider in what proportion the tanks and infantry be used. It is rather very difficult to forecast the damage inflicted or prevented and here the military commander is faced with a very difficult task. He has to depend entirely on his ability and experience to find out the damage that he can possibly inflict on enemy with a given weapon system.

Having explained what we mean by a weapon system, we shall now turn to establishing criteria for the selection of a weapon system from a class of weapon systems all of which can be used for a chosen military mission.

There are two bases on which weapon systems are in general compared—

(*i*) For a given level of effectiveness, the weapon system which costs the least is to be selected.

(*ii*) For a given military budget, the one which has the maximum effectiveness is to be preferred.

Whatever basis we use for comparison of weapon systems, cost cannot be ignored altogether. For example, in (*i*) cost is taken as the criterion of choice while in (*ii*) cost has to be calculated because the military budget is fixed.

Let us now suppose[1] that there are two weapon systems only and we are required to establish the criteria for three types of decisions in a given combat situation—

(*i*) which weapon system should we use;

(*ii*) in what proportion should the various weapon systems be used;

(*iii*) given the proportion of the weapon system how many of them should be employed;

Remembering the definition of the effectiveness of weapon system, the military effects will be judged by the enemy personnel killed, wounded or captured and material destroyed or captured. These effects naturally we want to maximize. On the other hand weapon system involves the use of manpower, scarce resources and these are the costs and we want to minimize them. The optimum solution will be given by the allocation of resources to those uses within the army which maximize its contribution to the military power of the nation. This can be done by considering two curves—one called the production function curves and the other known as the military effect curves. Let us suppose that the army is going to spend all its resources on two types of weapon systems—Rifle Companies and Howitzer Batteries. The production function curve represents the most productive use of the available resources. The cost of these weapon systems may either be increasing, constant or decreasing when considered with respect to

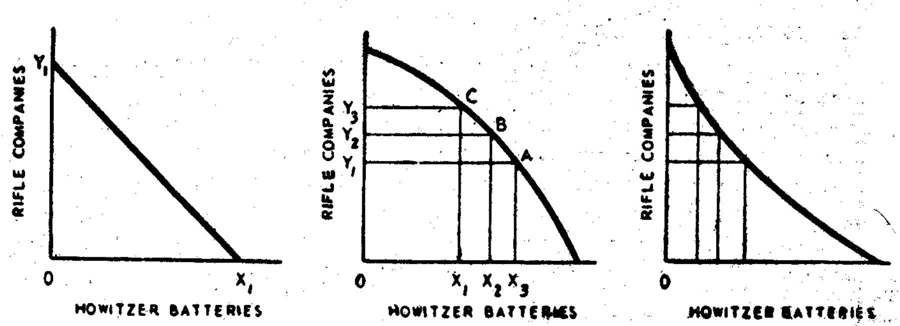


Fig 1

(a) (b) (c)

Production functions : (a) constant marginal cost ; (b) increasing marginal cost ; (c) decreasing marginal cost.

the availability of the materials etc. The military effect curves may also be similarly increasing, constant, or decreasing with respect to two types of weapon systems. As an illustration, let us consider now the military effect curves being convex with respect to the origin and the marginal cost increasing.

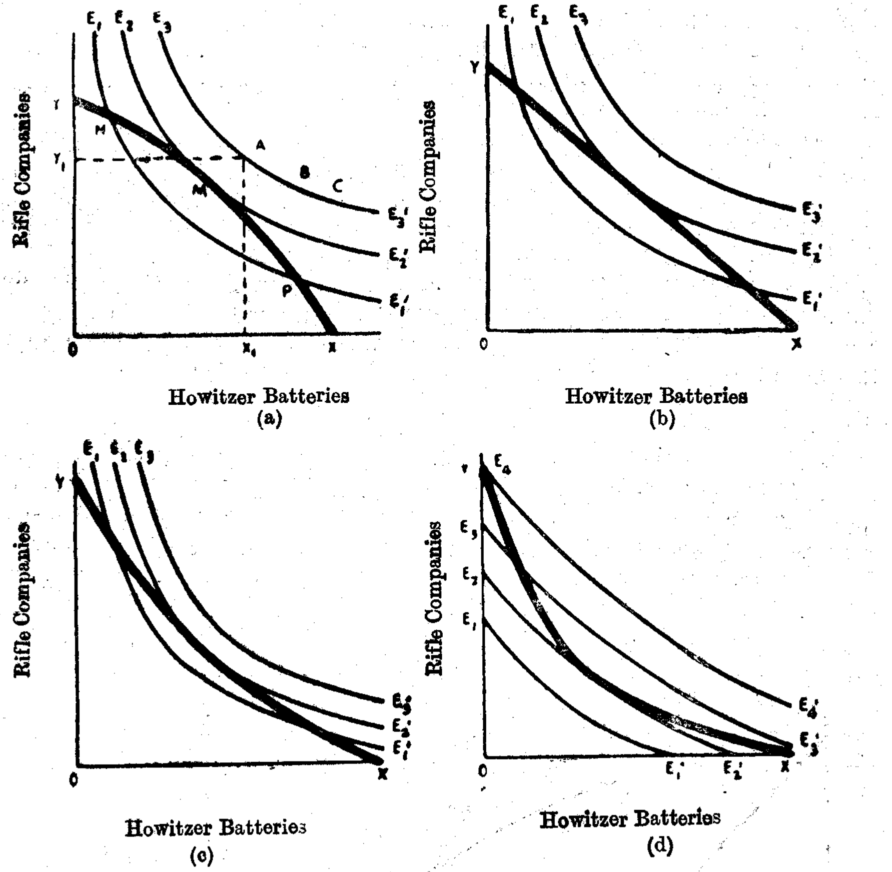


Fig. 2

Allocation under conditions of decrea ing marginal military effect and (a) increasing marginal cost, (b) constant marginal cost, (c) marginal cost decreasing less rapidly than marginal effect, and (d) marginal cost decreasing more rapidly than marginal effect.

The military effect curve E_3E_3' farther away from the origin represents greater military effect than E_2E_2' which is nearer to the origin. If we consider the point A on the military effect curve E_3E_3' we require a combination of OX_1 Howitzer Batteries and OY Rifle Companies. In order to achieve the same military effect at the point B we require more Howitzer Batteries and less of Rifle Companies. These points, however, do not lie on the production function XY which represents the most productive allocation of the resources in between Rifle Companies and Howitzer Batteries. At M, the point of tangency of the production function XY and the military effect curve E_2E_2' the military effect is greater than either at N or P, both of which lie on the production function. The optimum solution will be given by the coordinates of M.

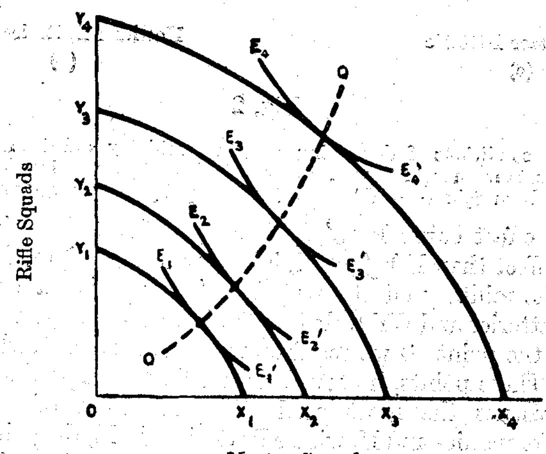
It should be noted that at the point M, the ratio of the marginal costs of rifle Companies and Howitzer batteries is equal to their marginal rates of substitution $E_2.E'_2$.

There are difficulties in finding out the optimum weapon system as outlined above. Even the production function is not easy to obtain—but to find the military effect there are enormous difficulties. From ordinary considerations the military effect curves can only be known if we have a true picture of enemy's capabilities and the nature of number of operations required to defeat him. If these informations were known with some degree of reliability then a model can be constructed.

This difficulty is partially overcome by a process known as Sub-Optimization which only means that we find an optimum solution at a lower level, that is to say, the large number of expected military individual combat operations are reduced to a small number of typical operations. The optimization in each of these typical operations is called Sub-Optimization. The notion of different levels can better be understood if we just remember the fact that destruction of an enemy bunker is a lower level operation than the capture of the enemy-held hill defended by several bunkers and other installations. Further a capture of the hill is at a lower level than a battle in which the attack on the hill is but a part.

Procedure, therefore, will be to estimate different military curves at different levels. We construct the production function X_1Y_1, X_2Y_2, X_3Y_3; Q,Q′ represent the points of tangency of the production function and military effect curves. The sub-optimization solution depends on the initial assumption made regarding the quantity of resources available.

In order to find the quantity of our weapon system we shall now draw two curves—the total cost and the total effect of our weapon system. In the attached figure QQ′ is the total effect curve and CC′ is the total cost curve. Initially when we put in a little cost the effect will be very little and the same will go on increasing as the cost increases till we arrive at a point which may be called the bonus of victory where we have the highest military

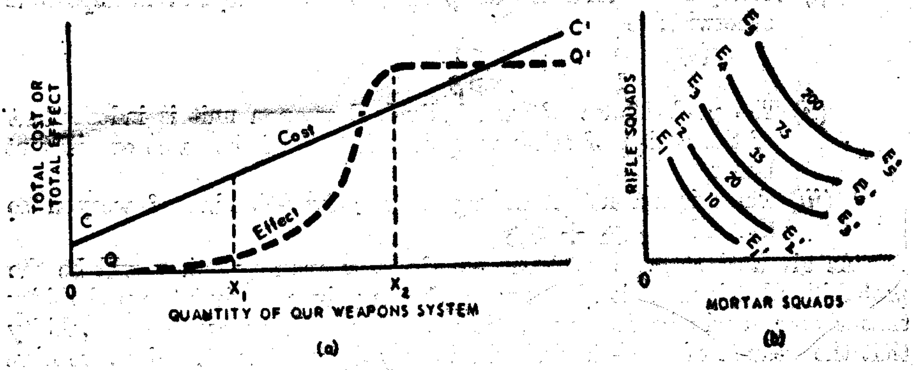


Schedule of suboptimum combinations.

Fig. 3

effect corresponding to a certain cost. Thereafter, with the increase of cost, the military effect will not increase as much.

The criterion for sub-optimization is, therefore, maximization of the difference between the effect and cost and this can only be accomplished where the marginal cost-effect ratios of each of our weapon systems in the operation are equal.



(a) The maximization of profit ; (b) the bonus of victory.

Fig. 4

It will be clear that the above discussions relate to a situation when it is possible to develop alternative weapon systems within the period under consideration. This static model has recently been extended by Messrs. F.P. Hoeber and A. Karchere[2] to cover the case in which some resource cannot be made available in the stated time, regardless of cost.

We will now develop a mathematical procedure[3] which consits in maximizing the expectation of military utility in order to select that weapon system or class of weapon systems, keeping the cost constant throughout and under certain simplifying assumptions. For convenience, we shall consider the case of two types of weapons from which to choose the optimum weapon system (the extension to the general case of n types of weapons presents no extra difficulty). Let us suppose that the number of enemy targets is given by x. Let $f(x)$ be the probability density function (p.d.f.) of x. Suppose that y_i is the quantity of the weapon units of the *i-th* kind. Then the p.d.f. $f(x)$ depends on the vector $y = (y_i, y_2)$. Let us further assume that the cost of producing y is $c(y)$ and the military utility of the damage (m.u.d.) is $u(x)$. Our problem then is to maximize the expected military utility given by

$$E\left[u(x)\right] = \int_0^\infty u(x)\ f(x/y)\, dx \equiv U\ (y)$$

subject to the conditions

$$y_1, y_2 \geqslant 0$$

$$\text{and } C\ (y) = K;$$

(In the latter condition, K denotes the total amount of money available for our purpose and under the integral we have written $f(x/y)$ instead of $f(x)$—this is merely to emphasize the dependence of $f(x)$ on the weapon system y). Optimum value of y and expected utility function $U(y)$ are obviously functions of cost K. In what follows we shall indicate a method of finding $U(y)$ and y under the following simplifying conditions and obtain certain interesting properties of the expected military utility.

(*i*) Victory is certain if at least β targets are destroyed and impossible otherwise: i.e

$$u(x) = 0 \qquad x<\beta$$
$$\quad\;\; = 1 \qquad x \geqslant \beta$$

(*ii*) The performance of each individual weapon unit is independent of the action of other weapons whether of the same or different kinds.

(*iii*) There is a constant average cost c_i for i-th kind of weapon so that $K = c_i y_1 + c_2 y_2$

As given above, the performance of the weapon system y will be the probability density function $f(x/y)$ which for any given x, gives the probability that x targets will be destroyed. Let us assume that p_i is the probability that the weapon of the i-th kind will destroy a given target. Since we have assumed the independence of the performance of each weapon, the probability p_i will be the product of three factors—

(1) the probability of getting to the target which under assumption of the independence of the performance of an individual weapon will depend on the properties of that weapon only;

(2) the probability of hitting the target which depends upon the probable aiming error and lethal radius of the weapon, and

(3) the reliability coefficient which depends on the probability of misfiring.

The probability that y_i units of the i-th kind will destroy x_i targets is given by the term

$$\binom{y_i}{x_i} p_i^{x_i} q_i^{y_i - x_i} \quad , \quad q_i \equiv 1 - p_i$$

of a binomial distribution where mean and variance are

$$\mu_i = p_i\, y_i, \qquad \sigma_i^2 = p_i\, q_i\, y_i$$

Let us assume further that the numerical magnitudes occurring in the above are such as to permit the approximation of the binomial distribution by a normal distribution. The distribution of the total number $x = \Sigma\, x_i$ of the targets destroyed will then be a normal distribution whose mean and standard deviation are given by

$$\mu = \Sigma p_i\, y_i, \qquad \sigma^2 = \Sigma p_i\, q_i\, y_i$$

This result is valid because of the normality of each x_i and because of the independent performance of each weapon unit. μ and σ are obviously functions of y. The expected military utility defined above becomes therefore

$$U(y) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} u(x)\, exp\left[- \frac{(x-\mu)^2}{2\sigma^2} \right] dx$$

If we now make use of the form of u(x) given by assumption (i), we find that

$$U(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(\mu-\beta)/\sigma} e^{-t^2/a}\, dt.$$

The above expression shows that U is a monotone increasing function of $(\mu-\beta)/\sigma$. It is obvious, therefore, that maximizing U is equivalent to maximizing $\omega = (\mu-\beta)/\sigma$. The function U($y$) has some interesting properties. For fixed σ, U increases monotonically with μ. Moreover if $\mu<\beta$, U is less than 1/2 and is monotone increasing with σ; if $\mu>\beta$, U > 1/2 and is monotone decreasing with σ. It follows that for low levels of budgets it may be preferable to concentrate on weapons with higher variance (*i.e.* with higher $p_i\ q_i$ and therefore with p_i near 1/2).

Uptill now we have found out the expected utility function U(y) by using assumptions (i) and (ii) and we also noted that maximizing U(y) is equivalent to maximizing

$$\frac{\mu-\beta}{\sigma} = \frac{p_1y_1 + p_2y_2 - \beta}{\sqrt{p_1q_1y_1 + p_2q_2y_2}}$$

with respect to y_1 and y_2 subject to the condition that $K = C_1y_1+C_2y_2$. This can be solved by elementary methods and need not be discussed further. We will, however, state one interesting result which gives the necessary and sufficient condition which the parameters of the problems must satisfy in order that the optimum weapon system may consist of a mixture of weapons. The required condition is

$$\left(\frac{p_1q_1}{c_1q_o} - \frac{p_o}{c_o}\right)\left(\frac{p_1}{c_1} - \frac{p_o}{c_o}\right) < 0$$

Illustrative Examples:

We will now give three examples as illustrations of the solution of the weapon system problem given above. Example 1 illustrates a case of two weapons in which mixed solutions occur. Example 2 is concerned with the two weapon problem for the case in which the best use of any budget is to expend it entirely on a single weapon.

Example 3 illustrates the choice problem for four weapons for the case in which mixed solutions occur for appropriate budget values.

In examples 1 and 2, three utility functions are considered, specified by the values β=50, 75, 100. In example 3, a single utility function, β=50 is considered.

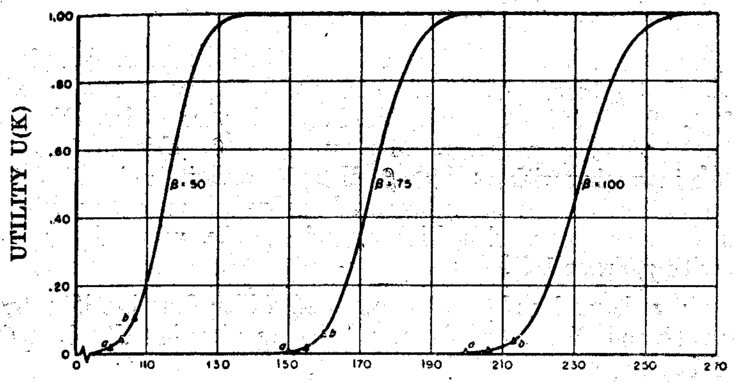
The solutions will be indicated by specifying the ranges in which the entire budget is expended on a single weapon, and, in the case of mixture, by specifying the budget value for which half of the budget is expended on each weapon of the mixture. The maximum expected utilities and the marginal maximum utilities are plotted in figures 5 to 7.

The maximum expected military utility (the utility of the best feasible weapon system) is plotted in the first Fig. as a function of the budget available

The adjacent figure shows the derivative of this function, the so-called "marginal utility" of the budget (or utility of an additional dollar), assuming that the budget is expended on the best feasible weapon system.

Example 1

OPTIMAL WEAPON SYSTEMS



BUDGET K

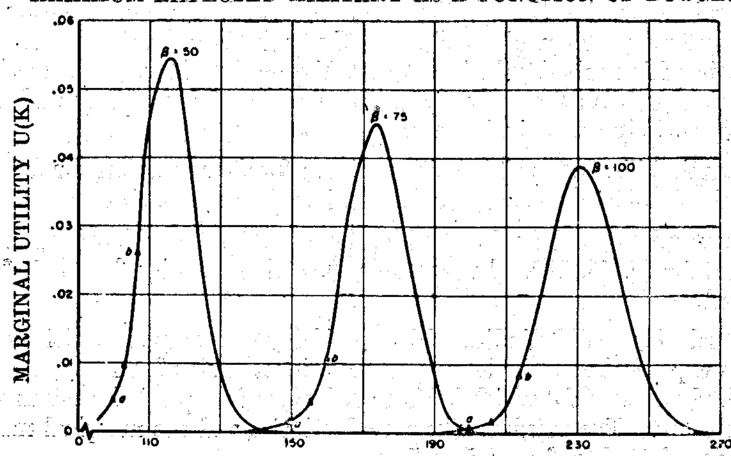
Weapon Characteristics

Weapon	Kill probability	Cost
A	0·4	1·00
B	0·8	1·85

Weapon Choice

Weapon Choice	Budget K		
	$\beta = 50$	$\beta = 75$	$\beta = 100$
A	K = 99·74	K=149·61	K=199·48
mix A and B 50-50	K=103·07	K=154·61	K=206·14
B	K=106·63	K=159·96	K=213·27

MAXIMUM EXPECTED MILITARY AS A FUNCTION OF BUDGET

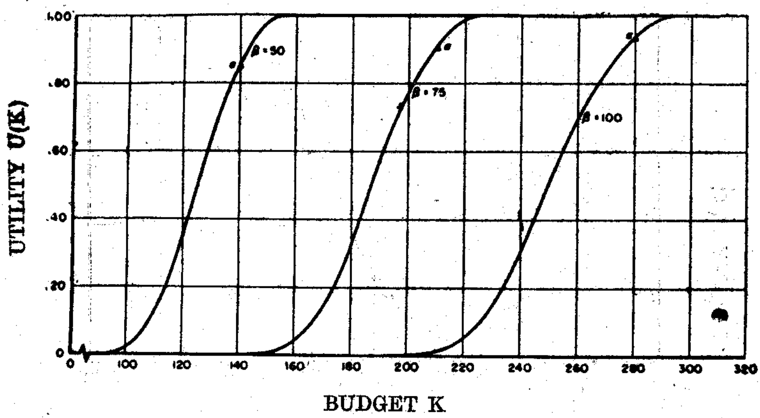


BUDGET K

MARGINAL UTILITY AS A FUNCTION OF BUDGET

Fig 5

Example 2



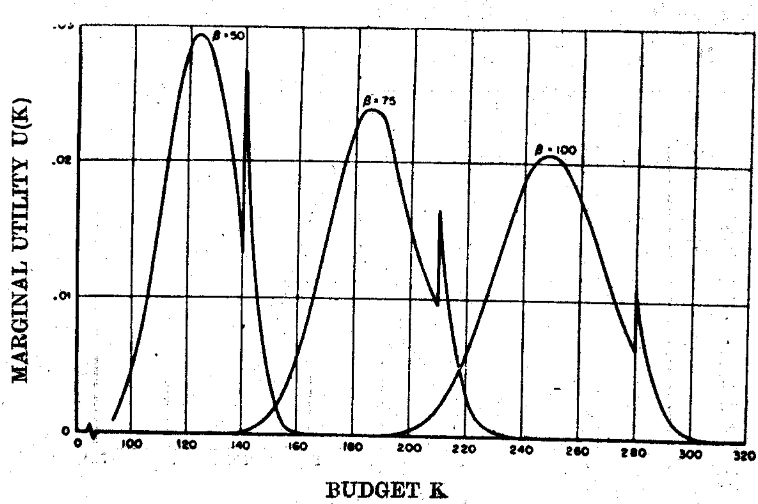
MAXIMUM EXPECTED MILITARY UTILITY AS A FUNCTION OF BUDGET

Weapon Characteristics

Weapon	Kill probability	Cost
A	0·4	1
B	0·8	2·10

Weapon Choice

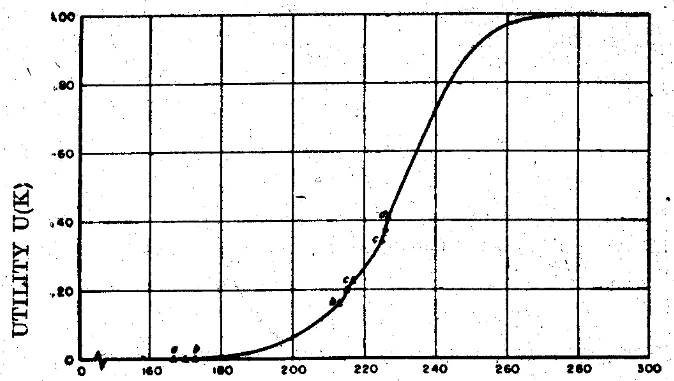
Weapon Choice	Budget K		
	β = 50	β = 75	β = 100
A	K= 139·66	K= 209·49	K= 279·31
B	K-139·64	K= 209·49	K= 279·21



MARGINAL UTILITY AS A FUNCTION OF BUDGET

Fig 6

Example 3



BUDGET K

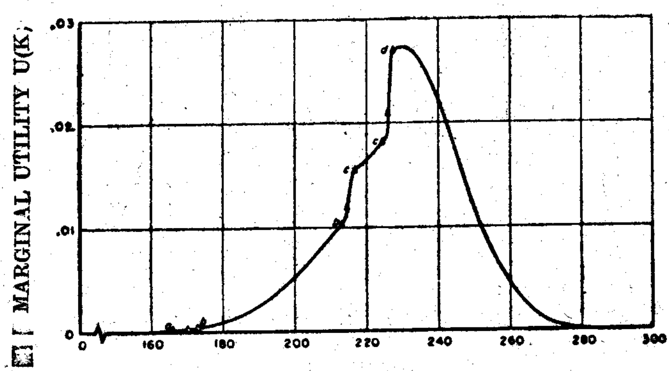
MAXIMUM EXPECTED MILITARY UTILITY AS A FUNCTION OF BUDGET. EXAMPLE 3 (β=50).

Weapon Choice

Weapon Choice	Budget
A	K = 166·83
mix A and B 50-50	K = 169·81
B	172·89 = K= 213·58
mix B and C 50-50	K = 215·33
C	217·11 = K = 225·23
mix C and D 50-50	K = 226·19
D	K = 227·17

Weapon Characteristics

Weapon	Kill probability	Cost
A	0·2	1·0
B	0·4	1·9
C	0·6	2·6
D	0·8	3·7



BUDGET K

MARGINAL UTILITY AS A FUNCTION OF BUDGET

Fig 7

Having dealt with the complete solution of our problem under a specified set of assumptions, we would like to indicate some directions along which the problem can be generalized. The above-mentioned assumptions which we made were such as to make our problem one in "linear programming". If we generalize the assumptions (*ii*) and (*iii*), then both the expected utility function $U(y)$ and the cost function $C(y)$ will be non-linear functions and to maximize $U(y)$ subject to the constraint $C(y)=0$, we will have to use general methods of non-linear programming. These have been discussed by Kuhn and Tucker[4] and Slater[5].

We shall now conclude this section with some very interesting observations on the first restriction.

It is quite a trivial thing that a linear function is a special case of a non-constant non-decreasing monotone function. Even by assuming the military utility to be a linear function, we are including in our discussion the following important points:—

(*a*) If 'x' is an element of a set of outcomes and 'y' an element of a set of decisions, it is known that for a rational decision maker there must exist a function $U(x)$ such that if y and y' are two divisions, and if $U(y)$ is greater than $U(y')$ then y is chosen in preference to y'. By taking the utility function to be linear, we are covering the rational decision maker's attitude to risk as well.

(*b*) If we take $u(x)$ to be linear, we have the advantage of assuming the additional interpretation of military utility as a probability of victory. Whether the enemy does or does not surrender, depends not only on the damage suffered but also on other factors. It is these other factors which are given a chance character. That is to say, given the damage suffered, surrender has a certain probability and we want to maximise the expected value of this probability. We could generalise the very special form of $u(x)$ that we have taken above by supposing that if not more than α targets are destroyed, utility will be zero. If β targets are more destroyed, the utility is equal to 1; for intermediate damage we interpolate linearly *i.e.* $u(x) = 0, \quad x < \alpha$

$$= \frac{x-\alpha}{\beta-\alpha}, \quad \alpha \leqslant x \leqslant \beta$$

$$= 1 \quad x > \beta.$$

(*c*) There is a school of thought which is in favour of maximising the expectation of the number of targets destroyed, namely,

$$E(x) = \int_{\sigma}^{\infty} x\, f(x)\, dx.$$

That is to say, we find out the cost of achieving the expectation for each kind of weapon or systems of weapons, and we choose the one that costs least. We have, however, in this paper taken the case of comparing cost of achieving a given expectation of utility with different weapons or weapon systems and chosing the one that costs least. The approach of our paper is more general. Maximisation of the expectation of the number of targets destroyed is equal to maximisation of the military utility $u(x)$ only when the military utility function is linear. Even when we assume the military utility to be linear, we are neglecting the fact that the utility of destroying an additional target after destroying, say, 150 targets, is not different from that of destroying the 11th target when the 10 targets have already been destroyed. It will be clear therefore that a school of thought which maximises the expectation of the number of targets destroyed is resorting to a waste in resources. The additional money available can, therefore, be invested more profitably in some other concern. There is, however, another disadvantage in assuming $u(x)$ to be non-decreasing. If our object is not only to see that the enemy surrenders but also to see how to deal with the people of the conquered country, we cannot assume the utility function to be non-decreasing. This complicates the mathematical procedure and work is still to be done in this connection.

Finally it is interesting to see that at the first International Conference on Operational Research held on 2—6 September 1957 at Oxford, Mr. H.K. Weiss[6] has shown, by using Lanchester's equations that against a weapon of great effectiveness, it is useful to employ small combat groups to prevent the weapon with large lethal area per round from destroying large number of men per round.

Weapon replacement policy

We have already defined earlier what we mean by efficiency of a weapon system—it is the ratio of its effectiveness to its cost of production. It is easy to see that the efficiency of a weapon system will vary with time. The cost variable is governed by the factors affecting the cost of capital goods in general, whereas effectiveness may change not only by the technical modifications but also due to the development of enemy countermeasures and the evolution of warfare. At this stage it will be better to compare the characteristics of replacement policy as applied to (*i*) a weapon system and (*ii*) an industrial equipment. We note two marked differences.

(*i*) The risks involved in planning a replacement policy by an industrial enterpreneur and the military are not of a comparable order. The former generally has complete information on the trends of the competing products, and the value of the goods or the services produced by an industrial equipment can be tested continuously in the market. The military policy maker is, however, not so fortunate. He cannot test the product of the weapon which is "kills" of men or material and this remains purely potential until combat takes place. Moreover, risk of war involves dependence on a given weapon against measures for defence and counter-attack of which the military has only fragmentary advanced intelligence.

(*ii*) A change in a weapon may alter its effectiveness disproportionately. For example, a 10 per cent. increase in the speed of a plane may increase its "kill probability" against an enemy plane from ·1 to ·4 *i.e.* up to 400 per cent. On the other hand, a change in the characteristics of a non-combatant equipment may be expected to result in change in output of a comparable order of magnitude in general.

It follows, therefore, that there are great uncertainties in military problems and any change that is necessary has to be weighed from several points of view and there may be impact of the industrial resources of the country and other relevant factors on their replacement policy.

In a very recent paper, Mr. J.C.R. Clapham[7] has dealt with the economic life of an equipment. According to him there is an optimum term for which an equipment should be kept before being scrapped. If the capital cost be C and the equipment is used for n years and then discarded (without scrap value), the average long term annual cost will be

$$\Gamma(n) = \frac{c}{n} + \frac{1}{n}\int_{0}^{n} f(t).dt$$

where $\Gamma(n)$ denotes the average annual total cost, depreciation and maintenance, over the years o to n and where the maintenance cost incurred in the interval $(t, t+\delta t)$ is f(t) dt (f(t) does not decrease. It is easy to show that minimum occurs when

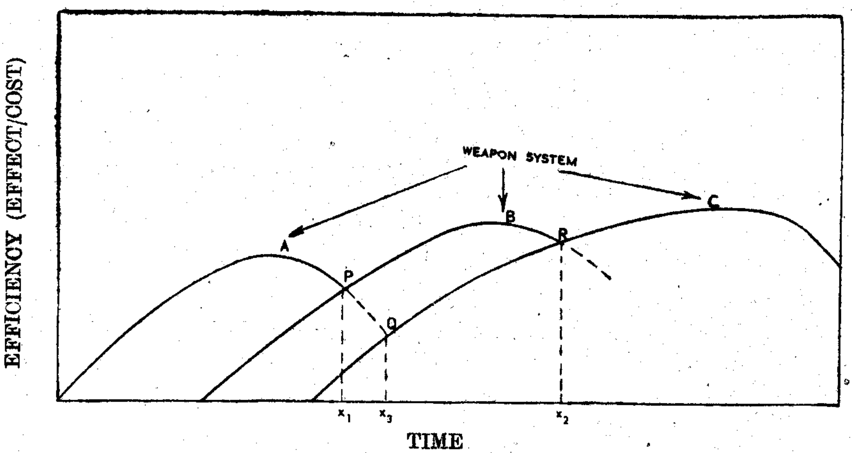
$$f(n) = \Gamma(n)$$

i.e., when the current rate of maintenance cost f(n), is equal to the average total cost to date $\Gamma(n)$. Simple mathematics gives that

$$\Gamma(n+h) = \frac{c}{n}\left(1 + \frac{h^2}{2n^2}\right)$$

From this it is easy to see that for $\frac{h}{n} = \cdot 1$, *i.e.*, if the equipment is kept 10 per cent longer (or shorter) than the economic life, the total cost is only increased by ½ per cent. This means that it may in practice be worthwhile scrapping before the economic life, since the improved version of the new model may well be worth the slight increase in cost.

We now state a criterion based on effectiveness of a weapon system which should govern the replacement policy[8]: "A system becomes subject to replacement when its marginal efficiency falls below that of a new substitute". If we refer to the figure (Fig. 8) wherein we have given a typical curve for efficiency versus time we see that it is convex and that it has a unique maximum. This is easy to explain. Initially, for a given weapon system, effectiveness rises continuously as the mechanics, training and tactical doctrine are perfected whereas the cost decreases as investment increases, techniques are learned and the volume increases. At same piont ratio E/C reaches maximum. Eventually, the effectiveness must decline in the face of counter-measures or changes in warfare and this decline in effectiveness will not be sustained by decline in cost for long so that the efficiency of the weapon falls down. The country has, then, to look to another weapon system that starts developing in stages which has its own curve with a summit. P is the point where these two curves intersect and it is here that we have to change over to the new weapon system; because if we look to the efficiency curve for the weapon 'A', we see that it is now continuing to fall below the point P whereas the efficiency of the new weapon 'B' increases at the point P. Similarly a time may come when we take to weapon system C. But if there are rapid fluctuations in the risk of war, then there is a possibility of a country switching on to the system C from the system A without ever adopting the system B.



THE SUCCESSION OF WEAPONS

Fig 8

Reference

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