EFFECT OF VARIATIONS IN LOADING CONDITIONS ON THE INTERNAL BALLISTICS OF GUNS

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ABSTRACT

In this paper tables have been constructed to show the variations in the position of all-burnt and pressure at all-burnt with variations in the Central Ballistic Parameter M and the shot-start pressure Z_0. It has been shown how the whole table for the variation of maximum pressure with variations in M and Z_0 (for tubular propellants) can be represented by a single graph. Also assuming zero shot-start pressure the internal ballistic equations have been expressed in a slightly different form and expressions have been obtained to relate pressure and velocity with shot-travel. Further the variations in maximum pressure and muzzle-velocity with respect to small variations in various parameters have been calculated.

Introduction

The main problem of internal ballistics is the calculation of the pressure-space-curve, maximum pressure and muzzle velocity for given loading conditions, *ie*, for various conditions which hold after the round has been loaded but before it is fired—and covers all the details of the charge, the shell and the gun, which can affect the ballistics. Various methods have been devised for the solution of the internal ballistic equations and they are solved by numerical integration and the results tabulated for various values of the many parameters involved. By suitable choice of assumptions relatively simple analytical solutions can be obtained. In a recent paper Venkatesan[1] has obtained an explicit expression for the relation between maximum-pressure and shot-start pressure for tubular propellants and which has been generalised by Aggarwal[2] for all values of θ. Their main assumption was that $B = i\theta$ *i.e.* the covolume of the gases equals the reciprocal of the density of the sold propellant. In another paper Mehta and Aggarwal[3] have discussed the effect of shot-start pressure on the pressure at all-burnt, all-burnt position and muzzle velocity etc.

In this paper the author has constructed tables to show the variations in the position of all-burnt and pressure at all-burnt with variations in the Central Ballistic Parameter M and the shot-start pressure Z_0. With the help of these two tables another table for the muzzle-velocity, for a given gun, can be prepared to show its variations with M and Z_0. Also for finding the pressure space curve from the solutions given by Venkatesan[2] and Aggarwal[1] we must know either the fraction of charge burnt or the fraction of web remaining at every instant during the burning period.

In this paper the equations of internal ballistics have been put in a slightly different form and an expression has been obtained to relate pressure directly as a function of shot-travel. Another important problem of internal ballistics is the study of the effects of small variations in one or more of the loading conditions on the ballistics of a gun. The effects of these variations can be discussed (*i*) qualitatively from fundamental principles of internal ballistics and (*ii*) quantitatively by using the mathematical solution of the internal ballistic equations. Thus the percentage changes in maximum pressure and muzzle velocity for unit percentage change in various parameters have been calculated.

The Basic Equations

The four fundamental equations of internal ballistics are

$$Z=\zeta(\xi-BZ)+\frac{\gamma-1}{2M}\eta^2 \quad \ldots \quad (1)$$

$$M\zeta=\eta\frac{d\eta}{d\xi} \quad \ldots \quad (2)$$

$$\zeta=-\eta\frac{df}{d\xi} \quad \ldots \quad (3)$$

$$Z=(1-f)(1+\theta f) \quad \ldots \quad (4)$$

where

$$\left.\begin{aligned} \xi&=1+\frac{x}{l} \\ \eta&=\frac{AD}{\lambda\beta c}u \\ \zeta&=\frac{Al}{\lambda c}p \\ M&=\frac{A^2D^2}{\lambda\beta^2cW} \end{aligned}\right\} \quad \ldots \quad (5)$$

so that ξ, η, ζ are non-dimensional variables corresponding to displacement, velocity and pressure respectively and M is a dimensionless constant called the Central Ballistic Parameter. If $\theta=0$ and $B=0$, then from (2), (3) and (4),

we have $\eta=M(Z-Z_o)$ $\quad \ldots \quad (6)$

Venkatesan[1] has obtained the following expressions:

$$\xi^{\gamma-1}\left[2MZ_o+2\eta-(\gamma-1)\eta^2\right]=2MZ_o\left[\frac{1+K_1\eta}{1+K_2\eta}\right]^{\frac{1}{K(\gamma-1)}} \quad \ldots \quad (7)$$

$$\zeta=\frac{\left[2MZ_o+2\eta-(\gamma-1)\eta^2\right]^{\frac{\gamma}{\gamma-1}}}{2M\left[2MZ_o\left(\frac{1+K_1\eta}{1+K_2\eta}\right)^{\frac{1}{K(\gamma-1)}}\right]^{\frac{1}{\gamma-1}}} \quad \ldots \quad (8)$$

where

$$K=\sqrt{\frac{1}{\gamma-1}\left[2MZ_{\circ}+\frac{1}{\gamma-1}\right]} \qquad \ldots \qquad (9)$$

and

$$K_1=\frac{1}{K-\frac{1}{\gamma-1}}, \quad K_2=\frac{1}{K+\frac{1}{\gamma-1}}$$

Also for $\gamma=1\cdot25$, the expression for maximum pressure $\left(\eta_1=\frac{1}{1\cdot25}\right)$ is given by,

$$\zeta_1=\frac{MZ_{\circ}+\cdot72}{M\xi_1} \qquad \ldots \qquad (10)$$

where

$$\xi_1^{\frac{1}{4}}=\frac{MZ_{\circ}}{MZ_{\circ}+\cdot72}\left\{\frac{5+\frac{4}{K-4}}{5-\frac{4}{K+4}}\right\}^{\frac{4}{K}} \qquad \ldots \qquad (11)$$

and

$$K=4\sqrt{1+\frac{MZ_{\circ}}{2}}$$

Now we notice that,

(*a*) When $Z_{\circ}=0$, $K=4$, equation (11) breaks down. In this case the relation between ξ and η is given by

$$\frac{d\xi}{\xi}=\frac{8d\eta}{8-\eta}(\text{for } \gamma=1\cdot25) \qquad \ldots \qquad (12)$$

Therefore

$$\xi=\left(\frac{8}{8-\eta}\right)^8 \qquad \ldots \qquad (13)$$

Thus the maximum pressure is given as

$$\zeta_1=\frac{\cdot3099364}{M} \qquad \ldots \qquad (14)$$

Hence if $\theta=0$, $Z_0=0$, $B=0$, we have

$$\zeta_1 \propto \frac{1}{M}$$

(*b*) If we divide both sides of (10) by Z_o we see that ζ_1/Z_0 is a function of MZ_0. This fact is of great help in preparing table for ζ_1 as a function of M and Z_0, because if we calculate the diagonal terms and the terms on the one side of the diagonal, the terms on the other side are automatically known.

From (*a*) and (*b*) above we observe that we need calculate only fourteen terms instead of twentyfive in Venkatesan's table for ζ_1 against values of M and Z_0. Fig. 1 illustrates the variation of ζ_1/Z_o against MZ_0. It must be understood that this is true only for the tubular propellants.

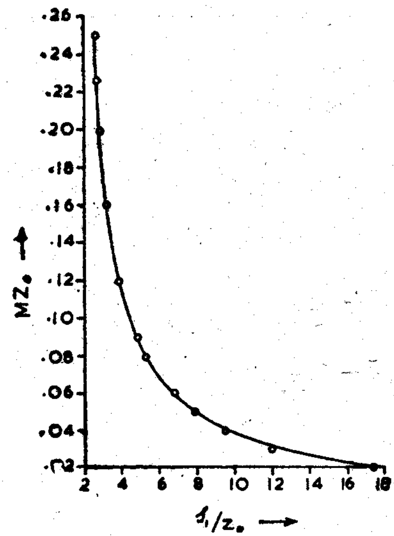

FIG. 1—Relation between ζ_1/Z_o & MZ_o for $\theta = 0$.

All-burnt position:—Quantities here will be denoted by suffix 2. From (6), we have

$$\eta_2 = M(1-Z_o) \qquad \ldots \qquad (15)$$

so that from (7), we have for $\gamma = 1{\cdot}25$

$$\xi_2^{\frac{1}{4}} = \frac{Z_o}{1-\frac{M}{8}\left(1-Z_o\right)^2} \times \left\{ \frac{1+\dfrac{M(1-Z_o)^2}{K-4}}{1-\dfrac{M(1-Z_o)^2}{K+4}} \right\}^{\frac{4}{K}} \qquad \ldots \qquad (16)$$

For Zo=0, from (13), we get

$$\xi_2 = \left(1-\frac{M}{8}\right)^{-8} \qquad \ldots \qquad (17)$$

Thus (16) and (17) determine ξ_2, the position of all-burnt as a function of M and Z_o. We can tabulate in double entry table form the values of $\xi^{\frac{1}{4}}$ against M and Z_o. This is done in the table given below:--

TABLE For $\xi_2^{\frac{1}{4}}$

TABLE I

Z_0 \ M	1	2	3	4
0·00 ..	1·3062	1·7778	2·5600	4·0000
0·02 ..	1·2714	1·6776	2·3216	3·4511
0·03 ..	1·2594	1·6392	2·2329	3·2494
0·04 ..	1·2476	1·6064	2·1571	3·0806
0·05 ..	1·2370	1·5774	2·0907	2·9311

The table shows that the position of all-burnt increases with increase in M but decreases as shot-start pressure increases.

Again from (8) and (15), we have for $\gamma=1\cdot25$ the pressure at all-burnt given by

$$\zeta_2 = \frac{1-\frac{M}{8}(1-Z_o)^2}{\xi_2} \quad \ldots \quad (18)$$

In (16) and (17), we have already expressed ξ_2 as explicit function of M and Z_o. Table II gives the values of ζ_2 for various values of M and Z_o.

TABLE FOR ζ_2

TABLE II

Z_o \ M	1	2	3	4
0·00 ..	·3006	·0751	·0147	·0020
0·02 ..	·3368	·0958	·0220	·0037
0·03 ..	·3507	·1059	·0261	·0048
0·04 ..	·3652	·1156	·0302	·0060
0·05 ..	·3788	·1250	·0346	·0074

A glance at the table shows that the pressure at all-burnt decreases as M increases but increases as the shot-start pressure increases.

Values after all-burnt :—After all-burnt Z=1, so that from (1), we have

$$1=\zeta\xi+\frac{\gamma-1}{2M}\eta^2 \qquad (19)$$

Also since the expansion of the gases is adiabatic

$$\zeta\xi^{\gamma}=\zeta_2\xi_2^{\gamma} \qquad (20)$$

Therefore from these equations, we have the muzzle velocity as

$$\eta_3^2=8M\left\{1-\frac{\zeta_2\xi_2^{\frac{5}{4}}}{\xi_3^{\frac{1}{4}}}\right\}(\text{when } \gamma=1\cdot25) \qquad (21)$$

This expresses η_3 as explicit function of M and Z_0, since ζ_2 and ξ_2 are already known functions of M and Z_0. Thus with the help of tables I and II we can construct double entry tables for given values of ξ_3.

Alternative Ballistic Equations

We now give the equations of internal ballistics in a slightly different form. Assuming zero shot-start pressure the equation for pressure-space curve, which is of significant importance for purposes of designing a gun, has been obtained. The four fundamental equations of internal ballistics are:

$$pv=\lambda cZ-\frac{\gamma-1}{2}W(12u)^2 \qquad (22)$$

$$D\frac{df}{dt}=-\beta p \qquad (23)$$

$$Z=(1-f)(1+\theta f) \qquad (24)$$

$$W\frac{d(12u)}{dt}=Ap \qquad (25)$$

Here v is the volume behind projectile minus the charge volume. Also if v_c is the chamber volume then the free volume of the chamber, v_o, is

$$v_o=v_c-bc \qquad (26)$$

where b is the average volume in cubic inches of one pound of propellant. Also

$$v=v_o+12Ax \qquad (27)$$

where x is the travel in feet.

From equations (23), (24) and (25), we have

$$\frac{\beta W}{AD}(12u)=\frac{1+\theta}{2\theta}\times\left[1-\sqrt{1-kZ}\right] \qquad (28)$$

where

$$k=\frac{4\theta}{(1+\theta)^2}$$

Let us now introduce the following two quantities,

$$\left.\begin{aligned} p_q &= \left(\frac{\beta c\lambda}{AD}\right)^2 \frac{W}{v_o} \\ p_c &= \frac{c\lambda}{v_o} \end{aligned}\right\} \quad \ldots \quad (29)$$

Both these quantities have dimensions of pressure. The latter quantity can be identified as the pressure which would be developed in the chamber if complete charge is burnt before shot begins to move.

Using equations (28) and (29), we have

$$c\lambda = \frac{2\theta}{1+\theta} \times \frac{(12u)}{1-\sqrt{1-kZ}} \times \sqrt{Wv_o p_q} = p_c v_o \quad \ldots \quad (30)$$

Thus

$$W(12u)^2 = \left(\frac{1+\theta}{2\theta}\right)^2 p_c v_o \left(\frac{p_c}{p_q}\right)\left\{1-\sqrt{1-kz}\right\}^2 \quad \ldots \quad (31)$$

With the help of (30) and (31), equation (22) can be written as

$$p\frac{v}{v_o} = p_c\left[Z - \frac{\gamma-1}{2}\left(\frac{1+\theta}{2\theta}\right)^2 \frac{p_c}{p_q}\left\{1-\sqrt{1-kZ}\right\}^2\right] \quad \ldots \quad (32)$$

Now

$$\frac{du}{dt} = 12Au\,\frac{du}{d(12Ax)}$$

Therefore from (25)

$$p = p_c v_o\left(\frac{p_c}{p_q}\right)\left(\frac{1+\theta}{2\theta}\right)^2\left[\left\{1-\sqrt{1-kZ}\right\}\frac{d\,[1-\sqrt{1-kZ}]}{dv}\right] \quad \ldots \quad (33)$$

Combining (32) and (33), we have

$$\log\left(\frac{v}{v_o}\right) = \int_0^Z \frac{\left\{1-\sqrt{1-kZ}\right\} d\left\{1-\sqrt{1-kZ}\right\}}{\left(\frac{2\theta}{1+\theta}\right)^2 \frac{p_c}{p_q} Z - \frac{\gamma-1}{2}\left\{1-\sqrt{1-kZ}\right\}} \, dZ \quad \ldots \quad (34)$$

$$= -\frac{1}{\left(\frac{\gamma-1}{2} + \theta\frac{p_q}{p_c}\right)} \log\left[1-\left(\frac{1}{2} + \frac{\gamma-1}{4\theta}\frac{p_c}{p_q}\right)\left(1-\sqrt{1-kZ}\right)\right]$$

Thus the relation between Z, the fraction of charge burnt, and $\frac{v}{v_o}$, the expansion-ratio is

$$\left(\frac{v}{v_o}\right) = \left[1-\left(\tfrac{1}{2}+\frac{\gamma-1}{4\theta}\frac{p_c}{p_q}\right)\left(1-\sqrt{1-kZ}\right)\right]^{-\frac{1}{\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}}} \qquad (35)$$

$$Z = \frac{(1+\theta)^2}{\theta+\frac{\gamma-1}{2}\frac{p_c}{p_q}}\left[1-\left(\frac{v}{v_o}\right)^{-\left(\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}\right)}\right] \times$$

$$\left[1-\frac{\theta}{\theta+\frac{\gamma-1}{2}\frac{p_c}{p_q}}\left(1-\left(\frac{v}{v_o}\right)^{-\left(\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}\right)}\right)\right] \quad \dots \quad (35A)$$

Putting the value of Z from (35*a*) in (32), we get the equation relating pressure to the expansion ratio as

$$p = \frac{(1+\theta)^2 p_c}{\left(\theta+\frac{\gamma-1}{2}\frac{p_c}{p_q}\right)}\left\{\left(\frac{v}{v_o}\right)^{\left(\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}\right)}-1\right\}\left(\frac{v}{v_o}\right)^{-\left(\gamma+2\theta\frac{p_q}{p_c}\right)} \qquad (36)$$

Again from (31) and (35*a*) the velocity in terms of expansion ratio is

$$u = \frac{p_c\,(1+\theta)}{12\left(\theta+\frac{\gamma-1}{2}\frac{p_c}{p_q}\right)}\sqrt{\frac{v_o}{Wp_q}}\left[1-\left(\frac{v}{v_o}\right)^{-\left(\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}\right)}\right] \qquad (37)$$

Now in a gun as the charge is burning the speed of the shell is increasing and so does the space behind the shell. Eventually there comes a time when the space behind the shell increases faster than the gases are produced. Just then the pressure is at its maximum and from then on it falls away while the speed of the shell continues to increase. Thus the behaviour of the product 'pu' with increase in shot-travel during the period of burning is given by

$$pu = \frac{p_c^2(1+\theta)^3}{12\left(\theta+\frac{\gamma-1}{2}\frac{p_c}{p_q}\right)^2}\sqrt{\frac{v_o}{Wp_q}}\left[1-\left(\frac{v}{v_o}\right)^{-\left(\frac{\gamma-1}{2}+\theta\frac{p_q}{p}\right)}\right]^2 \times \left(\frac{v}{v_o}\right)^{-\left(\frac{\gamma-1}{2}+\theta\frac{p_c}{p_c}\right)} \quad \dots \quad (38)$$

Chart[5] in his "Elements of Ammunition" indicates that the maximum value of 'pu' can be found either by trial or by calculation.

Now from (36), we find that pressure is maximum when

$$\frac{v_{max}}{v_o} = \left[\frac{\gamma + 2\theta \frac{p_q}{p_c}}{\frac{\gamma+1}{2} + \theta \frac{p_q}{p_c}} \right]^{\frac{1}{\left(\frac{\gamma-1}{2} + \theta \frac{p_q}{p_c}\right)}} \quad \ldots \quad (39)$$

$$Z_{max} = \frac{(1+\theta)^2 \left(\theta + \gamma \frac{p_c}{p_q}\right)}{\left(2\theta + \gamma \frac{p_c}{p_q}\right)^2} \quad \ldots \quad (39a)$$

and its value is

$$p_{max} = p_q (1+\theta)^2 \left\{ \gamma + 2\theta \frac{p_q}{p_c} \right\}^{\frac{\gamma + 2\theta \frac{p_q}{p_c}}{\frac{\gamma-1}{2} + \theta \frac{p_q}{p_c}}} \left\{ \frac{\gamma+1}{2} + \theta \frac{p_q}{p_c} \right\}^{-\frac{\frac{\gamma+1}{2} + \theta \frac{p_q}{p_c}}{\frac{\gamma-1}{2} + \theta \frac{p_q}{p_c}}}$$

This is true only so long as

$$\frac{p_c}{p_q} \geqslant \frac{1-\theta}{\gamma} \quad \ldots \quad (41)$$

Otherwise the maximum pressure occurs at all-burnt and its position is given by

$$\frac{v_b}{v_o} = 1 + \theta \left[1 - \frac{\gamma-1}{2} \frac{p_q}{p_c} \right]^{-\frac{1}{\left(\frac{\gamma-1}{2} + \theta \frac{p_q}{p_c}\right)}} \quad \ldots \quad (42)$$

while its value is

$$p_b = p_c \left(1 - \frac{\gamma-1}{2} \frac{p_c}{p_q} \right)^{\frac{\frac{\gamma+1}{2} + \frac{\theta p_q}{p_c}}{\frac{\gamma-1}{2} + \theta \frac{p_q}{p_c}}} (1+\theta)^{-\frac{1}{\left(\frac{\gamma-1}{2} + \theta \frac{p_q}{p_c}\right)}} \quad (43)$$

Also the velocity at all-burnt is

$$u_b = \frac{p_c}{12} \sqrt{\frac{v_o}{W p_q}} \quad \ldots \quad (44)$$

After the all-burnt position the expansion of the gases obeys the adiabatic law, so that at the muzzle the pressure is given as

$$p_m = p_b \left(\frac{v_b}{v_m}\right)^{\gamma} = p_b \left(\frac{v_b}{v_o}\right)^{\gamma} \left(\frac{v_o}{v_m}\right)^{\gamma} \quad \ldots \quad (45)$$

Putting the values of p_b and v_b, we obtain

$$p_m = p_c \left\{1 - \frac{\gamma-1}{2}\frac{p_c}{p_q}\right\}^{-\frac{\frac{\gamma-1}{2}-\theta\frac{p_q}{p_c}}{\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}}} \left(1+\theta\right)^{\frac{\gamma-1}{\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}}} \left(\frac{v_o}{v_m}\right)^{\gamma} \quad (46)$$

Therefore the muzzle energy is given by

$$E_m = \tfrac{1}{2}W\left(12u_m^2\right) = \frac{\lambda c}{\gamma-1}\left[1 - \left(\frac{v_o}{v_m}\right)^{\gamma-1} \left\{1 - \frac{\gamma-1}{2}\frac{p_c}{p_q}\right\}^{-\frac{\frac{\gamma-1}{2}-\theta\frac{p_q}{p_c}}{\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}}} \left(1+\theta\right)^{\frac{\gamma-1}{\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}}}\right] \quad (47)$$

and hence the muzzle velocity is known.

The temperature at the muzzle is given by

$$T_m = T_o\left(\frac{v_o}{v_m}\right)^{\gamma-1} \left(1 - \frac{\gamma-1}{2}\frac{p_c}{p_q}\right)^{-\frac{\frac{\gamma-1}{2}-\theta\frac{p_q}{p_c}}{\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}}} \left(1+\theta\right)^{\frac{\gamma-1}{\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}}} \quad (48)$$

where T_o is the adiabatic flame temperature of the propellant.

Also the shell will leave the gun before all-burnt position, unless

$$-\log\left[\frac{1-\frac{\gamma-1}{2}\frac{p_c}{p_q}}{1+\theta}\right] \leqslant r\left(\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}\right)$$

where

$$r = \log\frac{v_m}{v_o}$$

Effects of Variations in Loading Conditions on Ballistics

An important problem of internal ballistics is the study of the effects of small variations in one or more of the loading conditions on maximum pressure and muzzle velocity etc. of a gun. The loading conditions are:

(*a*) For Charge—Ballistic size D, charge weight c, shape or form factor θ, force constant λ and rate of burning constant β.

(*b*) For Gun—Chamber Capactiy v_o, shot travel or the length of the bore x_m and area of bore A.

(*c*) For Shot—Mass of shell w, shot-start pressure p_o. We will discuss the change in ballistics if only one of the loading conditions be changed.

(*i*) Propellant 'size' D and rate of burning constant β—For a constant charge weight an increase in D means a decrease of burning surface and so the pressure builds up more slowly. It has more time to affect the shell's motion and though the shell is moving less rapidly the all-burnt position is farther up the bore. If the 'size' is very large the pressure may go on increasing right upto the muzzle and the shell may leave the gun before the propellant is all-burnt and so there will be an increase in the irregularity. Since the burning constant β appears in ballistic equations in the denominator $\left(\text{e.g. } p_c/p_q=\frac{A^2D^2}{\lambda\beta^2 cW}\right)$ so the change in ballistic due to web D and rate of burning constant β are equal in magnitude but opposite in sign. The per cent. change in maximum pressure due to unit per cent change in D or β is given by

$$\frac{\partial(\log p_{max})}{\partial(\log \beta)}=-\frac{\partial(\log p_{max})}{\partial(\log D)}=2+\frac{2\theta\frac{p_q}{p_c}}{\left(\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}\right)^2}\log\frac{\gamma+2\theta\frac{p_q}{p_c}}{\frac{\gamma+1}{2}+\theta\frac{p_q}{p_c}}-\frac{2\theta\frac{p_q}{p_c}}{\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}} \quad \ldots \ldots \ldots (50)$$

Again since

$$\frac{p_c}{p_q}=\frac{A^2D^2}{\lambda\beta^2 cW}, \text{ where } W=1\cdot06\ m+\tfrac{1}{3}c \quad \ldots \ldots \ldots (51)$$

Now considering a gun for which $c=1\cdot5$, $m_1=16\cdot5$, $A=8\cdot831$, $\lambda=1810$ and $\beta=\cdot82$, the table III and Fig. 2 give the value of D for different values of p_c/p_q.

TABLE III

p_c/p_q	·5	1	2	3	4	5
D ..	·0193	·0272	·0385	·0472	·0545	·0609

Also from (51), we have

$$c=\frac{-3\cdot18m+\sqrt{(3\cdot18m)^2+\frac{12A^2D^2p_q}{\lambda\beta^2 p_c}}}{2} \quad \ldots \ldots \ldots (52)$$

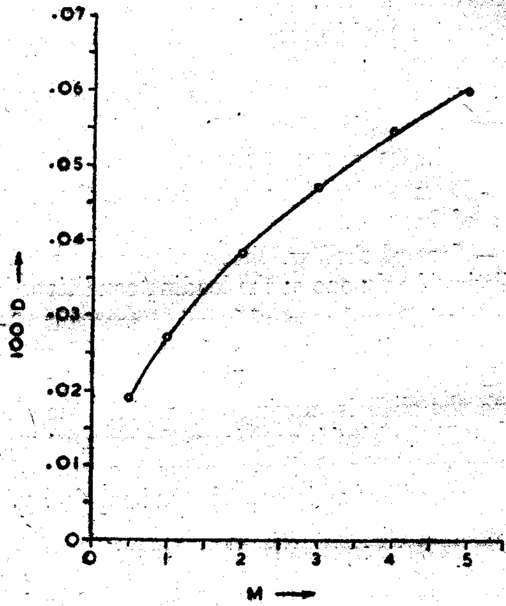

FIG. 2—Relation between M & D.

Thus if in the given gun we want to keep the value of p_c/p_q fixed then the relation between C and D for p_c/p_q =1, 2, 3, 4, 5 is given by table IV and Fig. 3.

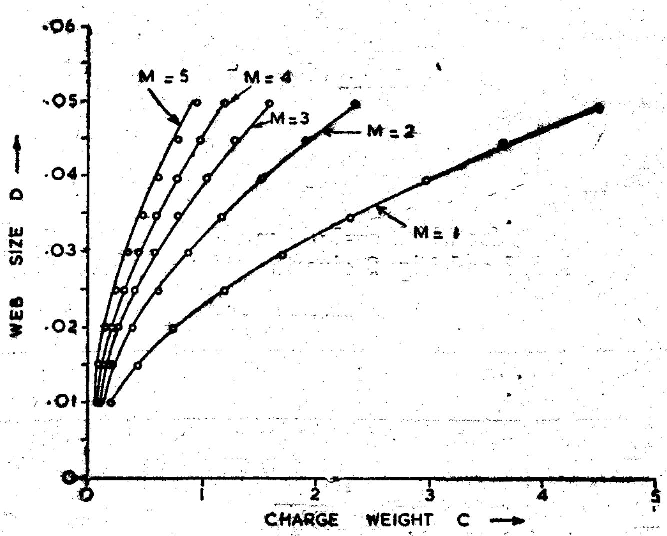

FIG. 3—Relation between C & D for different values of M.

TABLE IV

p_c/p_q \ D	·010	·015	·020	·025	·030	·035	·040	·045	·050
1	·1949	·4316	·7708	1·1945	1·7036	2·2931	2·9584	3·6433	4·4965
2	·0976	·2192	·3883	·6040	·8656	1·1665	1·5197	1·9091	2·3375
3	·0651	·1463	·2599	·4044	·5807	·7870	1·0313	1·2879	1·5811
4	·0489	·1098	·1949	·3038	·4369	·5922	·7708	·9718	1·1946
5	·0391	·0879	·1560	·2434	·3497	·4749	·6185	·7803	·9600

Again if all the loading conditions are constant, including the charge weight, the pressure space-curve for standard loading conditions and for increased web must be equal because the chemical energy released is constant and the area under pressure-space curve is a measure of muzzle energy of the shot). But the pressure-space curve for increased web is below that for standard and so the two curves must cross each other. Since the difference in areas before and after cross-over must be same in an infinitely long gun, it is apparent that with finite shot-travel the area under the curve corresponding to smaller 'size' will be greater than that for larger 'size' and thus the muzzle velocity is reduced by increasing the propellant thickness and the change is given by

$$\frac{\partial(\log E_m)}{\partial(\log \beta)} = -\frac{\partial(\log E_m)}{\partial(\log D)} = 2\frac{\partial(\log u_m)}{\partial(\log \beta)} = -2\frac{\partial(\log u_m)}{\partial(\log D)}$$

$$= \varepsilon\left[\frac{p_c/p_q}{1-\frac{\gamma-1}{2}\frac{p_c}{p_q}} \times \frac{\frac{\gamma-1}{2}-\theta\frac{p_q}{p_c}}{\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}} + \frac{2\theta\frac{p_q}{p_c}}{\left(\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}\right)^2}\log\frac{1+\theta}{1-\frac{\gamma-1}{2}\frac{p_c}{p_q}}\right] \quad \text{.. (53)}$$

where

$$\varepsilon = \frac{c\lambda}{E_m} - (\gamma - 1)$$

(ii) Charge weight c—An increase in charge weight means an increase in chemical energy available and thus an increase in maximum pressure and muzzle velocity. Also since increased charge implies higher pressure so the rate of burning must be higher and though the projectile moves more rapidly still the all-burnt position occurs early in the bore. Thus with increased charge weight we expect improvement in the regularity. On the other hand

a reduction in charge weight will increase irregularity as the all-burnt position will be near the muzzle. The per cent. change in maximum pressure with unit per cent. change in charge weight is given by

$$\frac{\partial(\log p_{max})}{\partial(\log c)} = 1 + \frac{\theta p_q/p_c}{\left(\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}\right)^2} \log \frac{\gamma+2\theta\frac{p_q}{p_c}}{\frac{\gamma+1}{2}+\theta\frac{p_q}{p_c}} - \frac{\theta\frac{p_q}{p_c}}{\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}} + \left\{1-\frac{bc}{v_c}\right\}^{-1} \quad .. \quad (54)$$

Also the per cent change in muzzle energy and muzzle velocity with unit per cent. change in charge weight is given by

$$\frac{\partial(\log E_m)}{\partial(\log c)} = 2\frac{\partial(\log u_m)}{\partial(\log c)}$$

$$= 1+\varepsilon\left[\frac{\frac{\gamma-1}{2}-\theta\frac{p_q}{p_c}}{2\left(\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}\right)} \frac{p_c/p_q}{\left(1-\frac{\gamma-1}{2}\frac{p_c}{p_q}\right)} + \frac{\theta p_q/p_c}{\left(\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}\right)^2}\log\frac{1+\theta}{1-\frac{\gamma-1}{2}\frac{p_c}{p_q}} + \frac{bc}{v_c}\left\{1+\frac{v_c-bc}{12Ax_m}\right\}^{-1}\left\{1-\frac{bc}{v_c}\right\}^{-1}\right] \quad .. \quad (55)$$

Again if maximum pressure is kept constant by assuming that web or burning rate or both are varied then the change in muzzle velocity is given by

$$\frac{\partial(\log E_m)_p}{\partial(\log c)} = 2\frac{\partial(\log u_m)_p}{\partial(\log c)} = 1+\varepsilon\left(1-\frac{bc}{v_c}\right)^{-1}\left[\frac{bc}{v_c}\left(1+\frac{v_c-bc}{12Ax_m}\right)^{-1} + \left\{\frac{\theta}{\left(\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}\right)^2}\log\frac{1-\frac{\gamma-1}{2}\frac{p_c}{p_q}}{1+\theta} - \frac{p_c/p_q}{2\left(\frac{p_q}{p_c}-\frac{\gamma-1}{2}\right)} \cdot \frac{\frac{\gamma-1}{2}-\theta\frac{p_q}{p_c}}{\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}}\right\} \times \left\{\frac{p_c}{p_q} + \frac{\theta}{\left(\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}\right)^2}\log\frac{\gamma+2\theta\frac{p_q}{p_c}}{\frac{\gamma+1}{2}+\theta\frac{p_q}{p_c}} - \frac{\theta}{\frac{\gamma-1}{2}+\theta\frac{p_q}{p_c}}\right\}^{-1}\right] \quad .. \quad .. \quad (55a)$$