THE INFLUENCE OF METALLIC BODIES ON A COIL RADIATING AN ALTERNATING MAGNETIC FIELD

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ABSTRACT

The response of a metallic sphere placed in an alternating magnetic field has been studied in details and the conclusions derived therefrom have been verified experimentally. Similar investigations on cylinders and disc are also reported. The recent trend for developing non-metallic mines, in which the metallic components are very few, makes it imperative that a more detailed study with very small cylinders should be carried out.

Introduction

A study of the influence of metallic objects of various sizes and shapes on a coil radiating an alternating magnetic field was undertaken with a view to correlate the nature and magnitude of the influence with (*a*) size of the object (*b*) shape of the object (*c*) the metal of which the object is made (*d*) the distance at which the object is situated from the detecting coil (*e*) the frequency of the magnetic field and (*f*) the dimensions of the detecting coil. The study is bound to be of interest to the operator in the mine ~~field and~~ also to the designer of the mine detector—the former wanting to form an idea of the buried object from the nature of the response and the latter to know what sort of characteristics his detector should have to give the maximum response to a given object. A detector working on the Heterodyne Principle was built here for making the measurements and the accuracy of these measurements was tested in the case of metallic spheres by theory. Having made sure about the soundness of the measuring technique employed, responses of metallic cylinders and discs of various sizes were found and tabulated. The most interesting result that has come out of these investigations is the behaviour of ferromagnetic metals, such as mild steel whose response depends on the frequency employed. In fact there is a critical frequency for each object at which the eddy current effect swamps the magnetic effect giving practically zero response. Here again the results calculated from theory agree very well with experimental values.

The Theoretical Problem

The disturbance produced by a spherical metallic particle in a homogeneous alternating magnetic field is a classical problem and it has been discussed by Poritsky, Clapp and others. The exact solution can be obtained as a series involving Bessel functions[1]

Let $i_{in} = Ie^{jwt}$ be the initial current in the detector coil *i.e.* when no disturbing body is present.

L=inductance of the detecting coil in c.g.s. units

R=Resistance of the coil in c.g.s. units

$a = \sqrt{r^2 + x^2}$

where

r=radius of the detecting coil in cms

x=distance of the centre of the sphere from the centre of the coil along the axis of the coil in cms.

$\omega = 2\pi \times$ frequency

N=No. of turns in the coil

$p = \dfrac{4\pi\mu\omega}{\tau}$ where μ =permeability of the material of the sphere.

τ =conductivity of the material of the sphere.

b=radius of the sphere

$P'_n(\cos\alpha)$= Associated Legendre function of order n

$\operatorname{Sin}\alpha = \dfrac{r}{(r^2 + x^2)^{\frac{1}{2}}}$

In (x) =Modified Bessel function of the first kind of order n.

In the presence of the body the current becomes

$$i_{dis} = I e^{j\omega t} - \frac{4\pi^2 \sin^2\alpha\, aj\omega\, I e^{j\omega t} (R - j\omega L)N^2}{R^2 + \omega^2 L^2} \times \frac{(\mu_n + n + \mu)\, I_n + \tfrac{1}{2}(b\sqrt{jp}) - b\sqrt{jp}\, I_n - \tfrac{1}{2}(b\sqrt{jp})}{(\mu - 1)n\, I_n + \tfrac{1}{2}(b\sqrt{jp}) + b\sqrt{jp}\, I_n - \tfrac{1}{2}(b\sqrt{jp})} \times \left(\frac{b}{a}\right)^{2n+1} \cdot \frac{[P'_n(\cos\alpha)]^2}{n(n+1)} \quad \ldots \quad (1)$$

In view of the actual values of the parameters involved, it will be sufficiently accurate to take only the first term of the series and to replace each of the Bessel functions by the leading term of its asymptotic expansion. Then,

$$i_{dis} = Ie^{j\omega t} - I e^{j\omega t} \frac{2\pi^2 \sin^4\alpha\, b^3\omega(\omega L + jR)N^2}{a^2(R^2 + \omega^2 L^2)} \times \frac{(2\mu + 1)\left(1 - \dfrac{1}{b\sqrt{2p}}\right) - b\sqrt{p/2} + j\dfrac{2\mu + 1}{b\sqrt{2p}} - jb\sqrt{p/2}}{(\mu - 1)\left(1 - \dfrac{1}{b\sqrt{2p}}\right) + b\sqrt{p/2} + j\dfrac{\mu - 1}{b\sqrt{2p}} + jb\sqrt{p/2}}$$

$$= I e^{j\omega t}\left[1 - (A + jB)\right] \quad \text{(say)} \quad \ldots \quad (2)$$

Thus

$$|i_{dis}| = I\sqrt{(1 - A)^2 + B^2} \quad \ldots \quad (3)$$

and

$$\text{Sensitivity (per cent)} = 100\,\frac{|i_{dis}| - |i_{in}|}{|i_{in}|}$$

$$= 100\left[\sqrt{(1 - A)^2 + B^2} - 1\right] \quad (4)$$

Experimental results

The arrangement employed initially for the measurements of change in inductance at low frequencies is shown in fig. 1

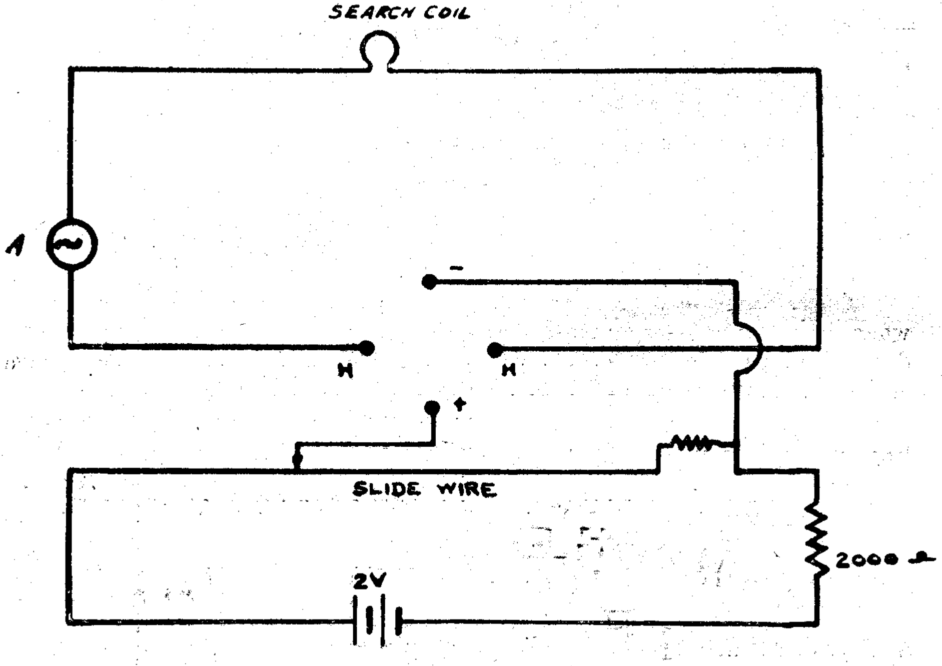

FIG. 1

A is the Audio frequency generator connected to the search coil and the current was measured by the Cambridge Vacuo-junction whose heater element was connected in series with the generator and the thermo-junction connected across a D.C. potentiometer. In the above arrangement the sensitivity of the potentiometer wire turned out to be 1/38 millivolt per cm length. Changes in e.m.f. upto ·01 m.v. could be accurately measured this way and the percentage change in current was found for Brass, Aluminium and Mild Steel spheres kept at various distances. The observed values agreed well with those calculated from theory, as the following Tables show.

Brass sphere 7·62 cms dia. *frequency of magnetic field 2,000 cps.*

Distance of object	Observed % decrease in inductance	Calculated % value
2 cms	4·70	4·54
5 cms	2·50	2·50
7·5 cms	1·50	1·54
10 cms	0·68	0·68
12·5 cms	0·40	0·37

Aluminium sphere 7·62 cms. dia., frequency of magnetic field 2,000 cps.

Distance of Object	Observed % decrease in inductance	Calculated % value
2	4·5	4·25
5	2·4	2·327
7·5	1·45	1·426
10	0·68	0·654
12·5	0·35	0·35

The arrangement however was not satisfactory at high frequencies when the current was obviously very small. A detector working on the heterodyne principle was therefore built in the laboratory and the percentage change in inductance was deduced by observing percentage change in frequency. The detector was quite sensitive to small objects and the circuit which was employed is shown below:

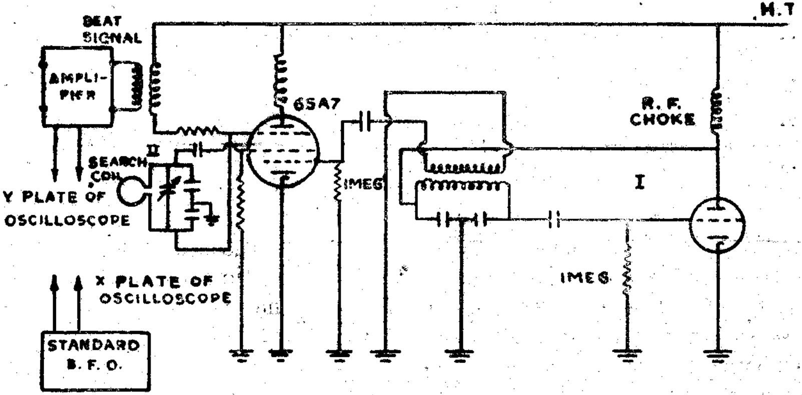

Fig. 2

I is a fixed oscillator of the Colpitt type, and II is a variable oscillator also of the Colpitt type, which has for its inductance the search coil. The mixed signal is fed through a step up transformer and amplifier to the 'Y' plates of the Cathode-ray-oscillograph where its frequency is compared to that produced in a standard beat frequency oscillator. The variable oscillator is initially adjusted so that there is no beat signal. Any object in the vicinity of the search coil gives a beat signal and the frequency of the signal is a measure of the change in inductance of the search coil.

This technique for measurements at high frequencies is obviously not as accurate as the former experiment which is a more direct method. The accuracy is limited by the following factors:

(*a*) The standard B.F.O. used for frequency comparison is calibrated for every hundred cycles and there is bound to be a 1% error in the measurement of frequencies of the order of 2,500 cycles.

(*b*) errors in the measurement of inductance of the search coil by the Universal Bridge.

*(*c*) errors in the measurement of the dimensions of (1) the object to be detected (2) the search coil.

*(*d*) errors in the measurement of the distance of the centre of the object from the centre of the search coil.

Within the limits of experimental errors listed above the results for Brass and Aluminium spheres compare favourably with those calculated from theory, as will appear from the following Tables.

Brass sphere 7·62 cms dia., frequency of the magnetic field 70,000 cps.

Distance of object	% decrease in inductance observed	% decrease as calculated
2	4·1	4·53
5	2·4	2·49
7·5	1·3	1·52
10	0·64	0·68
12·5	0·45	0·37

Aluminium sphere 7·62 cms dia., frequency of the magnetic field 70,000 cps.

Distance of object	% decrease in inductance observed	% decrease as calculated
2	3·9	4·24
5	2·25	2·32
7·5	1·26	1·42
10·0	0·6	0·65
12·5	0·4	0·35

* Also applicable to the first experiment.

The influence of frequency on sensitivity

The influence of frequency on sensitivity—In the case of non-magnetic metals like brass, copper and aluminium it was found that there is very little change of sensitivity with frequency. A mild steel sphere on the contrary behaved differently at different frequencies; the response at low frequencies was predominantly magnetic, while at high frequencies the eddy current had the better of the magnetic effect. The lowest sensitivity for the mild steel specimen under consideration was at forty thousand cycles per second, the corresponding theoretical figure being 35,000 cycles per second. The discrepancy between the theoretical and experimental curve may be attributed to the rather doubtful value of 150 which has been assumed as the permeability of the specimen under investigation.

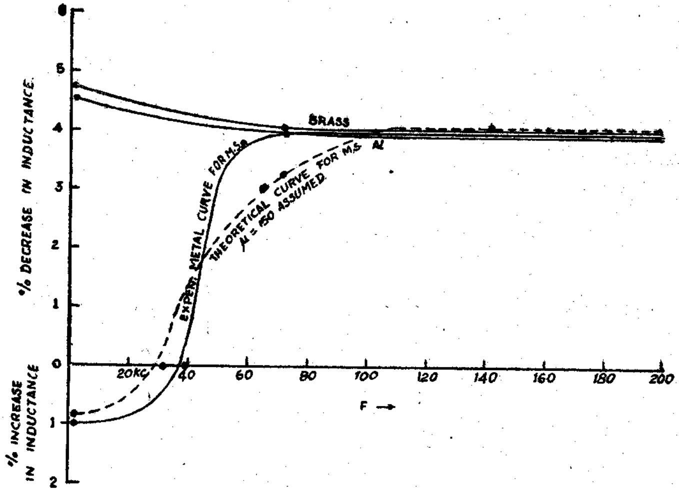

FIG. 3

Variation of Sensitivity with Distance

In the case of a non-magnetic metal for which the value of 'p' is large, the expression (2) can be further simplified to

$$i_{dis} = Ie^{j\omega t}\left[1 + \frac{2\pi^2 \sin^4\alpha\, b^3\omega\,(\omega L + jR)N^2}{a^2\,(R^2 + \omega^2 L^2)}\right] \quad .. \quad (5)$$

If further we assume that the resistance of the detecting coil is negligible in comparison with its inductive reactance, we have

$$i_{dis} = Ie^{j\omega t}\left[1 + \frac{2\pi^2\, b^3\, N^2\, r^4}{L\, a^6}\right] \quad .. \quad .. \quad (6)$$

It may be seen from this that the sensitivity at large distances of the object is proportional to the cube of the diameter of the sphere and inversely proportional to the sixth power of the distance of the object. The variation of sensitivity with distance is shown in the graphs below.

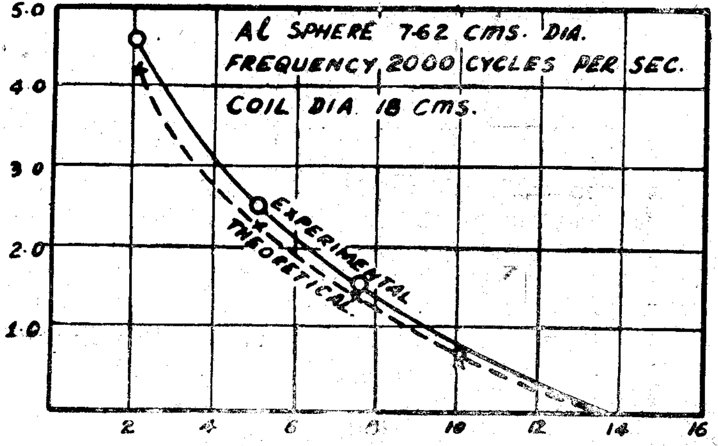

FIG. 4

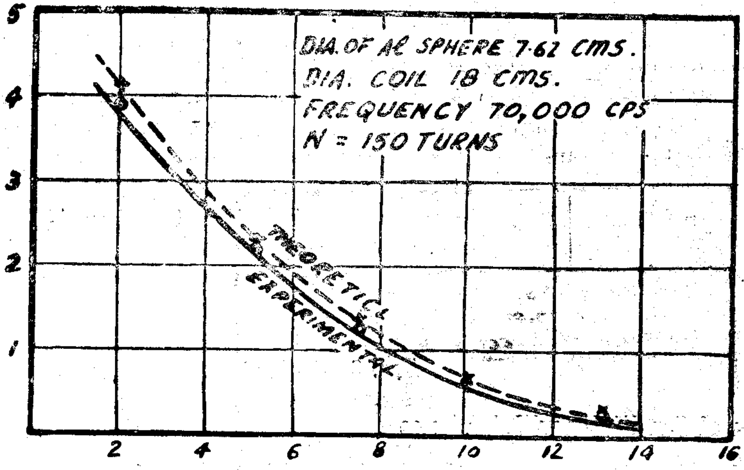

FIG. 5

Influence of coil diameter on sensitivity

It is clear from eqn. 6 para 6 that for $x << r$ the sensitivity falls off with the increase in the radius of the coil and for $x >> r$ sensitivity increases as the cube of the radius of the coil. With the object at a fixed distance from the centre of the detecting coil, the sensitivity for coils of varying radii is a maximum when the radius of the coil is equal to this distance. The graph below shows the variation of sensitivity with the diameter of the search coil for various distances of the object as calculated from the theory. The experimental values are also marked. It will be seen that experimental values within the limits of error agree fairly closely with the corresponding theoretical values.

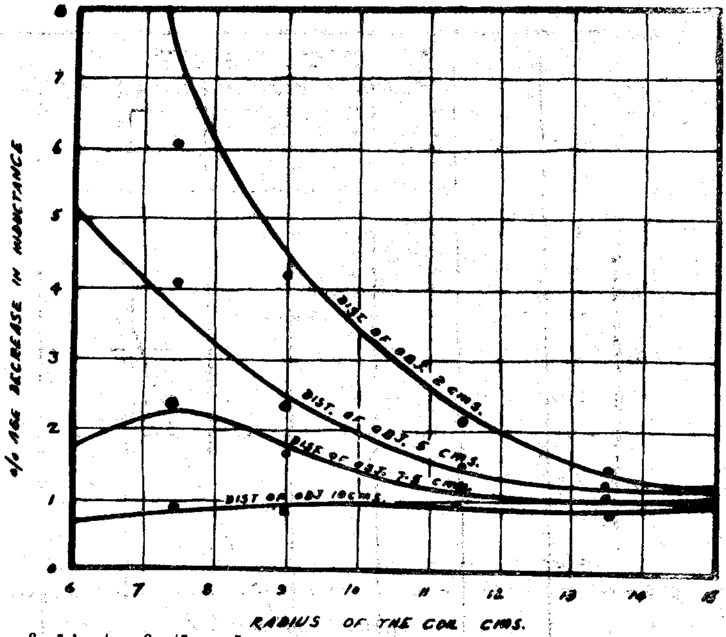

FIG. 6

Sensitivities of objects of other shapes

Experiments for the sensitivities of other objects such as brass cylinders (solid and hollow) of various diameters and lengths and discs of various diameters were done and results are expressed in figures 7, 8 and 9.

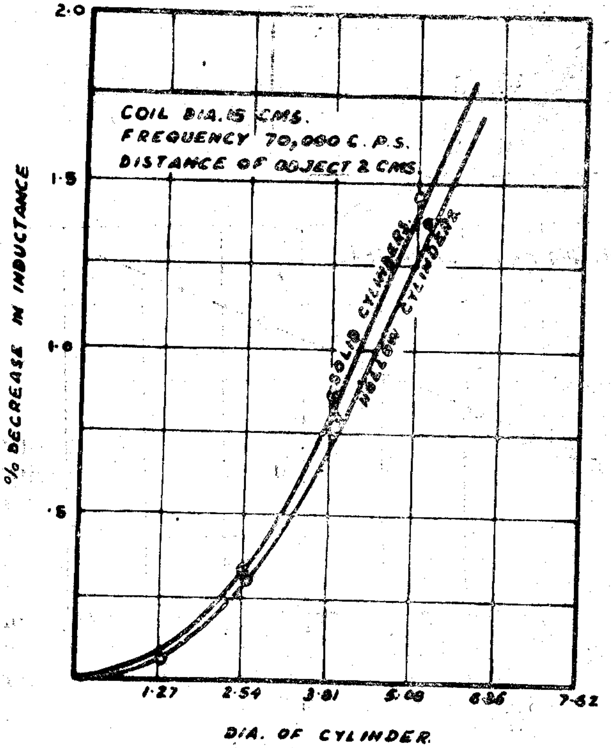

FIG. 7

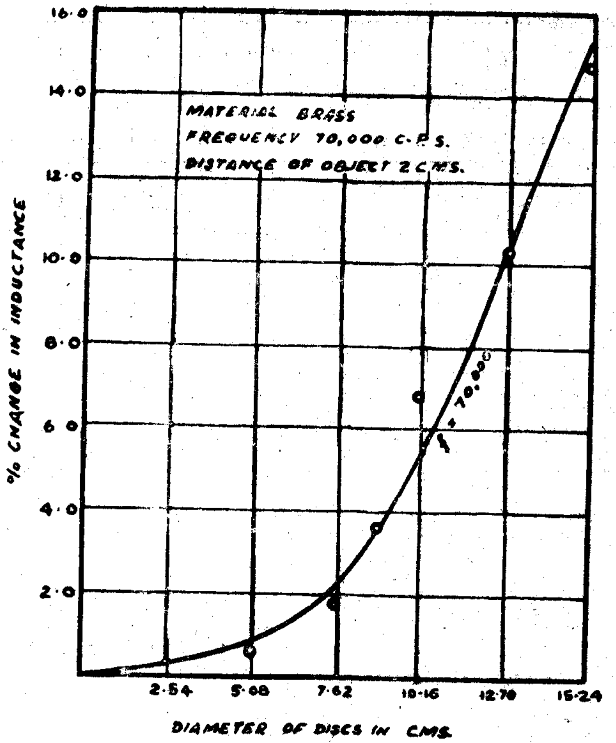
MATERIAL BRASS
FREQUENCY 70,000 C.P.S.
DISTANCE OF OBJECT 2 CMS.
% CHANGE IN INDUCTANCE
16·0
14·0
12·0
10·0
8·0
6·0
4·0
2·0
f = 70,000
2·54
5·08
7·62
10·16
12·70
15·24
DIAMETER OF DISCS IN CMS.

Fig. 8

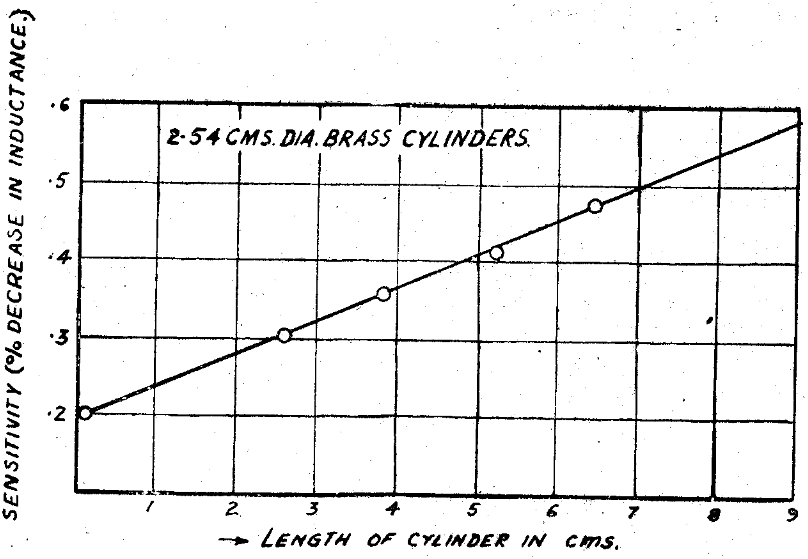
2·54 CMS. DIA. BRASS CYLINDERS.
SENSITIVITY (% DECREASE IN INDUCTANCE.)
·6
·5
·4
·3
·2
1
2
3
4
5
6
7
8
9
→ LENGTH OF CYLINDER IN CMS.

Fig. 9

Conclusion

A theoretical formula has been deduced for the response of a spherical metallic particle in an alternating magnetic field, and it has been verified by experiments performed on metallic spheres (1) for different coil diameters, (2) for various distances of the object and (3) for different frequencies. The analysis for a sphere is bound to give us at least a qualitative idea regarding the behaviour of objects of other shapes such as cylinders, discs etc. A few preliminary experiments have been done on cylinders of various lengths and diameters as well as discs of different diameters. The modern trend has been to develop non-metallic mines in which the igniter is perhaps the only metallic part. Design of metal detectors sufficiently sensitive to detect small cylinders seems therefore to be an urgent need. Further work on the theoretical and practical aspects of response to small cylinders is contemplated.

Acknowledgements

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References

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3. Curtiss—R. Shafer P. 86 Electronics Nov. 1951
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