THE POSSIBLE FORMS OF BORE RESISTANCE

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ABSTRACT

A method is proposed for calculating the bore resistance of a projectile by assuming as given the pressure-space curve. Applying this method to the Le Duc and the isothermal systems of internal ballistics, it is shown that these systems imply certain physically in admissible features at the beginning of the motion.

Introduction

The equations of ballistics at present do not involve the resistance to the motion of the projectile down the bore explicitly, though such a treatment has been attempted by assuming the nature of its variation with the gas pressure or the shot travel. Here is introduced a method of determining the nature or form of the resistance by inserting it explicitly in the equations of motion and state. The most general case shall be only outlined, while particularizations will be made for the complete solution.

The general equations to start with are:

$$D\frac{df}{dt} = -\beta p^{\alpha} \quad \text{(1,a)}$$

$$\phi = (1 - f)(1 + \theta f) \quad \text{(1,b)}$$

$$W_1 \frac{d^2x}{dt^2} = Ap\left\{W_1/(W_2 + \tfrac{1}{2}C)\right\} - Q \quad \text{(1,c)}$$

$$\text{and } A(W_1 + \tfrac{1}{3}C/W_1 + \tfrac{1}{2}C)\,p\,(x + l - l\,b\,\phi)/(\overline{\gamma} - 1)$$

$$+ (W_1 + \tfrac{1}{3}C)/_2 \left(\frac{dx}{dt}\right)^2 + \int_0^x Q\,dx = (Fc/\overline{\gamma} - 1)\phi \quad \text{(1,d)}$$

where Q is the resistance down the bore and the other symbols have their usual meaning.

Obviously, with the introduction of Q, an extra relation binding the variables is necessary so that a unique solution may be possible still. The normal approach here would be, and has been, to assume a relation between Q and p or x. This determines the solution uniquely. The approach considered in this paper is a semi-empirical one. The experimental pressure-space curve is made the starting point for an analytical treatment.

Suppose the pressure-space curve plotted experimentally for a certain class of guns is given the best fit of the form

$$p = p(x) \quad \text{(2)}$$

Then the only possible form of Q which can ensure the validity of (2) can be determined thus.

The equations above are, in dimensionless form,

$$\dot{f} = \frac{df}{dt} = - \zeta^{\alpha} \quad \ldots \quad (3, a)$$

$$\ddot{\xi} = M (\zeta - q) \quad \ldots \quad (3, b)$$

$$\zeta (\xi - b \phi) + (\bar{\gamma} - 1) \dot{\xi}^2/2M + (\bar{\gamma} - 1) \int_1^{\zeta} q \, d\xi/(1 + C/3W_1) = L\phi \quad \ldots \quad (3, c)$$

and $\zeta = \zeta (\xi)$ .. (3, d)

where $t = (\beta/D) (Fc/Al)^{\alpha} t,$ .. (4)

$L = (W_1 + \tfrac{1}{2}C) / (W_1 + \tfrac{1}{3}C)$ .. (5)

$M = (AD^2/L\beta^2) (Al/FC)^{2\alpha-1} / (W_1 + \tfrac{1}{2}C)$ .. (6)

$\zeta = (Al/FC)p,$ .. (7)

and $\xi = 1 + x/l$ .. (8)

From (3, c)

$$\phi = (L + b \zeta)^{-1} [\zeta \xi + (\bar{\phi} - 1)\dot{\xi}^2 / 2 M + (\bar{\gamma} - 1) \int_1^{\xi} q \, d \xi / (1 + c/3W_1)] \quad \ldots \quad (9)$$

From (3, b) and (9) $\dot{\phi} = (d\phi/d\xi)\dot{\xi}$

$$= \left[2M \int_1^{\xi} (\zeta - q) \, d\xi \right]^{1/2} d\left[(L + b\xi)^{-1} \left\{ \xi\xi + (\bar{\gamma} - 1) \int_1^{\xi} (\zeta - q)d\xi + (\bar{\gamma} - 1) \int_1^{\xi} q d\xi / (1 + c/3 W_1) \right\}^1 \right] \Big/ d\xi \quad \ldots \quad (10)$$

But by (1, b) and (3, a)

$$\dot{\phi} = (2 \theta f - \theta + 1) \dot{f} = [k_1 + (k_2 + k_3 \phi)^{1/2}]\zeta^{\alpha} \quad \ldots \quad (11)$$

where the K's are expressions in θ.

Substituting for ϕ from (9) and for $\dot{\phi}$ from (10), (11) transforms into an equation in ζ, ξ and q only. Using now (3,*d*), we arrive at a relation giving q in terms of ζ or ξ as is required.

Consider, for the sake of simplicity, a tubular propellant charge burning isothermally. Neglecting the co-volume factor, (1, b), (3, *a*) and (3, c) simplify to the form

$$\dot{\phi} = - \dot{f} = \zeta^{\alpha} \quad \ldots \quad (12)$$

and $\zeta\xi = L\phi$ .. (13)

and so (10) is simplified as

$$\dot{\phi} = [2M \int_1^{\xi} (\zeta - q) \, d\xi]^{\frac{1}{2}} \, d (\zeta\xi/L) /d\xi \quad \ldots \quad (14)$$

Rationalising (14) and using (12)

$$q = \zeta - (L^2/2M)\, d\,[\,\zeta^{\alpha} / \{d(\zeta\xi)/d\xi\}] \qquad \ldots (15)$$

Any $\zeta - \xi$ relation as given by (3, d) can now be used and q is got as a function of ζ or ξ.

For instance, let us assume the formula of Le Duc. Then (3, d) is to be taken as

$$\zeta = A(\xi - 1) / (\xi + B)^3 \qquad \ldots (16)$$

where A and B are constants. Assuming a linear law of burning, (15) and (16) lead to

$$q = A(\xi - 1)/(\xi + B)^3 + (L^2 A^{\alpha-1}/2M)\,(\xi - 1)^{\alpha}\,(\xi + B)^{4-3\alpha} \times (\xi^2 - 2\,\overline{B+1}\,\xi + B)^{-1}\,[\,(\alpha/\overline{\xi - 1}) + (4 - 3\alpha)/(\xi + B) - (2\xi - 2\,\overline{B+1})\,(\xi^2 - 2\,\overline{B+1}\,\xi + B)^{-1}\,] \qquad \ldots (17)$$

This result, for all its inelegance, still shows that Le Duc's system is wide off reality in the case of propellants with the rate of burning index less than unity. For $\alpha < 1$ implies q is infinite initially (at $\xi = 1$).

Instead of Le Duc's formula, let us assume the pressure-space curve to be of the form derived in Corner's 'Isothermal' system. Indeed that has been derived from equations not involving the q-terms. Still, q being small. let us accept the p-x relation obtained there as a good approximation to reality. The relation, for tubular charge, is got by combining his pre-burnt p-f and x-f equations, in the form

$$\zeta = A\,\frac{\log \xi}{\xi} \qquad \ldots (18)$$

This having been derived on the basis of a linear law of burning, α is made unity in (17) which is then combined with (18) to beget

$$q = \zeta - (L^2/2M)\,\xi \qquad \ldots (19)$$

ζ being zero initially by (18), initial bore-resistance is obtained as a negative quantity. This shows the discrepancy of Corner's solution from physical conditions at the initial stage.

Even in the absence of an exact equation to fit the pressure—space curve, one can study how the shape of the curve restricts in a general way the possible forms of bore-resistance. For instance the most general properties of a pressure-space curve may be expressed by the conditions

$$d\zeta/d\xi = o \text{ for some value } \xi_m \text{ of } \xi \qquad \ldots (20)$$

$$d^2\zeta/d\xi^2 < o \text{ for } \xi = \xi_i > \xi_m \qquad \ldots (21)$$

Now obviously it must be possible to combine (16), (20) and (21) and derive inequalities binding q, ζ and ξ . . .

References

1. Theory of Interior Ballistics of Guns—Corner.

2. Internal Ballistics—H. M. S. O., London.