ON THE INTERNAL BALLISTICS OF A GUN BY CROW'S METHOD TAKING COVOLUME INTO ACCOUNT

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ABSTRACT

In this paper the author has solved the equations of Internal Ballistics of the gun using propellant of any shape by Crow's method but has taken into account the covolume of the gases.

Introduction

There exist five different methods for the solution of Internal Ballistic equations—(1) R. D. 38 method (2) Hunt and Hind's method (3) Crow's method (4) Goldie's method and (5) Coppeck's method. There are different assumptions involved in the different methods. In this communication the author has followed the Crow's method but considered the covolume of the gases *i.e.* he has neglected the Kinetic energy term in the energy equation during the burning of the charge and assumed a zero shot start pressure. The effect of neglecting the Kinetic energy term during the burning of the charge is made up by decreasing F on the average by 10%. This is further reduced to allow for heat losses by dividing it by the factor $\left(1+\frac{1}{11d}\right)$ where d is the calibre in inches.

Solution

The equations of Internal Ballistics during the burning of the charge with the above assumptions become

$$F_1CZ = p\left\{A(x+l)-ZB\right\} \quad \ldots \quad (1)$$

$$w_1\frac{dv}{dt} = Ap \quad \ldots \quad (2)$$

$$Z = (1-f)(1+\theta f) \quad \ldots \quad (3)$$

$$D\cdot\frac{df}{dt} = -\beta p \quad \ldots \quad (4)$$

where the symbols have their usual meaning.

$$\xi = 1 + x/l$$

$$\eta = v\,AD/F_1C\beta$$

$$\zeta = p.A.l/F_1C$$

$$M = A^2D^2/F_1C\beta^2w_1$$

equations (1), (2), (3) and (4) become

$$Z = \zeta(\xi - BZ) \quad \ldots \quad \ldots \quad (5)$$

$$M\xi = \eta\frac{d\eta}{d\xi} \quad \ldots \quad \ldots \quad (6)$$

$$Z = (1-f)(1+\theta f) \quad \ldots \quad \ldots \quad (7)$$

$$\zeta = \frac{df}{d\xi} \quad \ldots \quad \ldots \quad (8)$$

from (6) and (8) we have

$$\eta = M(1-f) \quad \ldots \quad \ldots \quad (9)$$

with the help of (9)

$$Z = \frac{\eta}{M}\left[1+\theta-\frac{\theta\eta}{M}\right] \quad \ldots \quad \ldots \quad (10)$$

Making use of (10) and (6) in (5) we get.

$$\frac{\eta}{M}\left[1+\theta-\frac{\theta\eta}{M}\right] = \frac{\eta}{M}\frac{d\eta}{d\xi}\left[\xi - B\left\{\frac{\eta}{M}\left(1+\theta-\frac{\theta\eta}{M}\right)\right\}\right]$$

$$\text{or} \quad \frac{d\xi}{d\eta} - \frac{\xi}{\left(1+\theta-\frac{\theta\eta}{M}\right)} + \frac{B\eta}{M} = 0 \quad \ldots \quad \ldots \quad (11)$$

Integrating (11) we have—

$$\xi\left(1+\theta-\frac{\theta\eta}{M}\right)^{M/\theta} = \frac{B\eta}{\theta}\frac{\left(1+\theta-\frac{\theta\eta}{M}\right)^{\frac{M}{\theta}+1}}{\left(\frac{M}{\theta}+1\right)} + \frac{BM}{\theta^2}\frac{\left(1+\theta-\frac{\theta\eta}{M}\right)^{\frac{M}{\theta}+2}}{\left(\frac{M}{\theta}+1\right)\left(\frac{M}{\theta}+2\right)} + K.$$

The constant K is evaluated by putting the boundary condition that

$$\eta = 0 \qquad \text{when } \xi = 1$$

$$K = (1+\theta)^{M/\theta} - \frac{BM}{\theta^2}(1+\theta)^{\left(\frac{M}{\theta}+2\right)} \Big/ \left(\frac{M}{\theta}+1\right)\left(\frac{M}{\theta}+2\right)$$

Hence :—

$$\xi = \frac{B\eta\,(1+\theta-\theta\eta/M)}{(M+\theta)} + \frac{BM\left(1+\theta-\frac{\theta\eta}{M}\right)^2}{(M+\theta)(M+2\theta)} +$$

$$\left[(1+\theta)^{M/\theta} - BM\,(1+\theta)^{\frac{M}{\theta}+2}\right] \times \left(1+\theta-\frac{\theta\eta}{M}\right)^{-M/\theta} \quad \ldots \quad (12)$$

thus equation (12) expresses ξ in terms of η or say it gives us the shot travel in terms of the fraction of D remaining unburnt at any instant.

Maximum Pressure

The pressure at any instant is given by $\zeta = \dfrac{Z}{\xi - BZ}$ . . (13)

For getting maximum pressure we differentiate (13) and put $d\zeta = 0$

$$0 = d\zeta = \frac{dZ}{\xi - BZ} - \frac{Z\{d\xi - BdZ\}}{(\xi - BZ)^2}$$

$$\text{or } \xi \frac{dZ}{d\xi} = Z$$

$$\text{or } \xi\left[\frac{1}{M}\left(1+\theta - \frac{\theta\eta}{M}\right) - \frac{\theta\eta}{M^2}\right] = \frac{\eta}{M}\left(1+\theta - \frac{\theta\eta}{M}\right)\left[\frac{\xi}{\left(1+\theta - \frac{\theta\eta}{M}\right)} - \frac{B\eta}{M}\right]$$

$$\text{or } \eta\left\{\frac{1}{M} + \frac{2\theta}{M^2}\right\} = \frac{1+\theta}{M} + BF(\eta) \quad . \quad . \quad . \quad (14)$$

$$\text{where } F(\eta) = \frac{\eta^2}{M^2}\left[1+\theta - \frac{\theta\eta}{M}\right]\Big/\xi$$

and ξ as a function of η is given by Eqn. (12)

We solve equation (14) by the method of successive approximations.

For 1st order approximation

$$\eta_{\text{Max, 1}} = \frac{M(1+\theta)}{(M+2\theta)}$$

For 2nd order approximation.

$$\eta_{\text{Max, 2}} = \frac{\left(\frac{1+\theta}{M}\right) + BF\left(\eta_{\text{Max, 1}}\right)}{M+2\theta} . M^2$$

In general

$$\eta_{\text{Max, n+1}}\left[\frac{M+2\theta}{M^2}\right] = \frac{1+\theta}{M} + BF\left(\eta_{\text{Max, n}}\right) \quad . \quad . \quad (15)$$

where $\eta_{\text{Max}, \eta}$ is the value of η_{Max} at maximum pressure to the nth degree of approximation.

Thus equation (13) with the help of (15) gives the value of the maximum pressure.

All burnt

At the position of all burnt, $z=1$, $f=0$

$$\eta_b = M \qquad (16)$$

$$v_b = \eta F_1 C.\beta \Big/ A.D. \qquad (16A)$$

$$\xi_b = \frac{BM}{(M+\theta)} - \frac{BM}{(M+\theta)(M+2\theta)} + \left[(1+\theta)^{M/\theta} + \frac{BM(1+\theta)}{(M+\theta)(M+2\theta)}\right]^{M/\theta+2} \qquad (17)$$

$$\zeta_b = \frac{1}{\xi_b - B} \qquad (18)$$

After all burnt

For the solution after all burnt we can proceed as given in § 8·09 of Internal Ballistics (H.M.S.O. Publication).

Acknowledgement

The author is grateful to Dr. D. S. Kothari and Dr. R. S. Varma for their kind interest in the work.

Reference

1. Internal Ballistics, H.M.S.O. Publication, 108, 1951.