

# BURNING OF SINGLE DROPS OF FUEL IN AN OXIDIZING ATMOSPHERE

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## ABSTRACT

In their discussion of the burning of single drops in an oxidizing atmosphere Goldsmith and Penner have assumed conduction to be the sole mechanism of heat transfer. In the present paper the authors have employed the well known expression for the rate of evaporation of droplet due to Maxwell and Langmuir, instead of the above arbitrary assumption. The weight fraction of the fuel and oxidizer have been expressed as functions of the distance from the droplet centre. Expressions for the radius and temperature of the flame front have also been derived.

## INTRODUCTION

Godsave (1950, 1951 and 1952) and Goldsmith and Penner (1954) have carried out a preliminary theoretical investigation of the burning of a single drop of fuel in an oxidizing atmosphere. In technical combustion processes, with little interference between burning droplets the single drop analysis is a useful approximation.

The analysis of Goldsmith and Penner is based on the following assumptions—

- (1) The droplets are spherical.
- (2) The flame front surrounding the drop is represented by a spherical surface, concentric with the drop. All reactions take place instantaneously; the delivery rates of fuel and oxidizer are in stoichiometric proportions.
- (3) Steady state conditions are assumed for fixed droplet sizes.
- (4) The temperature of the liquid drop is assumed to be uniform and equal to the boiling point of the fuel.
- (5) A mean pressure and density have been assumed.
- (6) Conduction is the only mechanism of heat transfer.

The last assumption is far from truth since convection is mainly responsible for heat transfer in gases. Instead of this arbitrary assumption, we can assume the rate of evaporation of the droplet to be given by the well known expression due to Maxwell (1890) and Langmuir (1918) which has been verified amongst others by Morse (1910), Namekawa and Takahashi (1937) and recently by Ranz and Marshall (1952). This expression has been employed in the present paper.

Theory

The rate of evaporation of a fuel droplet of radius  $r_l$  is given by

$$\begin{aligned} \dot{m}_F &= - \frac{dm_F}{dt} = \frac{4\pi M}{R} \left( \frac{D_F p_F}{T} \right)_{T=T_1} r_l \\ &= \sigma r_l \dots \dots \dots (1) \end{aligned}$$

Where  $m_F$  is the mass of the fuel droplet,

$\dot{m}_F$  is the rate of production of fuel,

$t$  is the time,

$D_F$  is the coefficient of diffusion,

$T_1$  is the temperature of the fuel droplet equal to the boiling point of the fuel,

$p_F$  is the saturated vapour pressure of the fuel,

$r_l$  is the radius of the drop

and  $R$  is the Universal gas constant.

If  $Y_F$ ,  $Y_O$  and  $Y_1$  represent the weight fractions of the fuel, oxidizer and combustion products respectively and  $r$  the distance from the droplet centre the second assumption gives—

$$Y_O = 0, Y_F = 1 - Y_1 \text{ when } r_l < r < r_c \dots \dots \dots (2A)$$

$$Y_F = 0, Y_O = 1 - Y_1 \text{ when } r > r_c \dots \dots \dots (2C)$$

$$Y_F = Y_O = 0 \text{ when } r = r_c \dots \dots \dots (2C)$$

where  $r_c$  is the radius of the flame front

We also assume the physical properties of the product gases to be the same as those of the inert gas as has been assumed by Godsave and Goldsmith & Penner. If  $\dot{m}_O$  represents the delivery rate of oxidizer we also have

$$\dot{m}_O = - \gamma_o \dot{m}_F \dots \dots \dots (3)$$

from the second assumption; a negative sign for  $\gamma_o$  (the ratio of molecular weights of the oxidizer and the fuel) has been used because the oxidizer flows in a direction, opposite to that of the fuel.

The general continuity equation for the fuel vapour can be written in the form.

$$\dot{m}_F = Y_F \dot{m}_F - 4\pi r^2 \rho D_F \frac{dY_F}{dr} \dots \dots \dots (4)$$

where  $r$  is the distance from the centre of the droplet thus

$$- \frac{dY_F}{1 - Y_F} = \frac{\sigma r_l}{4\pi \rho D_F} \frac{1}{r^2} dr$$

$$\text{or } \log(1 - Y_F) = \frac{\sigma r_l}{4\pi \rho D_F} \left( - \frac{1}{r} \right) + \text{constant.}$$

Applying the boundary condition (2C)

$$\log(1 - Y_F) = \frac{\sigma r_l}{4\pi\rho D_F} \left( \frac{1}{r_c} - \frac{1}{r} \right) \dots \dots \dots (5)$$

The general continuity equation for the oxidizer may be put in the form

$$\dot{m}_o = 4\pi r^2 \rho Y_o \left\{ \frac{\dot{m}_F}{4\pi r^2 \rho} - \frac{D_o}{Y_o} \frac{dY_o}{dr} \right\}$$

Using Eqn (3) we get

$$4\pi r^2 \rho D_o \frac{dY_o}{dr} = \dot{m}_F (Y_o + \gamma_o) \dots \dots \dots (6)$$

$$\text{or } \frac{dY_o}{(Y_o + \gamma_o)} = \frac{\dot{m}_F}{4\pi\rho D_o} \frac{dr}{r^2}$$

$$\text{or } \log(Y_o + \gamma_o) = \frac{\sigma r_l}{4\pi\rho D_o} \left( -\frac{1}{r} \right) + \text{constant}$$

If  $k$  is the weight fraction of the oxidizer in the incoming gases we have  $Y_o = k$  at  $r = \infty$  and consequently

$$\log \left( \frac{k + \gamma_o}{Y_o + \gamma_o} \right) = \frac{\sigma r_l}{4\pi\rho D_o} \frac{1}{r} \dots \dots \dots (7)$$

Putting  $Y_o = 0$  at  $r = r_c$  Eqn (7) gives

$$r_c = \frac{\sigma r_l}{4\pi\rho D_o} \left\{ \log \left( \frac{k + \gamma_o}{\gamma_o} \right) \right\}^{-1} \dots \dots \dots (8)$$

From Eqns (5) and (8) we obtain

$$\log(1 - Y_F) = \frac{D_o}{D_F} \log \left( \frac{k + \gamma_o}{\gamma_o} \right) - \frac{\sigma r_l}{4\pi\rho D_F} \frac{1}{r} \dots \dots \dots (5A)$$

**Temperature of Flame Front**

In the steady state heat is produced in the system by burning of the fuel at the rate  $H\dot{m}_F$  where  $H$  is the heat of reaction of burning of a unit mass of the fuel.  $H$  is the difference of the enthalpies of the fuel and oxidizer gases and of the combustion products at a temperature  $T_c$ . The rate of heat utilisation in raising the temperature of the fuel from  $T_1$  to  $T_c$  and of oxidizer from  $T_c$  to  $T_c$  and also evaporating the fuel droplet is

$$\dot{m}_F \left\{ L + \int_{T_1}^{T_c} C_{pF} dT \right\} + \dot{m}_o \int_{T_c}^{T_c} C_{pO} dT$$

Where  $L$  is the latent heat of evaporation of the fuel. Hence we have

$$H = L + \int_{T_1}^{T_c} C_{pF} dT + \gamma_o \int_{T_c}^{T_c} C_{pO} dT \dots \dots \dots (9)$$

As a first approximation we may assume,  $H$ ,  $C_{pF}$  and  $C_{pO}$  as constant. Then

$$H = L + \overline{C_{pF}}(T_c - T_1) + \gamma_o \overline{C_{pO}}(T_c - T_c)$$

$$\text{or } T_c = \frac{(H - L + \overline{C_{pF}}T_1 + \gamma_o \overline{C_{pO}}T_c)}{(\overline{C_{pF}} + \gamma_o \overline{C_{pO}})} \dots \dots \dots (10)$$

The variation of  $Y_o$ ,  $Y_F$  and  $r_o$  with time can be studied by using the following relation which can be easily obtained from Eqn (1) —

$r_l = \sqrt{r_o^2 - (\sigma/2\pi\rho) t}$  where  $r_o$  is the initial fuel droplet radius.

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