THE INTERNAL BALLISTICS OF A HIGH-LOW PRESSURE GUN

By J. N. Kapur

Hindu College, Delhi University, Delhi.

ABSTRACT

In the present paper the system of equations for the Non-Isothermal Model for a H/L gun has been developed and from this, the systems of equations for Isothermal and Non-isothermal models for orthodox gun and solid fuel rockets have been deduced. The Non-Isothermal Model for H/L gun has been integrated for a tubular charge. For isothermal model, reduction to an equivalent non-leaking problem has been discussed and the partially non-isothermal model has been integrated for the linear law. After all-burnt modification needed in Corner's energy equation is obtained.

Introduction

Corner[1,2] first discussed the Internal Ballistics of a High-Low pressure gun. He assumed the temperatures in both the chambers to be the same and to be constant throughout the burning period. After all-burnt, he took the variation of temperature into account, but in writing the energy equation, he overlooked the energy passing from the First Chamber into the Second, and thus his energy equation and the subsequent integration of the equations in this case, require modification.

Later Aggarwal[3] discussed the Non-Isothermal model for the H/L gun. However his model was not completely non-isothermal as much as he took the temperature of the First Chamber to be still constant, and allowed for variations in the temperature of the Second Chamber only.

Even during the burning period, the isothermal and partially non-isothermal models are not completely satisfactory, in as much as they require *a posteriore* justifications by comparison with either more exact non-isothermal models or with experimental observations. But after all-burnt, these models break down completely, as apparently the temperatures of the two chambers decrease, since no more gases are produced by the burning of the propellant and there is at the same-time an increase in the available space for the gases.

It is proposed in the present paper, to discuss a model which takes into account the variation of temperatures in both the chambers even before 'burnt' and to obtain the modifications necessary in Corner's theory after 'burnt'.

It will be seen that both Corner's isothermal and Aggarwal's partially non-isothermal models follow as particular cases of our general model. For both these models, the problem of reducing the equations of First and Second Chambers of a H/L gun to those of an equivalent closed vessel or an equivalent orthodox gun, have been studied. An alternative method of integrating the equations of Internal Ballistics for the isothermal model is given; while the equations for the partially non-isothermal model have been integrated for the Linear Law.

The Fundamental Equation

The figure below gives the basic diagram for a H/L gun.

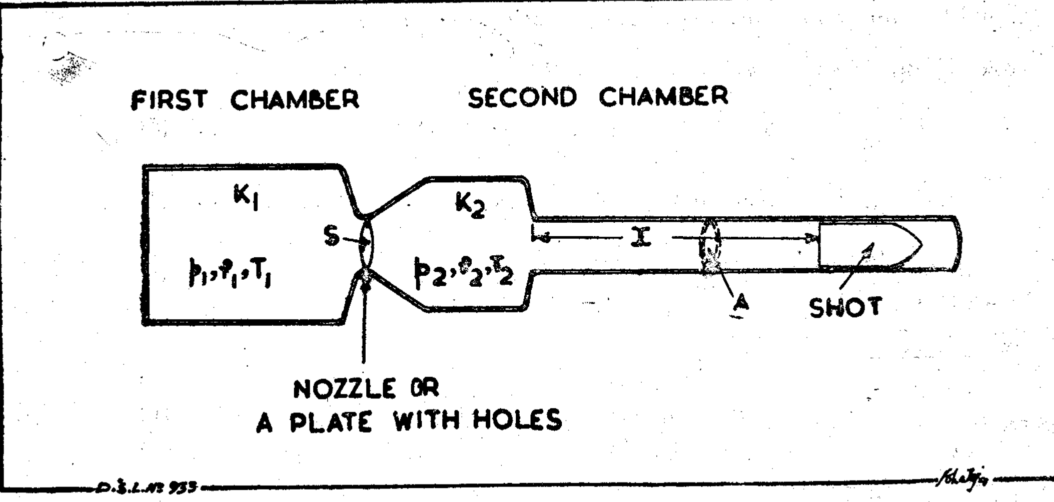

FIG. H/L GUN

At time t, let a fraction z of the charge mass C be burnt and let fractions N and $z-N$ be present in the two chambers. Let p_1, p_2 and T_1, T_2 be the pressures and temperatures in the two chambers whose capacities, when the shot is in its initial position are K_1, K_2. Let η be the covolume per unit mass, S the area of the throat of the nozzle and A the cross-section area of the bore. Then we have the following equations for the Non-Isothermal model for the H/L gun.

Equation of State for the First Chamber:

$$p_1\left[K_1 - \frac{C(1-z)}{\delta} - CN\eta\right] = CNRT_1 \qquad \ldots \quad \ldots \quad \ldots \quad (1)$$

Equation of State for the Second Chamber:

$$p_2\,[K_2 + Ax - C(z-N)\eta] = C(z-N)RT_2 \qquad \ldots \quad \ldots \quad \ldots \quad (2)$$

where x denotes the shot-travel.

Equation of Continuity:

$$\frac{dz}{dt} = \frac{dN}{dt} + \frac{\psi S p_1}{C\sqrt{RT_1}} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

where

$$\psi = \gamma^{\frac{1}{2}}\left(\frac{2}{+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

Here we have assumed that

$$\frac{p_2}{p_1} \leqslant \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \quad \ldots \quad (5)$$

If p_2/p_1 is greater than this critical value, we shall have to multiply the last term in (3) by a suitable back-pressure factor calculated in Corner[2]. This would be equivalent to decreasing S by a suitable factor.

Rate of burning equation:

$$D \cdot \frac{df}{dt} = -\beta p_1 \quad \text{(Restricted Linear Law)} \quad \ldots \quad (6)$$

$$\text{or } D \cdot \frac{df}{dt} = -\beta(p_1+p_o) \quad \text{(Linear Law)} \quad \ldots \quad (7)$$

$$\text{or } D \cdot \frac{df}{dt} = -\beta {p_1}^{\alpha} \quad \text{(Pressure-index Law)} \quad \ldots \quad (8)$$

Form-Function:

$$z = \, _{\iota}\,(f) \quad \text{(general form-function)} \quad \ldots \quad (9)$$

$$\text{or} \quad z = (1-f)\,(1+\theta f) \quad \text{(quadratic form-function)} \quad \ldots \quad (10)$$

$$\text{or} \quad z = 1-f \quad \text{(tubular charge)} \quad \ldots \quad (11)$$

Equation of Motion of the Shot:

Assuming that the conventional Lagrange's corrections for the orthodox gun apply, we have

$$W_2 \frac{d^2x}{dt^2} = W_2 v \frac{dv}{dx} = W_2 \frac{dv}{dt} = Ap_2 \quad \ldots \quad (12)$$

$$\text{where} \quad W_2 = W_1 + \tfrac{1}{3}C \quad \ldots \quad (13)$$

W_1 being the effective mass of the shot.

Equation of Energy for the First Chamber:

(*i*) In time dt, a mass Cdz with internal energy $CdzC_vT_o$ enters the gas. Let δT_1 be the change in temperature of the first chamber due to this additional gas, then

$$Cdz\,[C_v\,T_o = CN_1\,C_v\,\delta T_1 + Cdz\,C_v\,T_1]$$

$$\text{or} \quad \delta T_1 = \frac{dz}{N}(T_o - T_1) \quad \ldots \quad (14)$$

(*ii*) In time dt, a mass C(dz—dN) escapes through the nozzle. Since the expansion is adiabatic, we have, if $\triangle T_1$ is the change in temperature, due to this effect and ρ is gas density in the first chamber

$$d(\log T_1) = -(\gamma-1)\, d \log\left(\frac{1}{\rho} - \eta\right)$$

$$\text{or} \quad \frac{\triangle T_1}{T_1} = \frac{\gamma-1}{1-\eta\rho}\;\frac{\triangle\rho}{\rho}$$

Also $\dfrac{\triangle\rho}{\rho}=\dfrac{dN-dz}{N}$

$$\therefore\ \frac{\triangle T_1}{T_1}=(\gamma-1)\,(1+\varepsilon)\,\frac{dN-dz}{N} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

where $\varepsilon=\dfrac{\eta\rho}{1-\eta\rho}$

The change dT_1 in temperature due to both the factors is:

$$dT_1=\delta T_1+\triangle T_1$$

$$=\frac{dz}{N}\left(T_\circ-T_1\right)+\frac{T_1}{N_1}\left(\gamma-1\right)\left(1+\varepsilon\right)\left(dN-dz\right)$$

$$\text{or } \frac{d}{dt}\left(NT_1\right)=T_\circ\frac{dz}{dt}-\left[\gamma+(\gamma-1)\,\varepsilon\right]T_1\left(\frac{dz}{dt}-\frac{dN}{dt}\right)$$

Using (3), it gives

$$\frac{d}{dt}\left(NT_1\right)=T_\circ\frac{dz}{dt}-\left[\gamma+(\gamma-1)\varepsilon\right]\frac{\psi Sp_1}{CR}\left(RT_1\right)^{\frac{1}{2}} \quad \ldots \quad \ldots \quad (16)$$

or neglecting the covolume term

$$\frac{d}{dt}\left(NT_1\right)=T_\circ\frac{dz}{dt}-\frac{\gamma\psi Sp_1}{CR}\left(RT_1\right)^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

Energy Equation of the first and second chambers taken together

(*i*) energy added to the system in time dt

$$=\frac{CR}{\gamma-1}\,T_\circ\,\frac{dz}{dt}\,dt$$

(*ii*) Work done on the shot $=Ap_2dx$.

(*iii*) Increase in internal energy $=\dfrac{CR}{\gamma-1}d[NT_1+(z-N)T_2]$

$$\therefore\ \frac{CR}{\gamma-1}\,d\,[NT_1+(z-N)\,T_2]$$

$$=\frac{CR}{\gamma-1}T_\circ\,\frac{dz}{dt}\,dt-Ap_2\,dx$$

$$\text{or}\quad\frac{d}{dt}\,[NT_1+(z-N)\,T_2]$$

$$=T_\circ\,\frac{dz}{dt}-\frac{\gamma-1}{CR}\,Ap_2\,\frac{dx}{dt} \quad \ldots \quad \ldots \quad (19)$$

From (12) and (19)

$$\frac{d}{dt}\,[NT_1+(z-N)\,T_2]$$

$$=T_\circ\,\frac{dz}{dt}-\frac{1}{2}\,\frac{\gamma-1}{CR}\,W_2\frac{d}{dt}\left[\left(\frac{dx}{dt}\right)^2\right]$$

which on integration gives

$$\frac{CR}{\gamma-1}\left[T_{o}z-NT_{1}-(z-N)\,T_{2}\right] = \tfrac{1}{2}\,W_{2}\left(\frac{dx}{dt}\right)^{2} \qquad \ldots (20)$$

which simply expresses the fact that the total change in internal energy of the two chambers till time t is equal to the kinetic energy imparted to the shot. If, further, heat losses, are to be taken into account, (20) can be written as

$$T_{o}\,z-NT_{1}-(z-N)\,T_{2} = \tfrac{1}{2}\,\frac{\bar{\gamma}-1}{CR}W_{2}\left(\frac{dx}{dt}\right)^{2} \qquad \ldots (21)$$

$$\text{where } \bar{\gamma}-1=(\gamma-1)\,(1+\chi) \qquad \ldots (22)$$

where χ, assumed constant, is the ratio of heat losses up to any time to the kinetic energy at that time.

Summary of the Fundamental Equations:

$$\left.\begin{array}{lr}
p_{1}\left[K_{1}-\dfrac{C(1-z)}{\delta}-CN\eta\right]=CNRT_{1} & \ldots (1)\\
p_{2}\,[K_{2}+Ax-C\,(z-N)\,\eta]=C(z-N)RT & \ldots (2)\\
\dfrac{dN}{dt}+\dfrac{\psi Sp_{1}}{C\sqrt{RT_{1}}}=\dfrac{dz}{dt} & \ldots (3)\\
\dfrac{Ddf}{dt}=-\beta p_{1} & \ldots (6)\\
z=(1-f)\,(1+\theta f) & \ldots (10)\\
W_{2}\,\dfrac{d^{2}x}{dt^{2}}=Ap_{2} & \ldots (12)\\
T_{o}\,\dfrac{dz}{dt}-\left[\gamma+(\gamma-1)\varepsilon\right]\dfrac{\psi Sp_{1}}{CR}\sqrt{RT_{1}}=\dfrac{d}{dt}\,(NT_{1}) & \ldots (16)\\
T_{o}\,z-NT_{1}-(z-N)\;T_{2}=\tfrac{1}{2}\,\dfrac{\bar{\gamma}-1}{CR}\;W_{2}\left(\dfrac{dx}{dt}\right)^{2} & \ldots (21)
\end{array}\right\}\mathrm{I}$$

Integration of the equations before All-Burnt for a Tubular Charge

For simplicity, we consider the case of a tubular charge only, so that $\theta=0$ and (10) reduces to (11).

Also we substitute

$$RT_{o}=\lambda,\ \frac{T_{1}}{T_{o}}=\mathbf{T}_{1}\ ,\frac{T_{2}}{T_{o}}=\mathbf{T}_{2} \qquad \ldots (23)$$

$$\text{and } \frac{\psi SD}{\beta C\sqrt{RT_{o}}}=\frac{\psi SD}{\beta C\sqrt{\lambda}}=\Psi \qquad \ldots (24)$$

so that $\mathbf{T}_1$, $\mathbf{T}_2$ are the "reduced" temperatures and T_1, T_2 and ψ are dimensionless. The basic equations then become:

$$p_1\left[K_1 - \frac{Cf}{\delta} - CN\eta\right] = CN\lambda\mathbf{T}_1 \qquad \text{.. .. .. (25)}$$

$$p_2\left[K_2 + Ax - C(1-f-N)\eta\right] = C(1-f-N)\lambda\mathbf{T}_2 \qquad \text{.. .. (26)}$$

$$\frac{dN}{dt} - \frac{\Psi}{\sqrt{T_1}}\frac{df}{dt} = -\frac{df}{dt} \qquad \text{.. .. .. (27)}$$

$$-\frac{df}{dt} + \gamma\Psi\sqrt{\mathbf{T}_1}\frac{df}{dt} = \frac{d}{dt}(N\mathbf{T}_1) \qquad \text{.. .. (28)}$$

$$\frac{C\lambda}{\gamma-1}\left[1-f-N\mathbf{T}_1-(1-f-N)\mathbf{T}_2\right] = \tfrac{1}{2}W_2\left(\frac{dx}{dt}\right)^2 \qquad \text{.. (29)}$$

$$W_2\frac{d^2x}{dt^2} = Ap_2 \qquad \text{.. .. (30)}$$

$$D\frac{df}{dt} = -\beta p_1 \qquad \text{.. .. (31)}$$

From (27) and (28), on eliminating $\frac{df}{dt}$

$$\frac{dN}{N} = \frac{dT_1\left(1-\frac{\Psi}{\sqrt{\mathbf{T}_1}}\right)}{1-T_1-(\gamma-1)\Psi\sqrt{T_1}}$$

$$= \frac{-dT_1\left(1-\frac{\Psi}{\sqrt{\mathbf{T}_1}}\right)}{(\sqrt{\mathbf{T}_1}+\alpha)(\sqrt{\mathbf{T}_1}-\beta)} \qquad \text{.. (32)}$$

where

$$\alpha - \beta = \Psi(\gamma-1) \qquad \text{.. .. .. (33a)}$$

$$\alpha\beta = 1 \qquad \text{.. .. .. (33b)}$$

Integrating

$$N = K(\sqrt{\mathbf{T}_1}+\alpha)^{-A}(\sqrt{T_1}-B)^{-B} \qquad \text{.. .. .. (34)}$$

where K is a constant and

$$A + B = 2 \qquad \text{.. .. .. .. (33c)}$$

$$A\beta - B\alpha = 2\Psi \qquad \text{.. .. .. .. (33d)}$$

From (27) and (32)

$$\frac{d\mathbf{T}_1}{df} = \frac{(\sqrt{\mathbf{T}_1}+\alpha)(\sqrt{\mathbf{T}_1}-\beta)}{N} \qquad \text{.. .. (35)}$$

or $\quad df = K(\sqrt{\mathbf{T}_1}+\alpha)^{-1-A}(\sqrt{\mathbf{T}_1}-\beta)^{-1-B}\,d\mathbf{T}_1$

Integrating

$$f - f_o = K \int_{T_{10}}^{T_1} \frac{dT_1}{(\sqrt{T_1} + \alpha)^{1+A} \ (\sqrt{T_1} - \rho)^{1+B}}$$

$$= K \int_{\sqrt{T_{10}}}^{\sqrt{T_1}} \frac{2\,u\,du}{(u + \alpha)^{1+A} \ (u - \rho)^{1+B}} \quad .. \quad (36)$$

so that f can be expressed as a function of $\mathbf{T_1}$. Knowing N and f as functions of $\mathbf{T_1}$, (25) determines p_1 explicitly as a function of $\mathbf{T_1}$.

From (31)

$$\frac{D}{\rho} \ \frac{d\mathbf{T_1}}{dt} = - c\lambda \frac{(\sqrt{\mathbf{T_1}} + \alpha) \ (\sqrt{\mathbf{T_1}} - \beta)}{K_1 - \frac{CF}{\delta} - CN\eta} \quad .. \quad (37)$$

From (37), we obtain t as a function of T_1. From (26), (29) and (30),

$$\frac{c\lambda}{\gamma-1}\Big[\ 1 - f - NT_1\ \Big] - \frac{1}{\gamma-1} \ \frac{W_2}{A} \ \frac{d^2x}{dt^2}\Big[K_2 + Ax - c\eta(1-f-N)\Big]$$

$$= \tfrac{1}{2} W_2 \left(\frac{dx}{dt} \right)^2 \quad .. \quad .. \quad .. \quad (38)$$

Now since

$$\frac{dx}{dt} = \frac{dx}{d\mathbf{T_1}} \ \frac{d\mathbf{T_1}}{dt}$$

$$\frac{d^2x}{dt^2} = \frac{d^2x}{dT_1^2}\left(\frac{dT_1}{dt} \right)^2 + \frac{dx}{dT_1} \ \frac{d^2T_1}{dt^2}$$

from (33), (35), (37) and (38), we get an ordinary differential equation to get

x and $\dfrac{dx}{dT_1}\left(\text{or} \ \dfrac{dx}{dt}\right)$ as function of $\mathbf{T_1}$.

(29) determines T_2 as a function of T_1 and finally (26) will determine p_2 as a function of T_1.

Thus we have been able to express all the variables in terms of T_1, by two simple quadratures and the numerical in tegration of a differential equation of the second order. At this stage, we make the following observations:

(*i*) For most normal H/L guns, ψ lies in the neighbourhood of $\cdot 5$. Also for most modern gun propellants, γ lies in the neighbourhood of $1\cdot 25$. For $\psi = 0\cdot 5$ and $\gamma = 1\cdot 25$, (33) gives $\alpha = 1\cdot 064$, $\beta = \cdot 939$, $A = 1\cdot 56$, $B = \cdot 44$.

For most normal H/L guns, the values of α, β, A, B will lie in the neighbourhood of these values.

(*ii*) The small variations of γ from $1\cdot 25$ are not likely to effect the internal ballistics appreciably. Thus it would be advisable to take $\gamma = 1\cdot 25$ and to tabulate N and $f - f_0$ once for all as functions of **T**.

for

$$\Psi = \cdot 40,\ \cdot 45,\ \cdot 50,\ \cdot 55$$

$$K = \cdot 05,\ \cdot 10,\ \cdot 15,\ \cdot 20$$

(*iii*) K, f_0 are constants to be determined either from assumptions of a nozzle-start pressure and a shot-start pressure or better still by actually fitting the theoretical results with the observational data.

(*iv*) From (35), we find that $\dfrac{d\mathbf{T}_1}{df}$ is positive and since f is decreasing, this implies that T_1 decreases steadily from its initial value unity (or very near it). When $\mathbf{T}_1 = \beta^2$, $\dfrac{dT_1}{df}$ vanishes and after that $\mathbf{T}_1$ would remain constant till all-burnt; but (36) shows that if T_1 attains the value β^2, the integral in (36) becomes divergent for the normal values of A and B in H/L guns and f vanishes before $\sqrt{\mathbf{T}_1}$ reaches the value β. *Thus in normal H/L guns, the temperature in the first chamber falls steadily, but it never attains its lower limit which is about 88 per-cent of the ambient temperature. In a sense this provides the justification for the success of the Isothermal Model* and it gives us greater confidence in their use.

(*v*) From (32), since $\sqrt{\mathbf{T}_1}$ is greater than both β and Ψ, $\dfrac{dN}{dT_1}$ is always negative, but since T_1 is decreasing, *it shows that N will steadily increase till all-burnt* and as the Isothermal Model shows, it will attain the value of about $\cdot 5$ at all-burnt.

(*vi*) From (28)

$$\frac{d}{dt}\,(NT_1) = -\frac{df}{dt}\,(1 - \gamma\,\Psi\sqrt{T_1})$$

For $\gamma = 1\cdot 25$, $\Psi = \cdot 5$, $1 > \sqrt{\mathbf{T}_1} > \beta$ we find that $\dfrac{d}{dt}(NT_1)$ is always positive. Thus while T_1 decreases and N increases, NT_1 steadily increases till all-burnt. *Thus in normal H/L guns, the internal energy of the gas in the First Chamber goes on steadily increasing.*

(*vii*) If K_1, is sufficiently large so that effects of change of volume due to the burning of the propellant and of covolume η can be neglected, then from (25), p_1 is proportional to NT_1, and since NT_1 increases steadily throughout the burning period, p_1 will also steadily increase and the maximum pressure in the First Chamber will occur at all-burnt.

(*viii*) In general from (25), (27) and (28)

$$\frac{1}{C\lambda}\left[\left\{K_1 - \frac{Cf}{\delta} - CN\eta\right]^2 \frac{dp_1}{dt}\right.$$

$$= \left\{\left[K_1 - \frac{Cf}{\delta} - CN\eta\right]\left[-1 + \gamma\psi\sqrt{T_1}\right]\right.$$

$$\left. + NT_1\frac{C}{\delta} + NT_1C\eta\left(\frac{\psi}{\sqrt{T_1}} - 1\right)\right\}\frac{df}{dt} \quad .. \qquad (39)$$

For most propellants η is very nearly equal to $\frac{1}{\delta}$. Using this approximation, we have, if B denotes the suffix corresponding to all-burnt

$$\frac{1}{C\lambda}\left[K_1 - CN_B\,\eta\right]^2\left[\frac{dp_1}{df}\right]_B$$

$$= \left[K_1 - \frac{C}{\delta}N_B\right]\left[-1 + \gamma\,\psi\,\sqrt{T_{1,B}}\right]$$

$$+ N_B\,T_{1,B}\,\frac{C}{\delta}\,\frac{\psi}{\sqrt{T_{1,B}}}$$

$$= K_1\left[-1 + \gamma\,\psi\sqrt{T_{1,B}}\right]$$

$$+ \frac{CN_B}{\delta}\left[\psi\sqrt{T_{1,B}} + 1 - \gamma\,\psi\,\sqrt{T_{1,B}}\right]$$

Or

$$\frac{1}{C\lambda K_1}\left[K_1 - CN_B\,\eta\right]^2\left[\frac{dp_1}{df}\right]_B$$

$$= -\left[1 + \gamma\psi\,\sqrt{T_{1,B}}\right] + \frac{C/\delta}{K_1}\left[1 - (\gamma - 1)\,\psi\sqrt{T_{1,B}}\right]N_B$$

For most normal H/L guns, $\gamma = 1\cdot25$, $\psi = \cdot5$, $\sqrt{T_{1,B}}$ lies between $\cdot9$ and 1 and N_B lies between $\cdot4$ and $\cdot5$ and therefore for normal densities of loading, $\frac{dp_1}{df}$ will be still negative at all-burnt. *Thus in general, for a tubular charge, the maximum pressure will always occur at all-burnt.*

(*ix*) The value $T_{1,B}$ of T_1 at all-burnt is determined from (36) by putting f = 0. Knowing $T_{1,B}$ we determine N_B from (34) and then $p_{1,B}$ is determined from (25)

$$p_{1,B} = \frac{C\lambda NT_{1,B}}{K_1 - C\eta N_B}$$

(39) then determines the sign of $\frac{dp_1}{df}$ at this instant. If it is negative, the maximum pressure occurs at all-burnt. If, however, it comes out to be positive by putting $\frac{dp_1}{df} = 0$, we determine the value of T_1 at which pressure is

maximum and from (25), we know the value of the maximum pressure. In general, however, since p_1, will have to be tabulated as a function of T_1, the table will itself determine both the value of maximum pressure and the value of T_1 at which it occurs.

The Isothermal Model

In this section, we discuss the Isothermal Model as a particular case of our more general model.

The basic Equations

In the isothermal model, a mean temperature of the propellant gases in both the chambers is assumed, so that RT_1 and RT_2 are to be replaced in our equation by their mean value λ. This reduces the number of variables by two and at the same time two of the equations viz., the energy equations of the two chambers have to be dispensed with.

From the system of equations I, we then get the following equations for this Model.

$$
\left.
\begin{aligned}
& p_1 \left[K_1 - \frac{C(1-z)}{\delta} - C N \eta \right] = C N \lambda \quad \cdots \quad \cdots (40) \\
& p_2 \left[K_2 + Ax - C(z - N)\eta \right] = C(z - N)\lambda \quad \cdots \quad \cdots (41) \\
& \frac{dN}{dt} + \frac{\psi S p_1}{C\sqrt{\lambda}} = \frac{dz}{dt} \quad \cdots \quad \cdots \quad \cdots (42) \\
& W_2 \frac{d^2 x}{dt^2} = A p_2 \quad \cdots \quad \cdots \quad \cdots (43) \\
& D \frac{df}{dt} = -\beta p_1 \quad \cdots \quad \cdots \quad \cdots (44) \\
& z = (1-f)(1+\theta f) \quad \cdots \quad \cdots \quad \cdots (45) \\
& \text{Or } z = \phi(f) \quad \cdots \quad \cdots \quad \cdots (46)
\end{aligned}
\right\} \text{III}
$$

These are the equations for the Isothermal Model first obtained by Corner[1]. He discussed the solutions of these equations for the particular case $\theta = 0$. Later the solution of these equations was given by Aggarwal [4,5] for $\theta = 0$ and for the form (46).

An alternative method of solution

Here we give an alternative method of solving the system of equations III, which is comparatively simpler.

From (24), (42) and (44) $\quad \cdots \quad \cdots \quad \cdots$ (47)

$$z = N + \Psi(1-f) \quad \cdots \quad \cdots$$

$\therefore$ from (40)

$$p_1 = \frac{C\lambda\left[\;\;(f) - \Psi(1-f)\right]}{K_1 - \frac{C}{\delta} + \frac{C}{\delta}\varphi(f) - C\eta\,\varphi(f) + C\eta\,\Psi(1-f)} = P_1(f) \quad \cdots \quad (48)$$

This equation had been discussed earlier by the author and it had been shown that

(*a*) while for a tubular charge, maximum pressure in the First Chamber occurs at all-burnt; for a chord charge, for moderate densities of loading, maximum pressure occurs when about three-fourths of the ballistic size has been burnt through[6],

(*b*) maximum pressure is unique for the case of composite charges[7].

Conditions for the maximum pressure to occur before or at all-burnt for form-function (45) or (46) were also obtained[6].

From (44) and (48)

$$D \frac{df}{dt} = -\beta P_1(f) \qquad \ldots \qquad (49)$$

This can be integrated to give t as a function of f. From (41), (43) and (47)

$$W_2 \frac{d^2x}{dt^2} = \frac{AC\lambda\ \Psi(1-f)}{K_2 + Ax - C\Psi\ (1-f)\eta} \qquad \ldots \qquad (50)$$

On using (49), this gives

$$\frac{d}{df}\left[P_1(f)\frac{dx}{df}\right] = \frac{AC\lambda D^2}{\beta^2 W_2}\ \frac{\psi(1-f)}{K_2 + Ax - C\psi(1-f)}\ \frac{1}{P_1(f)} \qquad \ldots \qquad (51)$$

a second-order differential equation between x and f.

For the quadratic form-function (45), on putting

$$X = (K_2 + Ax)\frac{\beta C\lambda}{AD}\ \frac{1+\theta-\psi}{K - \dfrac{C}{\delta}}\left(\frac{W_2}{C\lambda\Psi}\right)^{\frac{1}{2}} \qquad \ldots \qquad (52)$$

$$z = (1-f) \qquad \ldots \qquad (53)$$

(51) becomes

$$\frac{d}{dz}\left[\frac{z - L_1 z^2}{1 + M_1 z + N_1 z^2}\ \frac{dX}{dz}\right] = \frac{1}{X - \nu z}\ \frac{1 + M_1 z + N_1 z^2}{1 - L_1 z} \qquad \ldots \qquad (54)$$

where

$$L_1 = \frac{\theta}{1 + \theta - \Psi} \qquad \ldots \qquad (55)$$

$$M_1 = \frac{\left(\dfrac{C}{\delta} - C\eta\right)(1 + \theta + C\eta\Psi)}{K - \dfrac{C}{\delta}} \qquad \ldots \qquad (56)$$

$$N_1 = \frac{C\theta\left(\eta - \dfrac{1}{\delta}\right)}{K - \dfrac{C}{\delta}} \qquad \ldots \qquad (57)$$

$$\nu = \frac{\eta}{A}\left(\frac{W_1 C\Psi}{A}\right)^{\frac{1}{2}}\frac{\beta C\lambda}{D}\ \frac{1 + \theta - \Psi}{K - \dfrac{C}{\delta}} \qquad \ldots \qquad (58)$$

(54) is similar to the corresponding equation of Aggarwal, but is not exactly identical with it, since our X and ν, unlike his, are dimensionless. This fact helps in the discussion of the convergence of the series solution which can easily be obtained for X in terms of Z.

The comparative simplicity of our solution arises from the fact that we use f as the independent-variable throughout; while Aggarwal works through the variable z, and later again introduces f through the variable Z′ which is nothing but our variable $Z = 1-f$.

The Equivalent Closed Vessel (for the first Chamber)

Substituting in (40) from (45) and (47),

$$p_1\left[K_1 - \frac{C}{\delta} + \frac{C}{\delta}(1-f)(1+\theta f) - c\eta(1-f)(1+\theta f - \Psi)\right]$$
$$= C\lambda(1-f)(1+\theta f - \Psi) \quad \ldots \quad \ldots \quad \ldots \quad (59)$$

For a closed vessel of capacity K′ in which a charge with force constant λ of mass C′, density δ' covolume η' and form-factor θ' burns, the pressure builds up according to the equation

$$p\left[K' - \frac{c'}{\delta'} + \frac{c'}{\delta'}(1-f)(1+\theta' f) - c'\eta'(1-f)\ (1+\theta' f)\right]$$
$$= c'\lambda(1-f)(1+\theta' f) \quad \ldots \quad \ldots \quad \ldots \quad (60)$$

Comparing (59) and (60), we see that if $\theta = o$, the two equations would become identical if

$$\left.\begin{array}{ll} K' = K_1 \ , & c' = c(1-\Psi) \\ \eta' = \eta \ , & \delta' = \delta(1-\Psi) \end{array}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (61)$$

Thus the pressure in the first chamber of a high-low pressure gun, in the case of tubular charge builds up as if it were a closed vessel with charge $C(1-\Psi)$ and density $\delta(1-\Psi)$ and covolume per unit mass. This result was first noticed by Corner[1].

For a charge with any general value of θ, the two equations would become *almost* identical, except at high densities of loading, if

$$K' = K_1,\ C' = C(1-\Psi),\ \delta' = \delta(1-\psi),\ \eta' = \eta \quad \ldots \quad \ldots \quad \ldots \quad (61)$$

$$\text{and } \theta' = \frac{\theta}{1-\Psi} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (62)$$

This result is of about the same accuracy as that for the Recoil-less gun established by Corner[1] and as in that case shows that *a charge which is degressive in an orthodox gun is much more so in the case of a H/L gun.* Actually since Ψ is of the order of $\frac{1}{2}$, θ' is almost double of θ. Thus it is not advisable to use cord in high-low pressure guns and for the same reason the virtues of a progressive shape are especially pronounced in high-low pressure guns.

If we want (59) and (60) to become exactly identical, it is obvious that a single charge does not provide the answer. We try a compositecharge consisting of two component charges with same λ. Let C_1, C_2 be the masses of the two component charges; δ_1, δ_2 be their densities; η_1, η_2 be their covolumes per unit mass and θ_1, θ_2 be the two form-factors. We shall assume, without loss of generality, that each has the same 'effective' ballistic size (ratio of the ballistic size to the rate-of-burning constant) as the original charge so as not to disturb the rate of burning equations and also in order that the two component charges may burn out simultaneously[8]. This would imply that the two equations

$$\frac{D_1}{\beta_1}\frac{df_1}{dt}=-p_1,\ \frac{D_2}{\beta_2}\frac{df_2}{dt}=-p_1$$

give $$\frac{df_1}{dt}=\frac{df_2}{dt},$$

or $1-f_1 = 1-f_2 = 1-f$ (say) .. .. (61)

so that

$$z_1 = (1-f_1)(1+\theta_1 f_1) = (1-f)(1+\theta_1 f) \quad (62)$$

$$z_2 = (1-f_2)(1+\theta_2 f_2) = (1-f)(1+\theta_2 f) \quad (63)$$

Now the equation corresponding to (60), for the composite charge is

$$p\left\{K-\frac{C_1}{\delta_1}-\frac{C_2}{\delta_2}+\frac{C_1z_1}{\delta_1}+\frac{C_2z_2}{\delta_2}-C_1\eta_1 z_1-C_2\eta_2 z_2\right\}$$
$$=\lambda\{C_1 z_1 + C_2 z_2\} \quad (64)$$

Using (62), (63), we find that (59) and (60) would be identical if

$$K_1-\frac{C}{\delta}=K-\frac{C_1}{\delta_1}-\frac{C_2}{\delta_2} \quad (65)$$

$$C_1+C_2 = C(1-\Psi) \quad (66)$$

$$C_1\theta_1 + C_2\theta_2 = C\theta \quad (67)$$

$$\frac{C}{\delta}-C\eta(1-\Psi)$$
$$=\left(\frac{C_1}{\delta_1}-C_1\eta_1\right)+\left(\frac{C_2}{\delta_2}-C_2\eta_2\right) \quad (68)$$

$$\frac{C}{\delta}\theta - C\eta\theta = \left(\frac{C}{\delta}-C_1\eta_1\right)\theta_1$$
$$+\left(\frac{C_2}{\delta_2}-C_2\eta_2\right)\theta_2 \quad (69)$$

It is easily verified that these equations are satisfied if we choose

$K = K_1,\quad \eta_1 = \eta_2 = \eta$ .. .. (70a)

$\theta_1 = 1,\quad \theta_2 = 0$ .. .. (70b)

$C_1 = C\theta,\quad C_2 = C(1-\Psi) - C\theta$ .. .. .. (70c)

$\delta_1 = \delta,\quad \delta_2 = \delta\left(1-\frac{\Psi}{1-\theta}\right)$ .. .. .. (70d)

Thus pressure in the First Chamber of a high-low pressure gun builds up as if it were a closed vessel, and a composite charge consisting of two component charges with same force constants, effective ballistic sizes and covolumes per unit mass burnt in it. One component charge of mass $C\theta$ is in the form of cords and the other component charge of mass $C(1-\Psi)-C\theta$ is in the form of tubes, the densities being δ and $\delta\left(1-\frac{\Psi}{1-\theta}\right)$ respectively.

If the original charge is in tubular form and the composite charge consists of one component viz., tubular charge of mass $C(1-\Psi)$ and of density $\delta\,(1-\Psi)$. This was the result proved by Corner.

If the densities had been equal, the composite charge would have behaved as a single charge with mass $C\,(1-\Psi)$ and form-factor

$$\theta' = \frac{C\,\theta'\cdot 1 + [\,C\,(1-\psi) - C\,\theta]\cdot o}{C\,\theta + [\,C\,(1-\psi) - C\,\theta]} = \frac{\theta}{1-\Psi}$$

which is the same as the result established in (62). The solution (70d) fails if $\theta = 1$ i.e. if the original charge is in cord form. The reason is obvious, since no composite charge consisting of cords and tubes can be equivalent to a charge, which, in this case is, by (62) more degressive than the cord. In this case, if we want a composite charge, we shall have to try components in the form of spheres, cubes and cords, but the problem is not being attacked here as, for reasons explained above, cords are not likely to be used in H/L guns.

Actually for solution (70d) to be valid

$$1-\Psi > \theta \quad \ldots \quad (71)$$

i.e. in general θ should be less than $\frac{1}{2}$. This is confirmed by (62), since for $\theta \geqslant \frac{1}{2}$, $\theta' \geqslant 1$.

The equivalent orthodox Gun (for the Second Chamber). Reason for failure.

From (41) and (47)

$$p_2\,[K_2 + Ax - C\psi\,(1-f)\,\eta\,] = C\psi\,(1-f)\,\lambda \quad \ldots \quad (72)$$

The equation of state for an orthodox gun of bore area A, chamber capacity K′, tubular charge of mass C′, force constant λ, density δ and covolume η' is

$$p[\,K' + Ax - \frac{C'}{\delta} + \frac{C'\,(1-f)}{\delta} - C'\eta'(1-f)] = C'\lambda\,(1-f) \quad \ldots \quad (73)$$

The two equations (72) and (73) become identical if

$$C' = C\psi \quad \ldots \quad (74a)$$

$$K' = K_2 + \frac{C\psi}{\delta} \quad \ldots \quad (74b)$$

$$\eta' = \eta + \frac{1}{\delta} \quad \ldots \quad (74c)$$

Thus the equation of state in the second chamber of a High-Low pressure gun is the same as the corresponding equation in an orthodox gun with chamber capacity, $K_2 + \frac{C\Psi}{\delta}$ and with charge in tubular form of mass $C\psi$, density δ and covolume per unit mass $\eta + \frac{1}{\delta}$. Thus the charge mass is about half, chamber capacity is slightly increased and covolume per unit mass is nearly doubled.

The equation of motion of the shot for the H/L and corresponding orthodox gun would also be the same.

In spite of this, the reduction to an equivalent orthodox gun cannot, however, be made since our equation (72) involves f which has to be determined from

$$D \frac{df}{dt} = - \beta p_1$$

for the H/L gun and from

$$D \frac{df}{dt} = - \beta p_2$$

for the corresponding orthodox gun.

If these two equations had become identical, the H/L gun would not have introduced anything new in Internal Ballistics. The main feature of a H/L gun is, of course, that while the shot moves under pressure p_2, burning takes place under a higher pressure p_1.

Partially Non-Isothermal Model

Basic Equations

Aggarwal[3] has discussed a non-isothermal model in which he has assumed the temperature of the First Chamber to be constant and has considered the variation in the temperature of the second chamber only. Thus in our system of equations I, one variable T_1 is reduced and the corresponding energy equation of the First Chamber is to be dispensed with. In this case, the equations become

$$\left.\begin{aligned}
&p_1 \left[K_1 - \frac{C(1-z)}{\delta} - CN\eta\right] = CN\lambda \quad \ldots \quad (75)\\
&p_2 \left[K_2 + Ax - C(z-N)\eta\right] = C(z-N)RT_2 \quad \ldots \quad (76)\\
&\frac{dN}{dt} + \frac{\Psi S p_1}{C\sqrt{\lambda}} = \frac{dz}{dt} \quad \ldots \quad (77)\\
&W_2 \frac{d^2x}{dt^2} = Ap_2 \quad \ldots \quad (78)\\
&CR(z-N)(T_o - T_2) = \tfrac{1}{2}(\gamma - 1) W_2 \left(\frac{dx}{dt}\right)^2 \quad \ldots \quad (79)\\
&D \frac{df}{dt} = - \beta p_1 \quad \ldots \quad (80)\\
&z = (1-f)(1+\theta f) \quad \ldots \quad (81)
\end{aligned}\right\} \text{IV}$$

These are the same equations as obtained by Aggarwal, who integrated them for law (80).

Another Law of burning, which is nearer the truth, though more difficult to handle analytically is the Linear Law (7). The Equations of the Isothermal Model for this law were integrated by Kapur[6]. In the next section, we integrate the equations of scheme IV for this law.

Integration for the Linear Law

From (7) and (77)

$$\frac{dz}{dt} = \frac{dN}{dt} + \frac{\psi S}{C\sqrt{\lambda}} \left[- \frac{D}{\beta} \frac{df}{dt} - p_o \right]$$

Integrating and using (24)

$$N = z - \Psi\,(1 - f) + \Psi\,\frac{\beta}{D}\,p_o t \quad \ldots \quad (82)$$

since initially $t = 0, f = 1, z = 0, N = 0$ .. .. .. (83)

From (7), (9), (75) and (80)

$$-\frac{D}{\beta}\,\frac{df}{dt} = p_o + \frac{C\lambda\left[\phi\,(f) - \Psi\,(1 - f) + \Psi\,\frac{\beta}{D}\,p_o t\right]}{\left\{\begin{array}{l} K_1 - \frac{C}{\delta} - C\eta\,\Psi\,\frac{\beta}{D}\,p_o t \\ + \left(\frac{C}{\delta} - C\eta\right)\phi\,(f) + (1 - f)\,C\,\eta\psi \end{array}\right\}} \quad \ldots \quad (84)$$

For the quadratic form-function (81), we get

$$-\frac{D}{\beta}\,\frac{df}{dt} = p_o + \frac{C\lambda\left\{(1 - f)\,(1 + \theta f) - \Psi\,(1 - f) + \Psi\,\frac{\beta}{D}\,p_o t\right\}}{\left\{\begin{array}{l} K_1 - \frac{C}{\delta} + \frac{C}{\delta}\,(1 - f)\,(1 + \theta f) \\ - C\eta\,(1 - f)\,(1 + \theta f) + C\eta\,\Psi\,(1 - f) \\ - C\eta\,\Psi\,\frac{\beta}{D}\,p_o t \end{array}\right\}} \quad \ldots \quad (85)$$

and for tubular charge,

$$-\frac{D}{\beta}\,\frac{df}{dt} = p_o + \frac{C\lambda\,[(1 - f)\,(1 - \psi)] + C\lambda\psi\,\frac{\beta}{D}\,p_o t}{\left[K_1 - \frac{C}{\delta} + (1 - f)\left(\frac{C}{\delta} - C\eta + C\eta\Psi\right) - C\eta\Psi\,\frac{\beta}{D}\,p_o t\right]} \quad \ldots (86)$$

(85) and (86) can be easily integrated in finite terms by means of simple integrations. For the more general function, numerical integration may be necessary to get f as a function of t.

Knowing f as a function of t, (82) determines N as a function of t and then (75) determines p_1, as a function of t.

From (76), (78), and (79)

$$W_2\,\frac{d^2x}{dt^2}\left[K_2 + Ax - C\eta\left(\Psi\,(1 - f) - \Psi\,\frac{\beta}{D}\,p_o t\right)\right] = C\left[\Psi(1 - f) - \Psi\,\frac{\beta}{D}\,p_o t\right]\lambda - \tfrac{1}{2}\,(\bar{\gamma} - 1)\,W_2\left(\frac{dx}{dt}\right)^2 \quad \ldots \quad (87)$$

Since f is a known function of t, (87) gives an ordinary differential equation to solve for x and $\frac{dx}{dt}$ as functions of t. (79) and (76) then determine T_2 and p_2 as functions of t.

From (75)

$$p_1 = \frac{C\lambda N}{K_1 - \frac{C}{\delta} + \frac{Cz}{\delta} - CN\eta}$$

$$\therefore \left[K_1 - \frac{C}{\delta} + \frac{Cz}{\delta} - CN\eta\right]^2 \frac{1}{C\lambda}\frac{dp_1}{df}$$

$$= \left[K_1 - \frac{C}{\delta} + \frac{C}{\delta}\varphi\,(f)\right]\left[\phi'(f)\frac{df}{dt} + \psi\frac{df}{dt} + \psi\frac{\beta}{D}p_o\right]$$

$$- \left[\psi(f) - \psi\,(1-f) + \psi\frac{\beta}{D}p_o t\right]\left[\frac{C}{\delta}\,\psi'(f)\frac{df}{dt}\right] \quad \cdots \quad (88)$$

From (84) or (85) or (86) we know f and $\frac{df}{dt}$ as functions of t. Substituting these values in (88) we determine the instant of maximum pressure. Then the maximum pressure is determined by eqn. (75).

Putting $f = 0$ in the solution of (84) or (85) or (86) we get the instant of all-burnt position.

Reduction to an equivalent closed vessel or orthodox gun

Since equations (75), (77), (80) and (81) of this model are the same as equations (40), (42), (44) and (46) for the Isothermal Model and these are the only equations referring to the First Chamber, our discussion of the Isothermal Model remains valid here also.

The equation of energy for an orthodox gun for a tubular charge is given by

$$p\left[K' - \frac{C'}{\delta} + Ax - C\left(\eta' - \frac{1}{\delta}\right)(1-f)\right]$$

$$= C'\,RT\,(1-f) \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (89)$$

where T is given by

$$C'\,R\,(1-f)\,(T_o - T) = \tfrac{1}{2}\,(\overline{\gamma}-1)\,W_2\left(\frac{dx}{dt}\right)^2 \quad \cdots \quad \cdots \quad \cdots \quad (90)$$

Using (47), we get from (76) and (79)

$$p_2\,[K_2 + Ax - C\psi\,(1-f)] = C\psi\,(1-f)\,RT_2 \quad \cdots \quad \cdots \quad \cdots \quad (91)$$

$$\text{and}\quad CR\psi\,(1-f)\,(T_o - T_2) = \tfrac{1}{2}\,(\overline{\gamma}-1)W_2\left(\frac{dx}{dt}\right)^2 \quad \cdots \quad \cdots \quad (92)$$

Comparing (89) and (90) with (91) and (92), we find that substitution (74) would make the two pairs identical. Thus these substitutions reduce (76), (78) and (79) to the corresponding equations of an orthodox gun. The reduction, however, is again not complete as (80) will be different from the corresponding equation of an orthodox gun.

Motion after All-Burnt

Neglecting covolume η and putting $z = 1$, the equations for this case are

$$p_1 K_1 = CNRT_1 \quad \ldots \quad (93)$$

$$p_2[K_2 + Ax] = C(1 - N)RT_2 \quad \ldots \quad (94)$$

$$\frac{dN}{dt} = -\frac{\psi S p_1}{C\sqrt{RT_1}} \quad \ldots \quad (95)$$

$$\frac{d}{dt} NT_1 = -\frac{\gamma\psi S p_1}{CR}\sqrt{RT_1} \quad \ldots \quad (96)$$

$$\frac{CR}{\gamma - 1}[T_\circ - NT_1 - (1 - N)T_2] = \tfrac{1}{2} W_2 \left(\frac{dx}{dt}\right)^2 \quad \ldots \quad (97)$$

$$W_2 \frac{d^2x}{dt^2} = Ap_2 \quad \ldots \quad (98)$$

From (95) and (96)

$$\frac{dT_1}{T_1} = (\gamma - 1)\frac{dN}{N}$$

Or $$\frac{T_1}{T_{1,B}} = \left(\frac{N}{N_B}\right)^{\gamma - 1} \quad \ldots \quad (99)$$

where suffix B refers to the position of all-burnt.

From (93) and (99)

$$\frac{p_1}{p_{1,B}} = \frac{N}{N_B}\,\frac{T_1}{T_{1,B}} = \left(\frac{N}{N_B}\right)^{\gamma} = \left(\frac{T_1}{T_{1,B}}\right)^{\frac{\gamma}{\gamma - 1}} \quad \ldots \quad (100)$$

From (78) and (80)

$$\frac{dN}{dt} = -\frac{\psi S}{C\sqrt{R}}\,\frac{CNR\sqrt{T_1}}{K_1} = -\frac{\psi S}{K_1}\sqrt{RT_{1,B}}\,N\left(\frac{N}{N_B}\right)^{\frac{\gamma - 1}{2}} \quad \ldots \quad (101)$$

Integrating

$$\frac{N}{N_B} = \left[1 + \frac{t - t_B}{\theta_1}\right]^{-\frac{2}{\gamma - 1}} \quad \ldots \quad (102)$$

where

$$\theta_1 = \frac{2K_1}{(\gamma - 1)\,\psi\,S\sqrt{RT_{1,B}}} \quad \ldots \quad (103)$$

From (100) and (102)

$$\frac{p_1}{p_{1,B}} = \left[1 + \frac{t - t_B}{\theta_1}\right]^{-\frac{2\gamma}{\gamma - 1}} \quad \ldots \quad (104)$$

and $$\frac{T_1}{T_{1,B}} = \left[1 + \frac{t - t_B}{\theta_1}\right]^{-2} \quad \ldots \quad (105)$$

From (94), (97) and (98)

$$C_1T_\circ - CRNT_1 - [K_2 + AX]\frac{W_2}{A}\frac{d^2x}{dt^2} = \tfrac{1}{2}\left(\bar{\gamma} - 1\right)W_2\left(\frac{dx}{dt}\right)^2$$

Or

$$[K_2 + AX]W_2\frac{d^2x}{dt^2} + \tfrac{1}{2}A(\bar{\gamma} - 1)W_2\left(\frac{dx}{dt}\right)^2 = \bar{A}CRT_\circ - AK_1p_{1,B}\left[1 + \frac{t - t_B}{\theta_1}\right]^{-\frac{2\gamma}{\gamma - 1}} \quad .. \quad (106)$$

This is an ordinary differential equation of the second order to determine x and $\frac{dx}{dt}$ as functions of time, the initial conditions being

$$t = t_B\,,\ x = x_B\,,\ \frac{dx}{dt} = \left(\frac{dx}{dt}\right)_B \quad .. \quad .. \quad (107)$$

Modification in Corner's energy equation

From (97), at all-burnt

$$\frac{CR}{\bar{\gamma} - 1}[T_\circ - N_B T_{1,B} - (1 - N_B)T_{2,B}] = \tfrac{1}{2}W_2\left(\frac{dx}{dt}\right)_B^2 \quad .. \quad .. \quad (108)$$

From (97) and (108)

$$\frac{CR}{\bar{\gamma} - 1}[N_B T_{1,B} - NT_1 + (1 - N_B)T_{2,B} - (1 - N)T_2] = \tfrac{1}{2}W_2\left[\left(\frac{dx}{dt}\right)^2 - \left(\frac{dx}{dt}\right)_B^2\right] \quad .. \quad .. \quad (109)$$

In Corner's theory (Isothermal Model before burnt)

$$RT_{1,B} = RT_{2,B} = \lambda. \quad \text{Also } N_B = 1 - \psi \quad .. \quad .. \quad (110)$$

so that (109) gives

$$\frac{C\lambda}{\bar{\gamma} - 1}\left[(1 - NT_1) - (1 - N)T_2\right] = \tfrac{1}{2}W_2\left[\left(\frac{dx}{dt}\right)^2 - \left(\frac{dx}{dt}\right)_B^2\right] \quad .. \quad .. \quad (111)$$

which can also be written as

$$\frac{C\lambda}{\bar{\gamma} - 1}\left[(1 - NT_1 - \psi) + (\psi - (1 - N)T_2)\right] = \tfrac{1}{2}W_2\left[\left(\frac{dx}{dt}\right)^2 - \left(\frac{dx}{dt}\right)_B^2\right] \quad .. \quad .. \quad (112)$$

The corresponding equation of Corner is

$$\frac{C\lambda}{\gamma-1}\left[\Psi-(1-N)T_2\right]=\tfrac{1}{2}W_2\left[\left(\frac{dx}{dt}\right)^2-\left(\frac{dx}{dt}\right)^2_B\right] \quad .. \quad (113)$$

Comparison of (112) and (113) shows that Corner had neglected the term NT in the energy equation corresponding to the energy of the First Chamber. He has also apparently not taken heat losses into account.

Thus, in order to get the correct equation from Corner's equation, we have to replace γ by $\bar{\gamma}$ and replace Ψ by

$$1-NT_1=1-N\frac{T_1}{T_o}=1-N_B\left(\frac{T}{T_B}\right)^{\frac{1}{\gamma-1}}\frac{T}{T_B}$$

$$=1-N_B\left(\frac{T}{T_B}\right)^{\frac{\gamma}{\gamma-1}}$$

$$=1-(1-\psi)\left(1+\frac{t-t_B}{\theta_1}\right)^{-\frac{2\gamma}{\gamma-1}}$$

$$=1-\frac{N}{\left(1+\frac{t-t_B}{\theta_1}\right)^2} \quad .. \quad .. \quad (114)$$

Thus (111) can also be written as

$$\frac{C\lambda}{\bar{\gamma}-1}\left[1-\frac{N}{\left(1+\frac{t-t_B}{\theta_1}\right)^2}-(1-N)T_2\right]$$

$$=\tfrac{1}{2}W_2\left\{\left(\frac{dx}{dt}\right)^2-\left(\frac{dx}{dt}\right)^2_B\right\} \quad .. \quad .. \quad (115)$$

In any H/L gun with all-burnt occurring fairly early in the travel, this error would make a difference of the order of 15 to 20% in the muzzle velocity, since as t tends to infinity, the energy that could be obtained by complete expansion of the gas would be $\frac{C\lambda}{\gamma-1}\psi$ by Corner's equation instead of the obviously correct $\frac{C\lambda}{\bar{\gamma}-1}$. As in a typical case the energy extracted is of the order of 30% of this amount and as presumably, the energy would be computed correctly in the phase before 'burnt' and as Ψ is of the order of $\frac{1}{2}$, one would expect the square of the muzzle velocity to be not more than 30% out. Even then this would be a serious error, specially for H/L guns with burnt early in the travel. Fortunately, however, the advantages of a H/L gun are associated with flat pressure-space curves and so with "burnt" near the end of the travel and in this case, the error would be comparatively smaller.

It is obvious that our modified equation (115) gives the correct value $\frac{C\lambda}{\bar{\gamma}-1}$ as t tends to infinity.

Another way of judging the correctness of the Energy Equation is to try to reduce the isothermal model as a particular case of the general model by making γ or $\bar{\gamma}$ approach unity. If this is done in Corner's equation (113), it gives $(1—N)\ T_2$ as constant. Since, however, N is varying, it will imply the variation in T_2 which shows that the model will not reduce to isothermal one. If, however, $\bar{\gamma}$ is made to approach unity in our equation (111), it will be satisfied if $T_1 = T_2 =$ constant, as is the case for the Isothermal Model.

Acknowledgements

I am grateful to Prof. P.L. Bhatnagar, Head of the Department of Applied Mathematics, Indian Institute of Science, Bangalore and to Dr. R.S. Varma, Officer-in-Charge and Principal Scientific Officer, Defence Science Laboratory, for their interest, encouragement and valuable suggestions. I am also grateful to Dr. S.P. Aggarwal, for his lectures at the Defence Science Laboratory which first created interest in the subject in me. I am particularly grateful to Dr. J. Corner, who, when the error in his energy equation was pointed out to him, not only accepted it and agreed to make the necessary correction in his book, but went out of his way to give an estimate of the error involved and to give a number of other valuable suggestions which have led to a substantial improvement in the present paper.

REFERENCES

1. Corner, J. — Internal Ballistics of a H/L gun. Journ. Frank. Inst., 246 (1948).
2. Corner, J. — Theory of Interior Ballistics of Guns (1950) John Wiley & Sons, New York.
3. Aggarwal, S.P. — Internal Ballistics of a H/L gun Non-Isothermal Model, Proc. Nat. Inst. Sci., 22A (1957).
4. Aggarwal, S.P. — Internal Ballistics of a H/L gun with quadratic form-function, Proc. Nat. Inst. Sci., 21A (1956).
5. Aggarwal, S.P. — Internal Ballisties of a H/L gun with general form-function, Proc. Nat. Inst. Sci., 22A (1956).
6. Kapur, J.N. — Internal Ballistics of a H/L gun—Proc. Nat. Inst. Sci., 22A (1956).
7. Kapur, J.N. — Uniqueness of Maximum Pressure for composite charges for H/L guns etc. Proc. Inst. Cong. Th. App. Mech. I (1956), 227—238.
8. Kapur, J.N. — The Equivalent-charge Method in the general theory of composite charges. Proc. Nat. Inst. Sci., 22A (1956), 63—81.