PURSUIT COURSES IN AERIAL GUNNERY

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ABSTRACT

In this paper, several methods of attack on a bomber made by a fighter, equipped with fixed guns, are discussed, with particular reference to the curve of pursuit attack. The importance of pursuit course of attack in aerial gunnery, the various types of pursuit courses, the characteristics of the above courses and their limitations, have been discussed. The difficulties of adopting a curve of pursuit attack on present day high-speed bombers have been explained, leading to an introduction of interception or collision course of attack.

Introduction

During the past few years a great deal of attention has been devoted to improving and assessing methods of aerial gunnery. In this paper the methods of attack by a fighter, equipped with fixed guns, on a bomber are discussed. The three basic methods considered in fixed-gun deflection shooting are (1) 'Fly Through' method (2) 'Lengths Ahead' method and (3) the Curve of Pursuit. In fixed gun deflection shooting the gun is rigidly fixed to the aircraft and in order to change the direction of fire, the direction of flight of the aircraft as a whole has to be changed.

A Comparative study of the three methods of attack in Aerial Gunnery

Fly through method

In this method aim is taken ahead of a moving target along its line of flight, held on a point at a distance well ahead of the target, fire is opened and maintained. Obviously the target will pass through the bullet Pattern after a certain elapsed time. But the time taken for the target to pass through a "distance", equal to the sum of the diameter of the bullet pattern at the shooting range and the length of the target, during which hits are possible on the target, is only a small fraction of a second. So the density of the bullet pattern is very small. Since the area of the bullet pattern which the target covers is only a fraction of the total area of the bullet pattern, the number of probable hits on the target is negligibly small to produce the desired result. Since fire is opened even before the nose of the target enters the bullet pattern, there is a large wastage of ammunition with little success. Further, there is a tendency on the part of the pilot to allow the target to fly through his sight at the end of a burst of fire.

'Lengths Ahead' method

For every target speed and range there is a fixed distance A B to aim ahead to hit the target which is obtained by the product of the target speed and the time of flight of the bullet or $\frac{\text{Target speed x range}}{\text{mean bullet velocity}}$

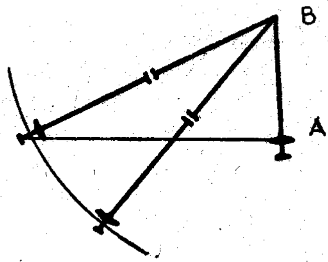

FIG A

Provided the range is kept constant, the time of flight of the bullet and hence the distance ahead remains constant, whatever be the angle off. The distance ahead is expressed in lengths of target as unity. The disadvantages of the method are (1) range must be kept constant and (2) the number of "lengths ahead" to be allowed for will vary with different target speeds and for a given target speed, with different target lengths.

The curve of pursuit and its importance in aerial combat

The famous 'dog and master' problem is merely the problem of determining the equation to a curve of pursuit. Historically it goes back to the days of Leonardo-da-Vinci. The dog traces a curve of pursuit as it heads towards its master all the time, till it catches him from behind (fig. 1).

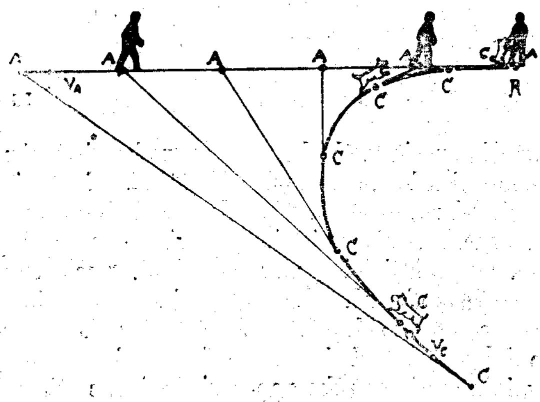

FIG 1

In defence, importance of this problem was realised in air to air combat. In order for the fighter to keep the target under continuous fire it must fly some kind of a pursuit course. For this purpose a pursuit course is defined as the path traced out by some fixed point in the fighter plane (usually, the centre of gravity) as it tracks a target moving along a specified course. The target usually considered is a bomber moving at a constant speed in a straight horizontal line.

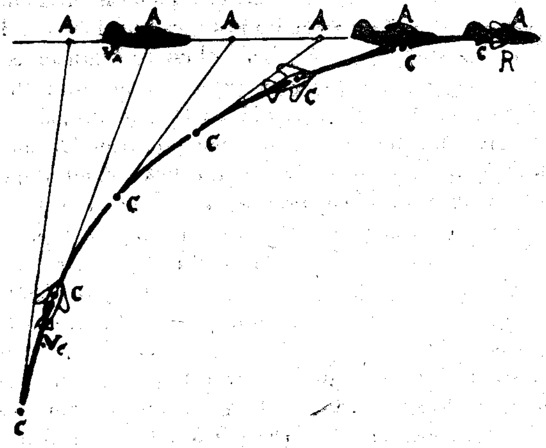

FIG 2

Fig. 2 shows the path of the fighter C moving with constant speed, keeping the target A under continuous fire. The pilot needs merely to head constantly towards the aircraft to be attacked. Seen by an observer in aircraft A, the curve of pursuit described by the fighter becomes the swimmer's curve fig. 3.

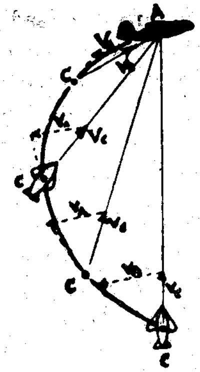

FIG 3

In the curve of pursuit method the guns are bearing throughout the fire. It is the basis of all fixed-gun deflection shooting. The pilot in the fighter aircraft, armed with fixed-guns, must understand the curve of pursuit course in order to know whether or not his aircraft, with its aerodynamic characteristics, can fly such a course and in order to feed the necessary inputs to his fire control system, to successfully hit the target. The gunner in the bomber should understand such a course, so that he can determine the future position of the fighter and give the necessary deflection to his guns for counterfire or he can take the necessary avoiding action at the appropriate time so that the fighter finds it difficult to bear his guns on the bomber and breaks away. The problem of pursuit curves came into greater importance with the development of homing missiles.

Various types of pursuit curves in aerial combat

There are four types of pursuit courses in air to air combat. A fighter, equipped with fixed guns, is in combat with a bomber, equipped with movable guns. Both the fighter and the bomber are regarded as points moving with constant speeds. The fighter's guns are assumed to point in the direction of the flight path of the aircraft. If the fighter's path is such that the tangent vector of its path is directed towards the bomber or, in other words, the guns are constantly directed towards the bomber, then the fighter is said to fly a *pure pursuit course.* This course does not take into account the speeds of the fighter and the bomber which are comparable with the muzzle velocity of the projectile. Accordingly, the fighter pilot must not aim at the instantaneous position of the bomber but must add a certain lead which depends upon the relative positions of the fighter and the bomber and their velocities. If the fighter flies so that its guns are always directed at a point ahead of the bomber by the required amount to secure a hit it is said to fly a *lead pursuit course* or a *deviated pursuit course.* If a fixed lead is made, it is said to fly a *fixed lead pursuit course.* In the above mentioned courses, it is assumed that the aircraft moves in the direction in which it is pointing. But the velocity vector of the fighter does not coincide with the bore axis of the gun. It is well known, from aerodynamic considerations that there exists an angle of attack. If the fighter flies in such a way that its guns are always pointed directly at the bomber and its flight path is determined from the angle of attack and other aerodynamic considerations, it is said to fly an *aerodynamic pursuit course.* If the lead is taken into consideration, besides the flight path being determined from aerodynamic considerations, the fighter is said to fly an *aerodynamic lead pursuit course.* These courses become more and more complex in the order stated above.

Mathematical expressions for the pure and lead pursuit courses

There are two types of courses to consider (1) the actual course traversed by the pursuer (2) the path of the pursuer relative to the pursued. The relative course of the pursuer is considered in the automatic adjustment of the fire control equipment. For the relative course, a polar coordinate system which has its origin at the bomber is chosen. From the relative polar coordinate, the actual space coordinate of the pursuer can be arrived at by using the usual transformation formula and by adding to the relative space coordinate, the coordinate of the bomber at the specified time.

Let V_B and V_F be the velocities of the fighter and bomber respectively, r is the present range, θ is the angle off, measured from the stern end of the longitudinal axis of the bomber aircraft. Let the bomber move from the origin along the γ axis. δ is the deviation of the guns to secure a hit. The rate of change of r (along the sight line) depends upon the rate of change of the component of the relative velocity of the fighter with respect to the bomber in that direction and the rate of change of θ depends upon the rate of change of

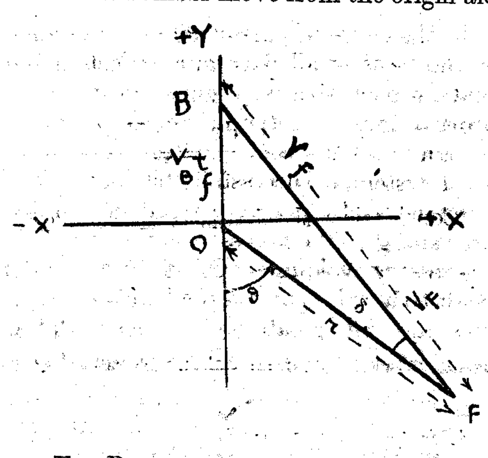

FIG B

relative crossing-speed or the rate of change of the component of the relative velocity of the fighter in a direction perpendicular to the sight line r, is introduced so

$$\frac{dr}{dt} = V_B \cos\theta - V_F \cos\delta \quad .. \quad .. \quad (1)$$

$$r\frac{d\theta}{dt} = V_F \operatorname{Sin}\delta - V_B \sin\theta \quad (2)$$

Dividing equation (1) by (2),

$$\frac{dr}{r} = \left(\frac{V_B \cos\theta - V_F \cos\delta}{V_F \sin\delta - V_B \sin\theta}\right)d \; .$$

For a pure pursuit curve $\delta = 0$

$$\text{so,} \quad \frac{dr}{r} = \left(\frac{V_B \cos\theta - V_F}{-V_B \sin}\right)d\theta$$

$$= \left(-\cot\theta + \frac{V_F}{V_B} \cdot \operatorname{Cosec}\theta\right)d\theta$$

Put $V_B/V_F = C$ (a constant)

Integrating, $\log r = \frac{1}{C}\log\tan\theta/2 - \log\sin\theta + k$

The value of k is found out from the initial conditions $r = r_O$ when

$$\theta = 90^\circ$$

$$k = \log r_o$$

$$\therefore r = \frac{r_o(\tan\theta/2)^{1/C}}{\sin\theta}$$

where r_o is the range on the beam.

Lead pursuit curve

In the analysis of the lead pursuit curves, a useful approximation is made by putting $\cos\delta = 1$, since δ is very small.*

Then

$$\frac{dr}{r} = \frac{V_F(c\cos\theta - 1)}{V_F(\sin\delta - C\sin\theta)}\,d\theta$$

Considering the ballistic triangle OBF

$$\frac{OB}{\sin\delta} = \frac{BF}{\sin\theta} \text{ or } \frac{V_B t_f}{\sin\delta} = \frac{r_f}{\sin\theta}$$

So, $\sin\delta = \frac{V_B t_f}{r_f}\sin\theta = \frac{V_B}{U}\sin\theta$ where U is the average bullet

*For rigorous evaluation, see appendix.

velocity over that range. For the combat range, it can be considered to be a constant.

$$\text{So } \sin \delta = C_1 \sin \theta \quad \text{where } C_1 = \frac{V_B}{U}$$

$$\text{So } \frac{dr}{r} = \frac{1 - C \cos \theta}{(C - C_1) \sin \theta}$$

$$\text{Integrating } \log r = \frac{1}{C - C_1} \log \tan \theta/2 - \frac{C}{C - C_1} \log \sin \theta + k$$

For initial conditions $\theta = 90°$, $r = r_o$, $k = \log r_o$

$$\text{So } r = r_o \left(\frac{\tan \theta/2}{\sin^C \theta} \right)^{\frac{1}{C - C_1}}$$

It is usual to open fire, to get good results, when the target is within combat range i.e. 300 yards and when the angle off is 30°, if a fixed reflector sight is used. With an automatic computing sight, the range can be increased to 600 yards or so. An ideal curve of pursuit with a fixed reflector sight is one which will bring the fighter to a range of 300 yards when the angle off is 30°. For a given value of C *i.e.* ratio of the bomber speed to the fighter speed, the value of r_o can be calculated which will make r = 300 yards when $\theta = 30°$. For V_F = c450 mph, and V_B 300 mph, r_o is calculated to be 3245 feet.

Figs. 4 and 5 show the positions of the fighter in terms of polar coordinates, and actual space coordinates of the fighter respectively, during a curve of pursuit attack. The fighter follows the curve of pursuit by

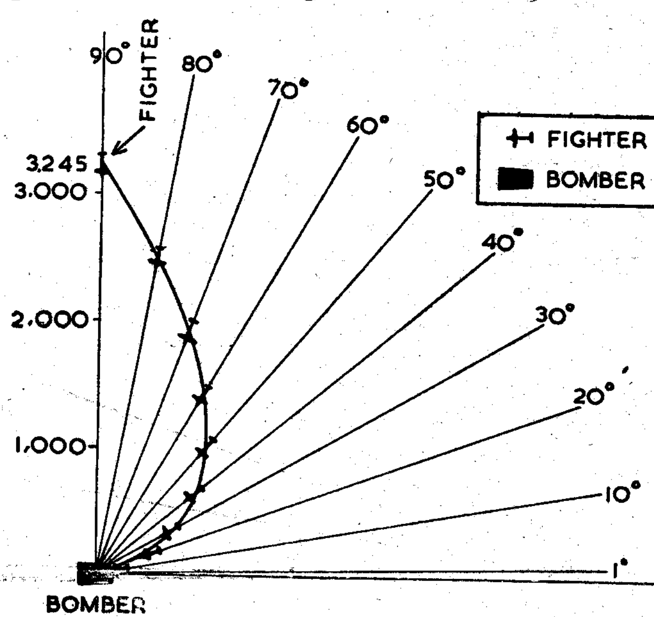

FIG. 4

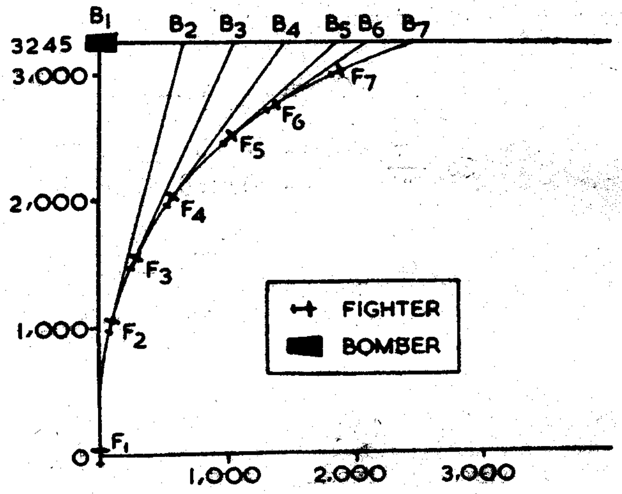

FIG 5

placing the bead of the sight on the tail of the target. The curve of pursuit need not necessarily be in the horizontal plane. If the curve of pursuit is initiated from the left or right beam of the bomber, then they are termed as *port beam* or *star-board beam attack* from the point of view of the bomber. Fig. 6 shows lead pursuit course of a fighter on a bomber initiating the attack when $\theta = 90°$ and $r_o = 1000$ yds.

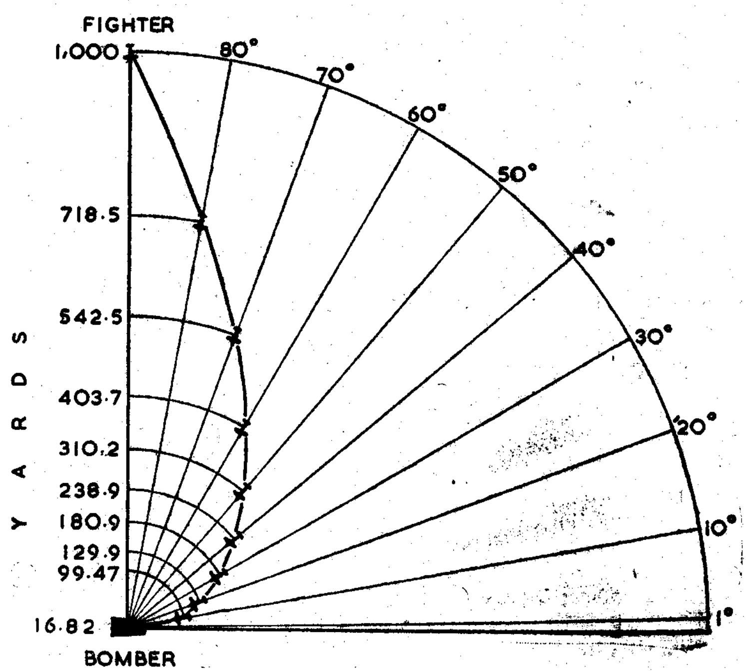

FIG 6

Characteristics of the pursuit curves

The shape of the curve may vary slightly depending upon the plane of attack because the speed of the fighter will increase in diving attacks and fall off in climbing attack. The radius of the curve of pursuit decreases up to an angle θ depending upon the ratio of the speed of the fighter to that of the bomber and then increases again up to the astern position. In other words the rate of turn thus steadily increases, reaches a maximum value and then decreases as it follows the curve of pursuit.

Normal acceleration of the fighter caused by the curvature of its space course

Normal acceleration $a = \frac{V^2_F}{R}$ where R is the radius of curvature.

In the polar coordinate system, *for a pure pursuit course*

$$R = -\frac{V_F}{\dot{\theta}}.$$

$$\text{from Eqn. (2), } \dot{\theta} = -\frac{V_B \sin\theta}{r},$$

$$\text{So } a = \frac{V_F V_B \sin\theta}{r} = \frac{V_F \cdot V_B \sin\theta}{rg} \qquad (3)$$

(expressed in units of gravity)

For a lead pursuit course $R = \frac{V_F}{\dot{\delta} - \dot{\theta}}$

using $\sin\delta = C_1 \sin\theta$ and $Z = \frac{C_1 \cos\theta}{\text{Cos}\,\delta}$, we can show

$$\dot{\delta} - \theta = \left(\frac{C_1 \text{Cos}\,\theta}{\text{Cos}\,\delta} - 1\right)\dot{\theta} \;:\; \delta - \theta = \frac{V_F}{\gamma}(C - C_1)\sin\theta\left(1 - \frac{C_1 \text{Cos}\,\theta}{\text{Cos}\,\delta}\right)$$

$$\therefore a = \frac{V^2_F (C - C_1)\sin\theta\,[1 - C_1 \text{Cos}\,\theta/\text{Cos}\,\delta]}{rg} \text{ (in units of gravity)} \quad (4)$$

The value of θ at which acceleration is maximum can be found out by equating the derivative of the above equations (3 or 4 as the case may be) with respect to θ to zero and solving for θ. For a pure pursuit course, acceleration is maximum when $\text{Cos}\,\theta = \frac{1}{2C}$. For $V_F = 450$ mph and $V_B = 300$ mph, acceleration is maximum when $\theta = 41° 24'$. In the case of reflector sights the 'pull through' of the bead of the sight at 30° is assisted by the fact that the rate of turn is lessened or otherwise the sight will overtake the target.

The shorter the opening range in a curve of pursuit than the ideal opening range, the higher is the rate of turn. This gives rise to a high value of 'g'— (normal acceleration due to curvature expressed in units of gravity). There is a limit to this value of 'g'—from the structural and aerodynamic characteristics of the aircraft. At high altitudes the aircraft structural load factor limits the acceleration to 2g after which the maximum permissible acceleration decreases, progressively to a value of zero at the aircraft's ceiling altitude. At low altitudes the physiological effect restricts the value of acceleration to 4g or 5g. The pilot suffers physically and the accuracy of his sighting and hence his shooting is impaired. Moreover, the range will be so short by the time 30° angle off position is reached that the length of burst will be exceedingly short, before breakaway, to have any lethal effect at the target.

If the range is too great, the curve will be gradual and by the time 30° angle off position is reached, the target will be far away and far out of range. As time elapses, the attack will develop into an astern chase and unless the fighter's speed is considerably greater than the target's speed, it may not be possible at all to close the range.

Distinct phases of a deflection attack

All deflection attacks develop through four distinct phases, namely

(1) Positioning for attack

(2) Turn in

(3) Curve of Pursuit and fighting

and (4) Break away.

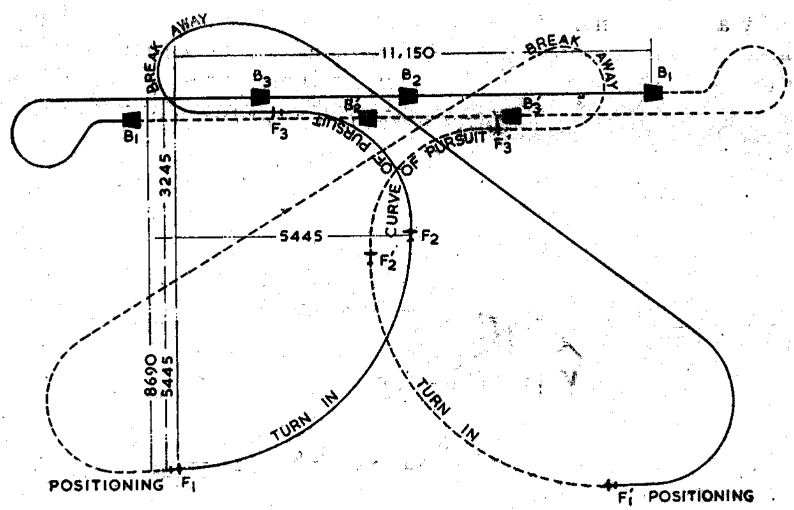

FIG 7

Fig. 7 shows the various aspects of a deflection attack when a fighter ($V_F = 450$ mph) engages a bomber ($V_B = 300$ mph) in a parallel course both for starboard beam attack and port beam attack. Positioning for attack controls the point of turn in and thus it eventually influences the whole attack. The turn will be made by a whole formation. It is assumed that the centrifugal force will not be more than 2·5 times the flying weight.

$$\text{The fighter's radius of turn} = \frac{V^2_F}{2\cdot 5\, g}$$

The distance a at which the aircraft must position himself in order that after 90° turn, it is at the ideal opening range is given by $3245 + \frac{V^2_F}{2\cdot 5\, g}$

The distance d from the bomber at which the fighter begins the turn in to reach the position at 90° angle off to the target at the ideal range, is given by $d = \frac{V_F}{2 \cdot 5g}\left(\frac{\pi}{2} V_B + V_F\right)$

From F_2 the fighter begins the curve of pursuit, opens the fire when the target is at an angle off 30° and at a range of 300 yards and then breaks away as shown in the figure. The same thing is shown for a starboard attack. The accuracy of the ensuing curve of pursuit therefore depends upon the correct judgement of positioning and turn in.

Aerodynamic pursuit courses

As already mentioned, the angle of attack must be taken into account when arriving at the aerodynamic pursuit courses of the fighter. The treatment of the problem is divided into two parts (1) the equations of motion of the aircraft and (2) the conditions of pursuit. To derive the equation of motion of the aircraft it is necessary to consider the usual force system acting upon the aircraft, *i.e.* forces of lift (L) thrust (T), drag (D) and weight (W). The conditions of pursuit then constrain this motion. An attack made in a vertical plane by a fighter on a bomber which moves in a straight and level flight path is considered below and an analysis of such an attack was made by G.H. Handleman, W.R. Heller and W. Prager of U.S.A. From the figure, resolving the forces along and normal to the flight line, the equations of motion of the aircraft can be written, by Newton's second law,

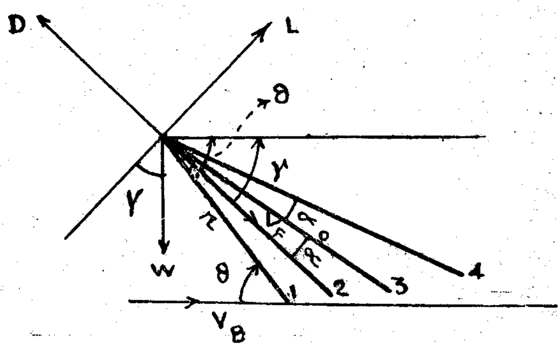

FIG. C

1. Sight line
2. Flight line
3. Thrust line (gun bore axis)
4. Zero lift line

Along the flight line

$$\frac{W}{g} \dot{V}_F = T \cos \alpha + W \sin \gamma - D \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

Normal to the flight line

$$\frac{W}{g} \frac{V_F^2}{R} = L + T \sin \alpha - W \cos \gamma$$

Where R is the radius of curvature of the space course of the fighter as it tracks the bomber.

$$R = -\frac{V_F}{d\gamma/dt}$$

substituting we get $$\frac{W}{g} V_F \frac{d\gamma}{dt} = W \operatorname{Cos} \gamma - T \sin \alpha - L \quad .. \qquad (6)$$

Equations of pursuit, expressed in terms of polar coordinates r and θ are, as before,

$$\frac{dr}{dt} = V_B \operatorname{Cos} \theta - V_F \operatorname{Cos} (\theta - \gamma) \quad .. \quad .. \quad .. \quad .. \qquad (7)$$

$$r\frac{d\theta}{dt} = -[V_F \sin (\gamma - \theta) + V_B \sin \theta] \quad .. \quad .. \quad .. \qquad (8)$$

The equations 7 and 8 are modified according to whether an aerodynamic pure pursuit course or an aerodynamic lead pursuit course is considered. In pure pursuit course, the gun bore axis (the thrust line) coincides with the sight line as shown in the figure.

$$\gamma - \theta = \alpha$$
$$\text{or } \theta = \gamma - \alpha$$

This value of θ is substituted in the equations. For a lead pursuit course, the gun bore axis is pointed so that the projectile leaving the gun at a muzzle velocity of V_o will travel the vector diagonal to the point of impact. At the point of impact the projectile's motion normal to the sight line should be equal to the bombers' motion normal to the sight line.

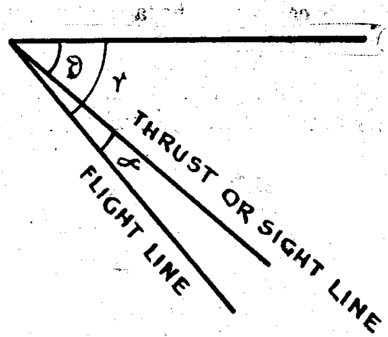

FIG. D

Hence $-V_F \sin (\gamma - \theta) + V_o \sin \triangle = V_B \sin \theta$

and $\gamma - \theta = \alpha - \triangle$

or $\triangle = \alpha + \theta - \gamma$

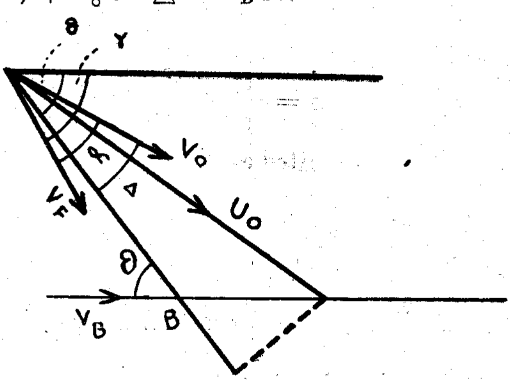

FIG E

So that △ may be eliminated. The above values are substituted in equations 7 and 8 to determine aerodynamic lead pursuit course.

The theory of aerodynamics gives formulae for the expressions L, D T. They are functions of aerodynamic variables

$$L = \frac{1}{2} \rho_a V^2_F SC_L$$

$$D = \frac{1}{2} \rho_a V^2_F SC_D$$

$$T = \frac{550 P\eta}{V_F}$$

where C_L and C_D are lift and drag co-efficients, S is the wing area, P, the break horse power of the engine, η the propeller efficiency and ρ_a is the air density.

$$C_L = k \sin (\alpha + \alpha_o) ; \quad C_D = A + \frac{C^2_L}{B}.$$

k, A and B are constants for a given plane.

$k = \dfrac{2\pi}{1 + \dfrac{2}{AR}}$ where AR is the aspect ratio which is determined by $\dfrac{b^2}{S}$

where b is the wing span. Parameters other than A, B, k and P are used in order to calculate the aerodynamic constants directly from the performance data of the fighter plane. The new constants C_1, C_2, C_3 and C_4 are defined by

$$C_1 = \frac{\rho_a k S}{2W}, \quad C_2 = \frac{\rho_a AS}{2W}, \quad C_3 = \frac{\rho_a k^2 S}{2BW}, \quad C_4 = \frac{550 P\eta}{W}$$

then

$$L = C_1 V^2_F W \sin (\alpha + \alpha_o)$$

$$D = [C_2 + C_3 \sin^2 (\alpha + \alpha_o)] V^2_F W$$

$$T = \frac{C_4 W}{V_F}$$

The constants C_1, C_2, C_3 and C_4 are determined from the fundamental performance equation

$$\frac{\sqrt{\rho}}{\lambda_t} = \frac{V^3_i}{\lambda_p} + \frac{\lambda_s}{V_i}$$

where $\rho = \dfrac{\rho_a}{\rho_o}$ (relative air density), $V_i = \sqrt{\rho}.V_F$

where V_i is the indicated air speed

$$\lambda_t = \frac{W}{550 \eta P}, \quad \lambda_p = \frac{2W}{\rho_o AS}, \quad \lambda_s = \frac{2W}{\rho_o BS} \text{ or } \frac{2W}{\rho_o \pi b^2}$$

$$\text{so that } C_4 = \frac{1}{\lambda_t}$$

$$C_2 = \rho/\lambda_p$$

$$\text{and } C_3 = \frac{C_1^2 \lambda_s}{\rho}$$

The description of the aircraft should supply the geometry of the plane, its gross weight W, the propeller efficiency η, maximum engine brake horse power P_{max} and the corresponding maximum level flight speed V_F for a certain altitude of density ratio ρ. With this information we can calculate λ_t, λ_p and λ_s by using the above equations. λ_p and λ_s depend only upon the geometry of the airplane and is therefore not affected by changes in ρ and V_F. With λ_p and λ_s determined, it is possible to calculate λ_t for any speed at any altitude. λ_t, λ_p, λ_s and k being known, C, C_2, C_3 and C_4 are determined. The differential equations are put in dimensionless form for easy solution. If V_{F_o} is some reference constant speed

$$\tau = \frac{gt}{V_{F_o}}, \quad v = \frac{V_F}{V_{F_o}}, \quad U_B = \frac{V_B}{V_{F_o}} \quad \text{and } S = \frac{g\,r}{V^2_{F_o}}$$

$$\text{and } K_1 = C_1 V^2_{F_o}, \; K_2 = C_2 V^2_{F_o}, \; K_3 = C_3 V^2_{F_o} \text{ and } K_4 = \frac{C_4}{V_{F_o}}$$

the four differential equations can be written as follows;—

$$\frac{dv}{d\tau} = \sin\gamma + \frac{K_4}{v}\cos\alpha - K_2 v^2 - K_3 v^2 \sin^2(\alpha + \alpha_o)$$

$$v\frac{d\gamma}{d\tau} = \cos v - K_1 v^2 \sin(\alpha + \alpha_o) + \frac{K_4}{v}\sin\alpha.$$

$$\frac{ds}{d\tau} = -v\cos(\gamma - \theta) + U_B\cos\theta$$

$$\frac{d\theta}{d\tau} = -\frac{1}{S}\left[v\sin(\gamma - \theta) + U_B\sin\theta\right]$$

The above equations can be solved numerically by the Runge-Kutta method or graphically for a suitable set of initial conditions, for the four variables v, s, γ and α. Fig. 8 is taken from H. G. Handleman's paper on "Aerodynamic pursuit curves for over-head attack". It shows the aerodynamic pursuit curve along with the pure pursuit curve for the same set of initial conditions. The fighter is a Republican Thunderbolt P 47 whose reference constant speed is $V_{F_o} = 260$ mph. The attack data is given by $S = \cdot 40$, $v = 1$, $U_B = \cdot 77$ and $\alpha_o = {}_o\cdot 035$ and $\theta = \cdot 524$ radians. K_1, K_2, K_3 and K_4 are found out to be 18·93, ·0618, 6·75 and ·0805 respectively.

The paths are plotted in terms of bomber's coordinates. In addition, position corresponding to increments in $\tau = \cdot 05$ have been marked. When the paths are considered as geometrical curves they are almost identical. If the time to reach a certain position along these practically coincident paths is considered, they are appreciably different. From the point of view of fire control therefore, there is a considerable difference in the target path depending on whether aerodynamic effects are considered or not.

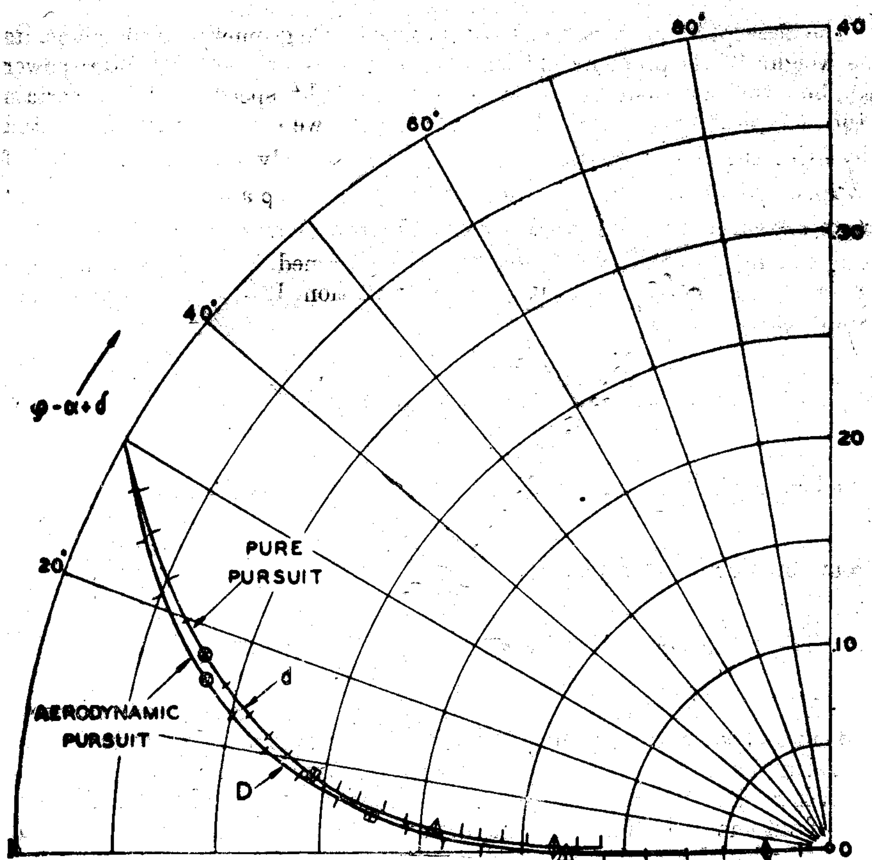

FIG 8

An introduction to the interception course of attack

Very great forces act on the wings of an aircraft in fast tight turns. In such manoeuvres, considerable loads and stresses may be set up on the aircraft structure. The aerodynamic restrictions on an aircraft may be such that the aircraft cannot fly a pursuit course. The formulae for the minimum radius of turn R in the horizontal and vertical planes are

$$R = \frac{V^2_F}{g\sqrt{n^2-1}} \quad \text{and} \quad R = \frac{V^2_F}{g\,(n - \cos\gamma)} \quad \text{respectively.}$$

where n is the load factor (Lift/Weight) and γ is the angle of climb. The radius of turn capable by the aircraft is a function of speed and load factor, which in turn, is a function of speed. The load factor falls off rapidly as the speed increases beyond the peak value at which 'n' is maximum. So radius of turn increases not only due to increase in speed but also due to the decreased value of the load factor, Consequently it becomes difficult to fly anything but a tail pursuit.

The present day bombers have speeds as high as the jet fighters and they can fly at such high altitudes as much as 50,000 feet. The advantages of a fighter over a bomber are its (1) speed superiority and (2) manouevrability. The speed of the fighter may be increased either by increasing the wing loading or decreasing the power loading or both. An increase in the wing loading decrease the manoeuvrability. Further at such high altitudes, the structural load factor limits the accelerations to 2g. So it is impossible for the fighters to adopt pursuit courses due to the high value of 'g' arising in such courses. So to meet the menace of the high speed bombers at such altitudes, a collision or interception course of attack or path of encounter is adopted. It is an evolution or a logical development of the curve of pursuit attack. Fig. 9 shows such an attack. The principle

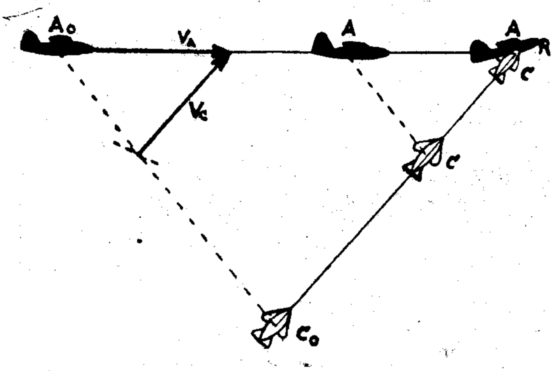

FIG 9

of attack is to fly in a straight path towards a point in advance of the target. This point is a collision point between the target and the attacker, as in the case of guided missiles, or a collision point between the target and the projectile that the attacker is firing. The success of the above method of attack depends on the degree of automatism achieved in the fire control system. The system is controlled entirely by electronic means.

Conclusion

Pursuit courses figure in the interception trajectories in spite of certain disadvantages such as (*a*) the target can be intercepted only when the speed of the fighter is much greater than that of the target unlike the case of the path of encounter where the target can be intercepted even when the fighter speed margin is less; (*b*) time of interception is greater in the curve of pursuit than in the case of path of encounter, and above all (*c*) tight turns are developed in pursuit course whereas a path of encounter is a straight path. Taking the analogy of dog and master problem, the dog follows a curve of pursuit only when the master moves slowly, and when it does not know the future position of the master, whether he is going to stop or make a turn. When the master moves at a great speed in a car, the dog follows the path of encounter to meet him. The same applies to aircraft, with added aerodynamical limitations, in aerial combat.

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APPENDIX A

If no approximation is made

$$\frac{dr}{r} = \left[\frac{(C \cos\theta - \cos\delta)}{(C_1 \operatorname{Sin}\theta - C \operatorname{Sin}\theta)}\right] d\theta$$

$$= \left[\frac{1}{C - C_1} \quad \frac{\operatorname{Cos}\delta}{\operatorname{Sin}\theta} - \frac{C}{C - C_1} \quad \operatorname{Cot}\theta\right] d\theta$$

Integrating $\log r = \frac{1}{C - C_1} \int \frac{\operatorname{Cos}\delta}{\operatorname{Sin}\theta} \; d\theta - \frac{C}{C_1 - C_1} \int \operatorname{Cot}\theta \, d\theta +$

$$\log r = \frac{1}{C - C_1} \int \frac{C \; s \; \delta}{\operatorname{Sin}\theta} d\theta - \frac{C}{C - C_1} \log \operatorname{Sin}\theta + k$$

$$\text{Put } Z = \frac{C_1 \; C \; s \; \theta}{C \; s \; \delta}$$

Differentiating $\operatorname{Cos}\delta \, dZ - \operatorname{Sin}\delta \, d\,\delta = - C_1 \operatorname{S n}\theta \, d\theta$

$$\operatorname{Sin}\delta = C_1 \operatorname{Sin}\theta \; ; \; \operatorname{Cos}\delta \, d\delta = C_1 \cos\theta \, d\theta$$

$$d\,\delta = C_1 \frac{\operatorname{Cos}\theta}{\operatorname{Cos}\delta} \; d\theta = Z \, d\theta.$$

Substituting this value,

$\operatorname{Cos}\delta \, dz = (Z^2 - 1) \; C_1 \operatorname{Sin}\theta \, d\theta.$

$$d\theta = \frac{1}{Z^2 - 1} \quad \frac{\operatorname{Cos}\delta}{C_1 \operatorname{Sin}\theta} \cdot dz.$$

But it can be shown that $\frac{\operatorname{Cos}\delta}{C_1 \operatorname{Sin}\theta} = \frac{\sqrt{1 - C^2}}{\sqrt{C_1^2 - Z^2}}$ as follows

$$\frac{\sqrt{1 - C_1^2}}{\sqrt{C_1^2 - Z^2}} = \frac{\sqrt{1 - C_1^2}}{C_1 \sqrt{1 - \frac{\operatorname{Cos}^2\theta}{\operatorname{Cos}^2\delta}}} = \frac{\operatorname{Cos}\delta}{C_1} \; \frac{\sqrt{1 - C_1^2}}{\sqrt{\operatorname{Cos}^2\delta - \operatorname{Cos}^2\delta}}$$

Substituting $\cos^2\delta = 1 - C_1^2 \operatorname{Sin}^2 0$ and simplifying

$$\frac{\sqrt{1 - C_1^2}}{\sqrt{C_1^2 - Z^2}} = \frac{\operatorname{Cos}\delta}{C_1 \operatorname{Sin}\theta}$$

A $\frac{1}{C - C_1} \int \frac{\operatorname{Cos}\delta}{\operatorname{Sin}\theta} d\theta$

$$= \frac{C_1}{C - C_1} \int \frac{1}{Z^2 - 1} \cdot \frac{1 - C_1^2}{C_1^2 - Z^2} \cdot dz.$$

$$= \frac{C_1 (C_1^2 - 1)}{C - C_1} \int \frac{dz}{(C_1^2 - Z^2)(1 - Z^2)}$$

By the usual method of partial fractions, when solved

$$\frac{1}{C-C_1}\int\frac{\cos\delta}{\sin\theta}\,d\theta=\frac{1}{C-C_1}\log\left[\left(\frac{C_1-Z}{C_1+Z}\right)^{\frac{1}{2}}\left(\frac{1+Z}{1-Z}\right)^{C_1/2}\right]$$

The value of k has been found out to be r_o by substituting initial values $r = r_o$ when $\theta = 90$.

Hence $\log r=\log r_o+\frac{1}{C_1-C_1}\log\left[\left(\frac{C_1-Z}{C_1+Z}\right)^{\frac{1}{2}}\left(\frac{1+Z}{1-Z}\right)^{C_1/2}\right]-\frac{C}{C-C_1}\log\sin\theta$

$$r=r_o\left\{\left[\left(\frac{C_1-Z}{C_1+Z}\right)\left(\frac{1+Z}{1-Z}\right)^{C_1}\right]^{\frac{1}{2}}\frac{1}{\sin^c\theta}\right\}^{\frac{1}{C-C_1}}$$

Substituting the value of Z and rearranging

$$\gamma=\gamma_o\left\{\left[\left(\frac{\cos\delta-\cos\theta}{\cos\delta+\cos\theta}\right)\left(\frac{\cos\delta+C_1\cos\theta}{\cos\delta-C_1\cos\theta}\right)^{C_1}\right]^{\frac{1}{2}}\frac{1}{\sin^c\theta}\right\}^{\frac{1}{C-C_1}}$$

where δ is a function of θ.

References

G. H. Handleman. Aerodynamic pursuit curves for overhead attacks. Journal of Franklin Institute March 1949.

Interavia Vol. 6 No. 2, 1951 Trejectories of interception.

Navord Report 1943. The mathematical theory of Air borne Fire Control.

Armament Notes on Fixed Gun sighting.