ON THE EQUILIBRIUM PRESSURE IN A ROCKET CHAMBER

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ABSTRACT

In this paper the author has derived an expression for the calculation of equilibrium pressure in a rocket motor chamber using

$$r = b + CP^n$$

as the law of burning.

Kershner[1] and Gupta and Mehta[2] have obtained expressions for the equilibrium pressure in a rocket motor chamber assuming the linear and power law of burning. However, for greater accuracy, specially when sufficient data on the burning of the propellant is available, the following relation[3] is employed

$$r = b + CP^n \quad \ldots \quad (1)$$

where r is the rate of burning, P the chamber pressure and b, c and n are constants. In this paper the author has derived an expression for the equilibrium pressure in a rocket motor chamber using the law of burning, given by equation (1).

In a rocket chamber, the mass rate of burning is equal to the mass rate of discharge plus the mass rate of accumulation in the chamber. Thus[1]

$$S\rho r = C_D \, A_t \, P + \frac{d}{dt}(V\rho_g) \quad \ldots \quad (2)$$

where S = constant area of the burning surface

ρ = density of the propellant

r = rate of surface regression in in./sec.

C_D = discharge coefficient

A_t = area of the throat

P = chamber pressure

ρ_g = density of the gas in the chamber

and V = volume of the chamber available to the gas.

But

$$\frac{dV}{dt} = S.r$$

hence equation (2) can be written as

$$S\rho' r = C_D \, A_t \, P + V \frac{d}{dt}(\rho_g) \quad \ldots \quad (3)$$

where $\rho' = (\rho - \rho_g)$ can be taken as constant since ρ_g is only two or three percent of ρ' for most of the propellants.

In the steady state we have

$$\frac{d}{dt}(\rho_g) = 0$$

Thus equation (3) gives

$$S\rho' r = C_D A_t P_{eq} \qquad \ldots \qquad (4)$$

where P_{eq} is the equilibrium pressure.

Combining equations (1) and (4), we have

$$P_{eq} = \left[\frac{S\rho'}{C_D A_t}\left(c + \frac{b}{P_{eq}^n}\right)\right]^{\frac{1}{1-n}} \qquad \ldots \qquad (5)$$

Equation (5) may be solved by the method of successive approximations. If P_m is the value of P_{eq} to the m_{th} approximation, then

$$P_{m+1} = \left[\frac{S\rho'}{C_D A_t}\left(c + \frac{b}{P_m^n}\right)\right]^{\frac{1}{1-n}} \qquad \ldots \qquad (6)$$

Starting with

$$P_o = \left[\frac{S\rho'}{C_D A_t} \cdot c\right]^{\frac{1}{1-n}} \qquad \ldots \qquad (7)$$

when b=O, we may evaluate P_{eq} to any desired degree of accuracy.

In the following three tables, the variation of $\frac{P_{eq} - P_o}{P_o}$ with b has been illustrated for different values of c and n, in the case of a rocket motor for which

$S = 519$ in^2

$A_t = 0{\cdot}43$ in^2

$\rho' = 0{\cdot}058$ lb/in^3

$C_D = 0{\cdot}007$ per sec.

TABLE I

Variation of $\frac{P_{eq} - P_o}{P_o}$ *with b when* c=0·00075.

n \ *b*	0	0·001	0·002	0·003	0·004	0·005
0·65	0	0·089	0·174	0·253	0·332	0·408
0·70	0	0·040	0·079	0·116	0·154	0·189
0·75	0	0·012	0·024	0·037	0·049	0·062

TABLE II

Variation of $\frac{P_{eq} - P_o}{P_o}$ *with b when* c=0·0008

n \ b	0	0·001	0·002	0·003	0·004	0·005
0·65	0	0·071	0·142	0·210	0·276	0·341
0·70	0	0·032	0·064	0·095	0·125	0·155
0·75	0	0·009	0·019	0·028	0·038	0·047

TABLE III

Variation of $\frac{P_{eq} - P_o}{P_o}$ *with b when* C=0·00085

n \ b	0	0·001	0·002	0·003	0·004	0·005
0·65	0	0·064	0·124	0·179	0·237	0·292
0·70	0	0·026	0·053	0·078	0·102	0·128
0·75	0	0·007	0·015	0·023	0·031	0·039

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