BALLISTIC METHOD BASED ON ISOTHERMAL MODEL FOR FINITE SHOTSTART PRESSURE

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ABSTRACT

This communication presents a generalisation of the 'R. D. 38 Method', given by Crow, to the case of finite shot-start pressure. Further here the problem of composite charge for isothermal model has been discussed and the expressions for the maximum pressure and muzzle velocity have been derived.

Introduction

The problem of internal ballistics has been discussed by **various authors,** under different valid assumptions. The most simple method was given **by Crow.** This method is based on the following two assumptions :—

(1) Isothermal approximation, and

(2) a zero shot-start pressure.

This paper consists of two parts. The first part gives an extension of the R. D. 38 Method, given by Crow, to the case of finite shot-start pressure. Second part deals with the problem of internal ballistics, based on isothermal model, for composite charge.

Notations,

c=charge weight

D=web-size

θ=form factor

F=force constant

β=rate of burning constant

γ=ratio of specific heats of gases

K_E=total capacity

K_o=chamber capacity

X_E=shot travel

$$A=\frac{(K_E-K_o)}{X}$$

W_1=effective mass of the projectile

$=1\cdot 06\ w+\frac{1}{3}c$, w being the actual mass of the projectile.

f= fraction of D remaining.

Z= fraction of the charge burnt.

First Part

Basic equations.

The four basic equations of internal ballistics for single charge and isothermal model are:

$$\frac{FcZ}{(x+l)} \frac{\left(1+\frac{c}{2w_1}\right)}{\left(1+\frac{c}{3w_1}\right)}=Ap \quad .. \quad .. \quad .. \quad (1)$$

$$w_1\left(1+\frac{c}{2w_1}\right)\frac{dv}{dt}=Ap \quad .. \quad .. \quad .. \quad (2)$$

$$Z=(1-f)(1+\theta f) \quad .. \quad .. \quad .. \quad .. \quad (3)$$

and $$D\frac{df}{dt}=-\beta p \quad .. \quad .. \quad .. \quad .. \quad (4)$$

Solution of the equations.

Eliminating p from (2) and (4) we have,

$$w_1\left(1+\frac{c_1}{2w_1}\right)\frac{dv}{dt}=-\frac{AD}{\beta}\frac{df}{dt} \quad .. \quad .. \quad (5)$$

Integrating this equation under the initial conditions,

$v=0$, $x=0$ and $f=f_o$, we obtain

$$v=\frac{AD(f_o-f)}{\beta w_1\left(1+\frac{c}{2w_1}\right)} \quad .. \quad .. \quad .. \quad (6)$$

From equations (2) and (5) we get,

$$Ap=-\frac{ADv}{\beta}\frac{df}{dx} \quad .. \quad .. \quad .. \quad (7)$$

With the help of equations (7) and (3), equation (1) becomes,

$$\frac{dx}{(x+l)}=\left[\frac{M(f-f_o)}{(1-f)(1+\theta f)}\right]df \quad .. \quad .. \quad (8)$$

where $$\frac{A^2D^2\left(1+\frac{c}{3w_1}\right)}{\beta^2 w_1 Fc\left(1+\frac{c}{2w_1}\right)^2}=M \quad .. \quad ... \quad .. \quad (8,a)$$

Integrating the equation (8) under initial conditions, we have

$$(x+l)=l\left[\frac{1-f_o}{1-f}\right]^{\frac{M(1-f)}{(1+\theta)}}\left[\frac{(1+\theta f_o)}{(1+\theta f)}\right]^{\frac{M(1+\theta f_o)}{\theta(1+\theta)}} \quad .. \quad (9)$$

Substituting the value of $(x+l)$ in (1) the pressure is given by,

$$p=K(1-f)^{\frac{M(1-f_o)}{(1+\theta)}+1} \cdot (1+\theta f)^{\frac{M(1+\theta f_o)}{\theta(1+\theta)}+1}$$

where

$$K=\frac{Fc}{Al}\left(\frac{1+\frac{c}{2w_1}}{1+\frac{c}{3w_1}}\right)(1-f_o)^{-\frac{M(1-f_o)}{1+\theta}} \cdot (1+\theta f_o)^{-\frac{M(1+\theta f_o)}{\theta(1+\theta)}} \quad .. \quad (10)$$

Thus, equations (6), (9) and (10) give the shot-velocity, shot-travel and pressure at any instant when the charge is burning.

Denoting the values at all-burnt position by the suffix B, we have,

$$v_B=\frac{ADf_o}{\beta w_1\left(1+\frac{c}{2w_1}\right)}=\frac{FcMf_o^2}{w_1\left(1+\frac{c}{3w_1}\right)} \quad .. \quad .. \quad (11)$$

$$(x_B+l)=l(1-f_o)^{M\left(\frac{1-f_o}{1+\theta}\right)} \cdot (1+\theta f_o)^{\frac{M}{\theta}\left(\frac{1+\theta f_o}{1+\theta}\right)} \quad .. \quad (12)$$

and $p_B=K$ .. .. (13)

where K has been defined above.

Maximum pressure

For maximum pressure $\frac{dp}{df}=o$. Therefore differentiating equation (10) and simplifying we get,

$$f=\frac{(Mf_o-1+\theta)}{(M+2\theta)}=f_m \text{ (say)} \quad .. \quad .. \quad .. \quad .. \quad (14)$$

Therefore, the maximum pressure is given by,

$$p_m=K(1-f_m)^{\frac{M(1-f_o)}{(1+\theta)}+1} \cdot (1+\theta f_m)^{\frac{M(1+\theta f_o)}{\theta(1+\theta)}+1} \quad .. \quad (15)$$

Muzzle velocity.

After all-burnt, since the expansion of gases is adiabtic, at any travel $x>x_B$, the pressure is given by,

$$p=p_B\left[\frac{x_B+l}{x+l}\right]^{\gamma} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (16)$$

Substituting the value of p from equation (16) in equation (2) we obtain,

$$w_1\left(1+\frac{c}{2w_1}\right)v\frac{dv}{dx}=Ap_B\left(\frac{x_B+l}{x+l}\right)^{\gamma}$$

Integrating this equation we get,

$$v^2=v^2_B+\frac{2Ap_B\,(x_B+l)}{w_1\left(1+\frac{c}{2w_1}\right)(\gamma-1)}\left[1-\left(\frac{x_B+l}{x+l}\right)^{\gamma-1}\right]$$

Now putting $\frac{2}{\gamma-1}\left[1-\left(\frac{x_B+l}{x+l}\right)^{\gamma-1}\right]=\varphi$ and making use of equations (11) and (1) at all burnt position, this equation reduces to,

$$v^2=\frac{Fc(Mf^2_o+\varphi)}{w_1\left(1+\frac{c}{3w_1}\right)} \quad .. \quad .. \quad .. \quad .. \quad (17)$$

Hence the muzzle velocity is given by,

$$v^2_E=\frac{Fc(Mf^2_o+\varphi_E)}{w_1\left(1+\frac{c}{3w_1}\right)} \quad .. \quad .. \quad .. \quad .. \quad (18)$$

where $\varphi_E=\frac{2}{\gamma-1}\left[1-\left(\frac{x_B+l}{x_E+l}\right)^{\gamma-1}\right]$ and (x_B+l) is given by equation (12)

Thus equations (15) and (18) give the maximum pressure and muzzle velocity, respectively.

Second Part

Basic equations

The basic equations of internal ballistics for composite charge based on isothermal model are:

$$\frac{(F_1c_1Z_1+F_2c_2Z_2)}{(x+l)}\;\frac{\left(1+\frac{c_1+c_2}{2w_1}\right)}{\left(1+\frac{c_1+c_2}{3w_1}\right)}=Ap. \quad .. \quad .. \quad (1)$$

$$w_1\left[1+\frac{c_1+c_2}{2w_1}\right]\frac{dv}{dt}=Ap. \quad .. \quad .. \quad (2)$$

$$Z_1=(1-f_1)(1+\theta_1f_1) \quad .. \quad .. \quad (3)$$

$$D_1\frac{df_1}{dt}=-\beta_1p \quad .. \quad .. \quad .. \quad (4)$$

$$Z_2=(1-f_2)(1+\theta_2f_2) \quad .. \quad .. \quad .. \quad (5)$$

$$D_2\frac{df_2}{dt}=-\beta_2p \quad .. \quad .. \quad .. \quad (6)$$

Where the suffix 1 refers to charge c_1 and suffix 2 refers to charge c_2.

Solution of the equations

From equations (4) and (6) we get,

$$\frac{df_1}{df_2} = \frac{\beta_1 D_2}{\beta_2 D_1} = K_1 \text{ (say)}$$

Integrating this equation under initial conditions, $f_1=1, f_2=1$ we obtain

$$(f_1 - f_{1,0}) = K_1(f_2 - f_{2,0}) \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

Conditions for simultaneous burning and non-simultaneous burning.

Now two cases arise.

Case 1. The two charges burn out simultaneously.

Case 2. The two charges burn out at different times.

If the two charges burn out simultaneously, then $f_1=0$, $f_2=0$ and the condition for simultaneous burning is $K_1=f_{1,0}/f_{2,0}$

If the charges burn out at different times then one charge which burns out first is, say, c_1. At that instant $f_2=f\ \ t_1$ (say), then

$$f_{1,0} = K(f_{2,0} - f_{2,t}) \text{ for } f_{2,t} = \left(f_{2,0} - \frac{f_{1,0}}{K_1}\right)$$

Since f_2,t_1 is a positive fraction, therefore condition for charge c_1 burning out first is $K_1 > \frac{f_{1,0}}{f_{2,0}}$

Similarly condition for charge c_2 burning out first is $K_1 < \frac{f_{1,0}}{f_{2,0}}$

Now eliminating p from equations (2) and (4) we obtain,

$$w_1\left(1+\frac{c_1+c_2}{2w_1}\right)\frac{dv}{dt} = -\frac{AD_1}{\beta_1}\frac{df_1}{dt} \quad \ldots \quad \ldots \quad (8)$$

Integration of this equation under the initial conditions, $f_1=f_{1,0}$, $f_2=f_{20}, v=0$ and $x=0$, give,

$$v = \frac{AD_1(f_{1,0} - f_1)}{\beta_1 w_1\left(1+\frac{c_1+c_2}{2w_1}\right)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

With the help of equations (2) and (8) equation (1) reduces to,

$$Z_1 + \frac{F_2 c_2}{F_1 c_1} Z_2 = -\frac{A^2 D_1^2\left[1+\frac{c_1+c_2}{3w_1}\right]}{\beta_1^2 w_1 F_1 c_1\left[1+\frac{c_1+c_2}{2w_1}\right]^2}(f_{1,0} - f_1)\frac{df_1}{dx} \quad \ldots \quad (10)$$

Substituting the value of Z_1, and Z_2 in terms of f_1 in this equation we get,

$$\frac{d}{x+l} = \frac{M_1(f_{1,0} - f_1)}{af_1^2 + bf_1 - e}\ df_1$$

where

$$a = \left(\theta_1 + \theta_2 \frac{F_2c_2}{F_1c_1K_1^2}\right)$$

$$b = \left(1 - \theta_1 - \theta_2 \frac{F_2c_2}{F_1c_1K_1} + \frac{F_2c_2}{F_1c_1K_1} + \frac{2\theta_2F_2c_2f_{2,0}}{F_1c_1K_1} - \frac{2\theta_2F_2c_2f_{1,0}}{F_1c_1K_1^2}\right)$$

$$e = \left(1 + \frac{F_2c_2Z_2}{F_1c_1}\right)$$

$$\text{or}\quad \frac{dx}{(x+l)} = \frac{M_1(f_{1,0} - f_1)}{a(f+\alpha)(f-\beta)} \qquad \ldots \qquad (11)$$

where $\alpha - \beta = \dfrac{b}{a}$.

$\alpha\beta = \dfrac{e}{a}$.

Equation (11) can be further written as,

$$\frac{dx}{x+l} = -\frac{M_1}{a}\ \frac{(f_{1,0}-\alpha)}{(\alpha+\beta)}\ \frac{1}{(\alpha+f_1)} + \frac{M_1\,(f_{1,0}-\beta)}{a\ (\alpha+\beta)}\cdot\frac{1}{(f-\beta)} \qquad \ldots \qquad (12)$$

Integrating this equation and applying initial conditions we get,

$$\left(\frac{x+l}{l}\right) = \left(\frac{f_{1,0}+\alpha}{f_1+\alpha}\right)^{\lambda}\left(\frac{f_1-\beta}{f_{1,0}-\beta}\right)^{\mu} \qquad \ldots \qquad (13)$$

where

$$\lambda = \frac{M_1}{a}\left(\frac{f_{1,0}+\alpha}{\alpha+\beta}\right),\quad \mu = \frac{M_1}{a}\left(\frac{f_{1,0}-\beta}{\alpha+\beta}\right)$$

The pressure is given by

$$p = \frac{K_2\ (e - bf_1 - a\ f_1^2)}{(x+l)} \qquad \ldots \qquad (14)$$

$$K_2 = \frac{F_1\,c_1\left(1 + \dfrac{c_1+c_2}{2w_1}\right)}{A\left(1 + \dfrac{c_1+c_2}{3w_1}\right)}$$

Case 1. Simultaneous burning of the two charges.

For simultaneous burning the condition is $K = \dfrac{f_{1,0}}{f_{2,0}}$, and all these equations hold good for $K = \dfrac{f_{1,0}}{f_{2,0}}$.

The values at the all burnt position are,

$$v_B = \frac{AD_1 \; f_{1,0}}{\beta w_1 \left(1 + \frac{c_1+c_2}{2w_1}\right)} = \frac{F_1 \, c_1 \, M_1 \, f^2_{1,0}}{w_1 \left(1 + \frac{c_1+c_2}{3w_1}\right)} \quad .. \quad (15)$$

$$(x_B + l) = l \left(\frac{f_{1,0}+\alpha}{\alpha}\right)^{\lambda} \left(\frac{\beta}{\beta - f_{1,0}}\right)^{\mu} \quad .. \quad .. \quad (16)$$

$$p_B = \frac{K_2 c}{(x_B + l)} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (17)$$

After all-burnt

After all-burnt the gases expand adiabatically, and the pressure is given by,

$$p = p_B \left(\frac{x_B + l}{x + l}\right)^{\gamma} \quad .. \quad .. \quad .. \quad .. \quad (18)$$

Putting this value of p in the dynamical equation we have,

$$w_1 \left(1 + \frac{c_1+c_2}{2w_1}\right) \frac{vdv}{dx} = Ap_B \left(\frac{x_B + l}{x + l}\right)^{\gamma} \quad .. \quad .. \quad (19)$$

Integration of this equation under initial conditions give,

$$v^2 = v^2_B + \frac{2Ap_B \, (x_B + l)}{w_1 \left(1 + \frac{c_1+c_2}{2w_1}\right)(\gamma - 1)} \left[1 - \left(\frac{x_B + l}{x + l}\right)^{\gamma - 1}\right] \quad (20)$$

The muzzle velocity is thus given by,

$$v^2_E = v^2_B + \frac{2Ap_B \, (x_B + l)}{w_1 \left(1 + \frac{c_1+c_2}{2w_1}\right)(\gamma - 1)} \left[1 - \left(\frac{x_B + l}{x_E + l}\right)^{\gamma - 1}\right] \quad (21)$$

For maximum pressure $dp = o$. Therefore differentiating (14) and simplifying we get,

$$f_1 = \frac{M_1 \, f_{1,0} - b}{M_1 + 2a} = f_{1,m} \text{ (say).}$$

Therefore, maximum pressure is given by,

$$p_m = \frac{K_2 \, (e - bf_{1,m} - af^2_{1,m})}{(x_m + l)}$$

where $(x_m + l)$ is obtained by, putting $f_1 = f_{1,m}$ in equation (13).

Case. 2. Non-simultaneous burning of the charges.

All our equations from (1) to (14) hold good for this case upto the instant when both the charges are burning. When charge c_1 is burnt out, our equations become :

$$\frac{F_1 c_1 + F_2 c_2 z_2}{x + l} \cdot \frac{1 + \frac{c_1 + c_2}{2w_1}}{1 + \frac{c_1 + c_2}{3w_1}} = Ap \quad .. \quad .. \quad .. \quad (22)$$

$$w_1 \left(1 + \frac{c_1+c_2}{2w_1}\right) \frac{dv}{dt} = Ap. \qquad (23)$$

$$Z_2 = (1-f_2)\,(1+\theta_2\, f_2) \qquad (24)$$

$$D_2 \frac{df_2}{dt} = -\,\beta_2 p. \qquad (25)$$

Eliminating p from (23) and (25) we get,

$$w_1 \left(1 + \frac{c_1+c_2}{2w_1}\right) \frac{dv}{dt} = -\frac{AD_2}{\beta_2} \frac{df_2}{dt}.$$

Integrating and applying these initial conditions $v=o$, $f_2=f_{2,o}$ we have

$$v = \frac{AD_2\,(f_{2,0}-f_2)}{\beta_2\, w_1 \left(1 + \frac{c_1+c_2}{2w_1}\right)}$$

Now with the help of equation (23), equation (21) reduces to,

$$\frac{dx}{(x+l)} = \frac{M_2\,(f_{2,0}-f_2)\,df_2}{(1-f_2)\,(1+\theta_2 f_2) + \frac{F_1 c_1}{F_2 c_2}}$$

$$\frac{dx}{(x+l)} = \frac{M_2\,(f_{2,0}-f_2)\,df_2}{\theta_2\,(\alpha_1+f_2)\,(\beta_1-f_2)} \qquad (27)$$

where

$$\alpha_1\,\beta_1 = \frac{1}{\theta_2}\left(1 + \frac{F_1 c_1}{F_2 c_2}\right)$$

$$\beta_1-\alpha_1 = \frac{1}{\theta_2}\,(\theta_2-1).$$

Integrating equation (27) and applying initial conditions that $x=x_b$, and $f_2=f_{2,0}-\frac{f_{1,0}}{K} = f'_2$ (say) we have,

$$\frac{(x+l)}{(x_b+l)} = \left(\frac{\alpha_1+f_2}{\alpha_1+f'_2}\right)^{\frac{M_2\,(f_{2,0}+\alpha_1)}{\theta_2\,(\alpha_1+\beta_1)}} \left(\frac{\beta_1-f'_2}{\beta_1-f_2}\right)^{\frac{M_2\,(f_{2,0}-\beta_1)}{\theta_2\,(\alpha_1+\beta_1)}} \qquad (28)$$

The pressure hereafter when charge c_1 is burnt out is given by,

$$p = \frac{K'_2\left[\frac{F_1\;c_1}{F_2\;c_2} + (1-f_2)\;(1+\theta_2\,f_2)\right]}{(x+l)} \qquad (29)$$

$$\text{where } K_2 = \frac{F_2\,c_2\left(1 + \frac{c_1+c_2}{2w_1}\right)}{A\left(1 + \frac{c_1+c_2}{3w_1}\right)}$$

and $(x+l)$ is given by equation (28)

The values at the all-burnt position are given by,

$$v_B = \frac{A\,D_2\,f_{2,0}}{\beta_2 w_1\left(1+\frac{c_1+c_2}{2w_1}\right)} = \frac{F_2\,c_2\,M_2\,f^2_{2,0}}{w_1\left(1+\frac{c_1+c_2}{3w_1}\right)} \quad .. \quad (30)$$

$$(x_B+l) = (x_b+l)\left(\frac{\alpha_1}{\alpha_1+f'_2}\right)^{\frac{M_2(f_{2,0}+\alpha_1)}{\theta_2(\alpha_1+\beta_1)}}\left(\frac{\beta_1-f'_2}{\beta_1}\right)^{\frac{M_2(f_{2,0}-\beta_1)}{\theta_2(\alpha_1+\beta_1)}} \quad (31)$$

and
$$p_B = \frac{K'_2\left[\frac{F_1\,c_1}{F_2\,c_2}+1\right]}{(x_B+l)} \quad .. \quad .. \quad .. \quad .. \quad (32)$$

For maximum pressure occurring in between the two charges, $dp = o$

Therefore differentiating (29) and simplifying we get,

$$f_2 = \frac{M_2\,f_{1,0}+\theta_2-1}{M_2+2\theta_2} = f_{2,m}.$$

Hence, the maximum pressure is given by,

$$p_m = \frac{K'_2\left[\frac{F_1\,c_1}{F_2\,c_2}+(1-f_{2,m})(1+\theta_2\,f_{2,m})\right]}{(x_m+l)} \quad .. \quad .. \quad (33)$$

where (x_m+l) is obtained from equation (28) by putting $f_2 = f_{2,m}$.

If the maximum pressure occurs at all burnt, then its value is given by equations (17) and (32) accordingly as both the charges burnt out simultaneously and non-simultaneously.

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