ON THE REMAINING VELOCITY OF OGIVAL AND HEMISPHERICAL PROJECTILES AFTER PENETRATING THROUGH A CERTAIN THICKNESS

By Shri V. K. Jain, Defence Science Laboratory, Ministry of Defence, New Delhi

ABSTRACT

Nishiwaki (1951) considered the penetration of conical projectiles. In this paper the author has extended his treatment to the case of ogival and hemispherical nosed projectiles and given an expression for the remaining velocity after penetrating through a certain thickness of the material concerned.

Introduction

Nishiwaki (1951) has discussed the penetration of conical projectiles. In this paper the author has extended his treatment to ogival and hemispherical nosed projectiles, which are actually used in operation against armour.

Equation of Motion

Consider a plane element $d\sigma$ at a point displaced in the material being penetrated along a direction $x'x$ included at an angle α with the element. Referring to Fig. 1, it experiences a resistance whose component along $x'x$ is—

$$dX = \left(\sin\alpha + f_{v\cos\alpha} \cos\alpha \right) dN \quad \ldots \quad \ldots \quad (1)$$

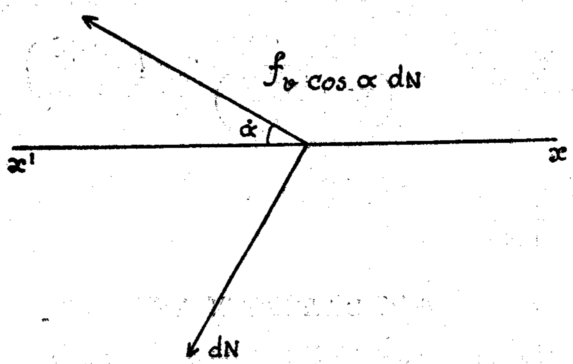

Fig. 1 Resistance to an element $d\sigma$

where dN is the normal reaction, $f_{v\cos\alpha}$ the co-efficient of steel friction on the material being penetrated at the velocity $v\cos\alpha$

We have—

$$\frac{dN}{d\sigma} = P_v \sin\alpha, \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

where P_v is the resistance to the penetration of the projectile at velocity v.

Hence Eqn (1) becomes

$$dX = P_v \sin\alpha \left[\sin\alpha + \frac{f}{v\cos\alpha} \cos\alpha \right] d\sigma \quad \ldots \quad (3)$$

Neglecting the term $\frac{f}{v\cos\alpha} \cos\alpha$, which is $<< \sin\alpha$

$$dX = P_v \operatorname{Sin}^2\alpha \; d\sigma \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

But Gabeand (1949) to our advantage gives the important relation

$$P_v = P(1+bv^2) \quad \ldots \quad \ldots \quad \ldots \quad (5$$

where b is a constant depending upon the metal to be penetrated given by—

$$b = \frac{\delta}{gP} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

where δ is the density of the material.

Using (5) and (4) the equations of motion becomes—

$$X = \frac{W}{g} v \frac{dv}{dx} = - P(1+bv^2) \int \sin^2\alpha \, d\sigma \; (7)$$

where W is the weight of the projectile

and g is the acceleration due to gravity.

Ogival Nosed Projectile

We denote the depth of penetration by x measured from the front surface of the material to the point of the projectile, the total thickness of the material penetrated by ϵ, the height of the ogive by h ($\geqslant \epsilon$).

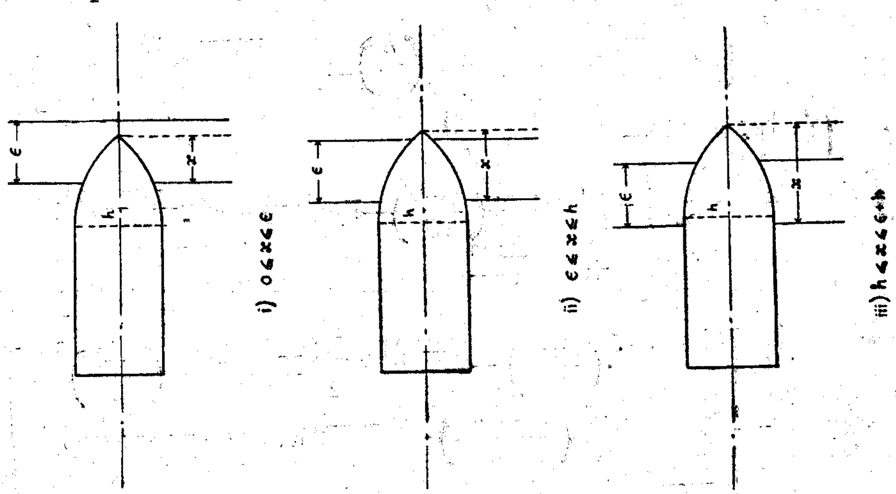

Fig. 2 Penetration by an ogival head projectile.

The equation of the circle whose arc is AB, referred to B, the nose of the projectile (n c.r.h) as the origin of the coordinates can be written as:

$$(x-h)^2 + \left[y - \left(\frac{d}{2} - nd \right) \right]^2 = n^2 d^2 \quad .. (8)$$

Differentiating:—

$$\frac{dy}{dx} = - \frac{x-h}{\sqrt{n^2d^2-(x-h)^2}} = \tan \alpha$$

$$d\sigma = 2\pi y \, ds$$

$$= \frac{2\pi nd \left[\left(\frac{d}{2} - nd \right) + \sqrt{n^2d^2 - (x-h)^2} \right] dx}{\sqrt{n^2d^2 - (x-h)^2}}$$

Referring to Fig. 2 we obtain X given by equation (7) in three steps

(i) $o \leqslant x \leqslant \epsilon$:—

$$\int \sin^2\alpha \, d\sigma = 2\pi nd \int_o^x \frac{(x-h)^2}{n^2d^2} \left\{ \frac{\left(\frac{d}{2} - nd \right) + \sqrt{n^2d^2-(x-h)^2}}{\sqrt{n^2d^2-(x-h)^2}} \right\} dx$$

$$= \frac{2\pi}{nd} \left[\frac{n^2d^2}{2} K \operatorname{Sin}^{-1} \left(\frac{x-h}{nd} \right) - \frac{K(x-h)\sqrt{n^2d^2-(x-h)^2}}{2} + \frac{(x-h)^3}{3} \right]_o^x$$

$$\text{Where } K = \frac{d}{2} - nd$$

$$= \frac{2\pi}{nd} \left\{ \begin{array}{l} \frac{n^2d^2K}{2} \sin^{-1}\left(\frac{x-h}{nd} \right) - \frac{(x-h)K\sqrt{n^2d^2-(x-h)^2}}{2} + \frac{(x-h)^3}{3} \\ + \frac{n^2d^2K}{2} \sin^{-1}\left(\frac{h}{nd} \right) - Kh \frac{\sqrt{n^2d^2-h^2}}{2} + \frac{h^3}{3} \end{array} \right\}$$

(ii) $\epsilon \leqslant x \leqslant h$:—

$$\int \sin^2 a \, d\sigma = \frac{2\pi}{nd} \left\{ \begin{array}{l} \frac{n^2d^2K}{2} \sin^{-1}\left(\frac{x-h}{nd} \right) - K \frac{(x-h)\sqrt{n^2d^2-(x-h)^2}}{2} \\ + \frac{(x-h)^3}{3} \end{array} \right\}_{x-\epsilon}^{x}$$

$$= \frac{2\pi}{nd} \left\{ \begin{array}{l} \frac{n^2d^2K}{2} \sin^{-1}\left(\frac{x-h}{nd} \right) - \frac{(x-h)K\sqrt{n^2d^2-(x-h)^2}}{2} + \left(\frac{x-h}{3} \right)^3 \\ - \frac{n^2d^2K}{2} \sin^{-1}\left(\frac{x-\epsilon-h}{nd} \right) + \frac{x-\epsilon-h}{2} K . \sqrt{n^2d^2-(x-\epsilon-h)^2} \\ - \frac{(x-\!-h)^3}{3} \end{array} \right\}$$

(iii) $h \leqslant x \leqslant h+\epsilon$:—

$$\int \sin^2 \alpha \, d\sigma = 2\pi/nd \left[\frac{n^2d^2K}{2} \sin^{-1}\left(\frac{x-h}{nd}\right) - \frac{K(x-h)\sqrt{n^2d^2-(x-h)^2}}{2} + \frac{(x-h)^3}{3} \right]_{x-\epsilon}^{h}$$

$$= 2\pi/nd \left[-\frac{n^2d^2K}{2} \sin^{-1}\left(\frac{x-\epsilon-h}{nd}\right) + \frac{K(x-\epsilon-h)\sqrt{n^2d^2-(x-\epsilon-h)^2}}{2} - \frac{(x-\epsilon-h)^3}{3} \right]$$

Introducing these expressions for $\int \mathrm{Sin}^2\alpha \, d\sigma$ into equation (7) we obtain the following results—

(i)

$$\frac{W}{2gbP} \left[\log(1+bv^2) \right]_{V_\circ}^{V_1}$$

$$= -\frac{2\pi}{nd} \left[\frac{n^2d^2k}{2} (\epsilon-h) \sin^{-1}\left(\frac{\epsilon-h}{nd}\right) + \frac{n^2d^2K}{2} \sqrt{n^2d^2-(\epsilon-h)^2} + \frac{\left[n^2d^2-(\epsilon-h)^2 \right]^{3/2} K}{6} + \frac{(\epsilon-h)^4}{12} + \frac{n^2d^2}{2} \epsilon K \sin^{-1}\left(\frac{h}{nd}\right) - \frac{hk\epsilon}{2} \sqrt{n^2d^2-h^2} + \frac{h^3}{3} \epsilon - \frac{n^2d^2}{2} K h \sin^{-1}\left(\frac{h}{nd}\right) - \frac{n^2d^2K}{2} \sqrt{n^2d^2-h^2} - \frac{(n^2d^2-h^2)^{3/2} K}{6} - \frac{h^4}{12} \right]$$

(ii)

$$\frac{W}{2gbP} \left[\log(1+bv^2) \right]_{V_1}^{V_2}$$

$$= -\frac{2\pi}{nd} \left[\frac{n^3d^3K}{2} + \frac{n^3d^3K}{6} - \frac{n^2d^2K.\epsilon}{2} \sin^{-1}\left(\frac{\epsilon}{nd}\right) - \frac{n^2d^2K}{2} \sqrt{n^2d^2-\epsilon^2} - \frac{\left\{ n^2d^2-\epsilon^2 \right\}^{3/2} K}{6} - \frac{\epsilon^4}{12} - \frac{n^2d^2 K (\epsilon-h) \sin^{-1}\left(\frac{\epsilon-h}{nd}\right)}{2} - \frac{n^2d^2}{2} K \sqrt{n^2d^2-(\epsilon-h)^2} - \frac{\left\{ n^2d^2-(\epsilon-h)^2 \right\}^{3/2} K}{6} - \frac{(\epsilon-h)^4}{12} + \frac{n^2d^2K}{2} h \sin^{-1} \frac{h}{nd} + \frac{n^2d^2K}{2} \sqrt{n^2d^2-h^2} + \frac{\left\{ n^2d^2-h^2 \right\}^{3/2} K}{6} + \frac{h^4}{12} \right]$$

(iii)

$$\frac{W}{2gb\,P}\Big[\log(1+bv)\Big]_{V_2}^{V'} = -\frac{2\pi}{nd}\Big[-\frac{n^3d^3K}{2}-\frac{n^3d^3K}{6}+\frac{n^2d^2}{2}K.\,\epsilon\sin^{-1}\Big(\frac{\epsilon}{nd}\Big)$$
$$+\frac{n^2d^2K}{2}\sqrt{n^2d^2-\epsilon^2}+\frac{\{n^2d^2-\epsilon^2\}^{3/2}}{6}K+\frac{\epsilon^4}{12}\Big]$$

Adding the three we get—

$$\Big[\log\ (1+bv^2)\Big]_{V_o}^{V'} = \frac{-2b\,Pg\varepsilon}{W}\left\{\begin{matrix}\frac{n^2d^2K}{2}\sin^{-1}\frac{h}{nd}-\frac{hK}{2}\sqrt{n^2d^2-h^2}\\ +\frac{h^3}{3}\end{matrix}\right\}\frac{2\pi}{nd}$$

$$= -\phi\,\epsilon \qquad ;$$

V_o and V' being the striking and remaining velocities respectively.

$$\text{or } V'^2 = \frac{-1+\left(1+bv_o^2\right)e^{-\phi\epsilon}}{b} \qquad \ldots \quad (9)$$

The same expression is obtained if we start with the assumption $h < \varepsilon$.

Hemispherical Nosed Projectile

The equation of the axial section of the head referred to the point of its nose as origin is

$$(x-r)^2 + y^2 = r^2$$

It can be shown that—

$$\frac{ds}{dx} = \frac{r}{y}$$

X may be obtained in three steps.

(i) $o \leqslant x \leqslant r$:—

$$\int \sin\alpha\ d\sigma = 2\pi r\int_o^x \frac{(x-r)^2}{r^2}\,dx = \frac{2\pi}{r}\Big[\frac{(x-r)^3}{3}+\frac{r^3}{3}\Big]$$

(ii) $\varepsilon \leqslant x \leqslant r$:—

$$\int \sin^2\alpha\ d\sigma = \frac{2\pi}{r}\int_{x-\epsilon}^{x}(x-r)^2\,dx = \frac{2\pi}{r}\Big[\frac{(x-r)^3}{3}-\frac{(x-\epsilon-r)^3}{3}\Big]$$

(iii) $r \leqslant x \leqslant r+\varepsilon$:—

$$\int \sin^2\alpha\ d\sigma = \frac{2\pi}{r}\int_{x-\epsilon}^{r}(x-r)^2dx = \frac{2\pi}{r}\Big[-\frac{(x-\varepsilon-r)^3}{3}\Big]$$

Introducing these expressions for $\int \sin^2\alpha \, d\sigma$ into equation (7) we obtain the following results.

$$\text{(i)} \quad \frac{W}{2bgP}\Big[\log(1+bv^2)\Big]_{V_o}^{V_1} = -\frac{2\pi}{r}\left[\frac{(\varepsilon-r)^4}{12}+\frac{r^3\epsilon}{3}-\frac{r^4}{12}\right]$$

$$\text{(ii)} \quad \frac{W}{2bgP}\Big[\log(1+bv^2)\Big]_{V_1}^{V_2} = -\frac{2\pi}{r}\left[\frac{-\epsilon^4}{12}-\frac{(\varepsilon-r)^4}{12}+\frac{r^4}{12}\right]$$

$$\text{(iii)} \quad \frac{W}{2bgP}\Big[\log(1+bv^2)\Big]_{V_2}^{V'} = -\frac{2\pi}{r}\left[\frac{\varepsilon^4}{12}\right]$$

Adding these three we get:—

$$\log\frac{(1+bV'^2)}{(1+bV_o^2)} = -\left(\frac{2bg\,P}{W}\;\frac{2\pi}{r}\cdot\frac{1}{3}r^3\right)\varepsilon$$

$$= -\psi\epsilon$$

$$\text{or } V'^2 = \frac{-1+(1+bV_o^2)e^{-\psi\epsilon}}{b} \qquad \ldots \qquad \ldots (10)$$

The same expression is obtained if we start with the assumption $\varepsilon > h$

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