SOLUTION OF THE PROBLEM OF COMPOSITE CHARGE USING 'R.D.38' METHOD

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ABSTRACT

In this paper the author has solved the problem of internal ballistics of composite charge using 'R.D. 38' method which is based upon the usual isothermal approximation. A linear law of burning has been assumed.

INTRODUCTION

The problem of composite charge can be solved either by reducing the charges to a single equivalent charge as has been done by Corner or by treating the problem directly. Corner has suggested how a composite charge, **weights** (C_1, C_2) of webs (D_1, D_2) and form factors (θ_1, θ_2) can be reduced to a single equivalent charge C of D web and form factor θ such that the single charge gives the same ballistic equations as the composite charge. He has given a method of finding the value of θ and suggests that having found the value of θ we can use it for finding the ballistic solution of composite charge by any of the well-known methods.

In this paper the author has given a direct treatment of the problem of composite charge using 'R.D. 38' method which is based upon the usual isothermal approximation i.e. the temperature of the gases during burning can be replaced by a mean value, corresponding to effective mean force constant λ assumed to be equal for the two charges. This is a fair approximation, since the continuing conversion of thermal energy of the gas to kinetic energy of the shot is largely compensated by the generation of energy by the reaction of more propellant.

Fundamental Equations

The equation of state of the gas in the gun is

$$P_1\left(U+Ax-C_1\frac{1-Z_1}{\delta_1}-C_2\frac{1-Z_2}{\delta_2}-C_1Z_1\eta_1-C_2Z_2\eta_2\right)=\lambda\left(C_1Z_1+C_2Z_2\right)$$

where P_1 is the mean pressure through the volume behind the shot. But from Lagrange's approximation the space mean pressure P_1 at the instant considered is

$$P_1=\frac{P\left(1+\dfrac{C_1+C_2}{3W_1}\right)}{\left(1+\dfrac{C_1+C_2}{2W_1}\right)} \qquad \cdots \qquad \cdots \qquad (2)$$

where P is the pressure at the breech and the pressure on the base of the shot is $P_1\left(1+\dfrac{C_1+C_2}{2W_1}\right)$ and W_1 is the effective mass of the shell.

Neglecting co-volume and using equation (2) in (1), we have

$$P\left(U + Ax - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2}\right) = (C_1Z_1 + C_2Z_2)\ \frac{\lambda\left(1 + \frac{C_1 + C_2}{2W_1}\right)}{\left(1 + \frac{C_1 + C_2}{3W_1}\right)} \quad \ldots (3)$$

We write

$$U - \left(\frac{C_1}{\delta_1} + \frac{C_2}{\delta_2}\right) = Al \quad \text{(initial free space behind the shot).}$$

So equation (3) becomes

$$P(x + l) = \frac{\lambda(C_1Z_1 + C_2Z_2)}{A}\ \frac{\left(1 + \frac{C_1 + C_2}{2W_1}\right)}{\left(1 + \frac{C_1 + C_2}{3W_1}\right)} \quad \ldots (4)$$

The form functions are

$$Z_1 = (1 - f_1)\ (1 + \theta_1 f_1) \quad \ldots \ldots (5a)$$

$$Z_2 = (1 - f_2)\ (1 + \theta_2 f_2) \quad \ldots \ldots (5b)$$

The rate of burning equations are

$$D_1 \frac{df_1}{dt} = -\beta_2 P \quad \ldots \ldots (6a)$$

$$D_2 \frac{df_2}{dt} = -\beta_2 P \quad \ldots \ldots (6b)$$

The dynamical equation of the motion of the shot is

$$\left(W_1 + \frac{C_1 + C_2}{2}\right)\frac{dV}{dt} = AP \quad \ldots \ldots (7)$$

Simultaneous and non-simultaneous burning of charges

From (6a) and (6b), we have

$$\frac{d\,f_1}{d\,f_2} = \frac{\beta_1\, D_2}{\beta_2\, D_1} = K(\text{say}) \quad \ldots \ldots (8)$$

Integrating (8) and applying the conditions that initially $f_1 = f_2 = 1$, we get

$$(1 - f_1) = K(1 - f_2) \quad \ldots \ldots (9)$$

Now two cases arise, viz

(*i*) The two charges may burn out simultaneously and

(*ii*) The two charges may burn out at different times.

Case (*i*):—If the two charges burn out simultaneously then at "all-burnt" $f_1 = f_2 = 0$ and so from (9) we have the condition for simultaneous burning as $K = 1$ i.e. $\beta_1\, D_2 = \beta_2\, D_1$.

Case (*ii*):—If the charge C_1 burns out first then if suffix '*b*' denotes this instant, we have from (9)

$$f_{2,b} = 1 - \frac{1}{K}$$

But $f_{2,b}$ must be a positive fraction, and so the condition for the burning out of charge C_1 first is, $K > 1$ i.e. $\beta_1 D_2 > \beta_2 D_1$.

If charge C_2 burns out first we must have similarly $K < 1$ i.e. $\beta_1 D_2 < \beta_2 D_1$.

For the sake of definiteness we will call that propellant which burns out first as C_1.

Solution of Equations

Eliminating P from (6A) and (7) and integrating, we get

$$V = \frac{AD_1\,(1-f_1)}{\beta_1\left(W_1 + \dfrac{C_1 + C_2}{2}\right)} \qquad \ldots \qquad \ldots \quad (10)$$

If we use equation (10) in equation (7) and eliminate P by using equation (4), we obtain

$$\frac{A^2D_1^2}{\lambda\beta_1^2\left(W_1 + \dfrac{C_1+C_2}{2}\right)} \quad \frac{df_1}{dx} = -\frac{C_1Z_1 + C_2Z_2}{(x+l)} \quad \frac{\left(1 + \dfrac{C_1 + C_2}{2W_1}\right)}{\left(1 + \dfrac{C_1 + C_2}{3W_1}\right)} \quad \ldots (11)$$

Putting the values of Z_1 and Z_2 and making use of relation (9) we get

$$\frac{dx}{df_1} = -M_1 \frac{(x+l)}{1 + \dfrac{C_2}{C_1K}\left\{1 + \theta_2\left(1 - \dfrac{1}{K}\right)\right\} + f_1\left\{\theta_1 + \dfrac{C_2\theta_2}{C_1K^2}\right\}}$$

$$\ldots \qquad \ldots \quad (12)$$

where

$$M_1 = \frac{A^2D_1^2\left(1 + \dfrac{C_1 + C_2}{3W_1}\right)}{\lambda C_1\beta_1^2W_1\left(1 + \dfrac{C_1 + C_2}{2W_1}\right)^2}$$

Integrating equation (12) and applying the initial conditions that $x = 0$ when $f_1 = 1$, we obtain

$$(x+l) = l\left[\frac{1 + \dfrac{C_2}{C_1K}\left\{1+\theta_2\left(1 - \dfrac{1}{K}\right)\right\} + \left\{\theta_1 + \dfrac{C_2\theta_2}{C_1K^2}\right\}}{1 + \dfrac{C_2}{C_1K}\left\{1+\theta_2\left(1 - \dfrac{1}{K}\right)\right\} + f_1\left\{\theta_1 + \dfrac{C_2\theta_2}{C_1K^2}\right\}}\right]^{\dfrac{M_1}{\theta_1 + \dfrac{C_2\,\theta_1}{C_1K_1}}} \quad \ldots \quad (13)$$

Equation (13) relates the travel to the parameter f_1

The pressure P can now be obtained from equation (4) as

$$P = \frac{C_1(1-f_1)(1+\theta_1 f_1) + \frac{C_2}{K}(1-f_1)\left\{1+\theta_2\left(1-\frac{1}{K}\right)+\frac{\theta_2 f_1}{K}\right\}}{A\,(x+l)} \cdot \frac{\lambda\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \qquad \text{.. .. (14)}$$

where $(x + l)$ is given by equation (13)

This completes the determination of V, X and P as functions of parameter f_1. The above equations hold so long as both the charges are burning.

Case (*i*):—Now we consider the case when both the charges burn out simultaneously. In this case all the above equations are true for K = 1. If suffix 'B' denotes the position of "all burnt", then the position at this instant is obtained from (13) by putting $f_1 = 0$ and $K = 1$. Thus

$$(x_B + l) = l\left[\frac{C_1(1+\theta_1)+C_2(1+\theta_2)}{(C_1+C_2)}\right]^{\frac{M_1 C_1}{(C_1\theta_1+C_2\theta_2)}} \qquad \text{.. .. (15)}$$

Also the pressure at "all-burnt" is

$$P_B = \frac{\lambda\,(C_2+C_1)}{A(x_B+l)} \cdot \frac{\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \qquad \text{.. .. (16)}$$

where $(x_B + l)$ is given by (15)

And the velocity at "all -burnt" is

$$V_B = \frac{AD_1}{\beta_1\left(W_1+\frac{C_1+C_2}{2}\right)} \qquad \text{.. .. .. (17)}$$

Case (*ii*):—Now we consider the case when charge C_1 burns out first and C_2 continues to burn. In this case all the equations from (1) to (14) hold good and the values when the first charge C_1 is burnt up are

$$V_b = \frac{AD_1}{\beta_1\left(W_1+\frac{C_1+C_2}{2}\right)} \qquad \text{.. . (18)}$$

$$(x_b + l) = l\left[\frac{1+\frac{C_2}{C_1K}\left\{1+\theta_2\left(1-\frac{1}{K}\right)\right\}+\left\{\theta_1+\frac{C_2\theta_2}{C_2K^2}\right\}}{1+\frac{C_2}{C_1K}\left\{1+\theta_2\left(1-\frac{1}{K}\right)\right\}}\right]^{\frac{M_1}{\theta_1+\frac{C_2\theta_1}{C_1K^2}}} \qquad \text{.. (19)}$$

and $$P_b = \frac{C_1 + \frac{C_2}{K}\left\{1+\theta_2\left(1-\frac{1}{K}\right)\right\}}{A(x_b+l)} \cdot \frac{\lambda\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \quad (20)$$

where $(x_b + l)$ is given by (19).

Now when charge C_1 is burnt out, $Z_1=1$ and so the equations of internal ballistics while C_2 is burning are

$$P(x+l) = \frac{\lambda(C_1+C_2Z_2)}{A} \cdot \frac{\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \quad \ldots (21)$$

$$Z_2 = (1-f_2)(1+\theta_2 f_2) \quad \ldots \quad \ldots (22)$$

$$D_2 \frac{df_2}{dt} = -\beta_2 P \quad \ldots \quad \ldots \quad \ldots (23)$$

$$\left(W_1 + \frac{C_1+C_2}{2}\right)\frac{dv}{dt} = AP \quad \ldots \quad \ldots (24)$$

Eliminating P from (23) and (24) and integrating, we get

$$V = \frac{AD_2(1-f_2)}{\beta_2\left(W_1+\frac{C_1+C_2}{2}\right)} \quad \ldots \quad \ldots (25)$$

Using equation (25) in equation (24) and eliminating P with the help of equation (21), we obtain

$$\frac{A^2D_2^2(1-f_2)}{\lambda\beta_2^2\left(W_1+\frac{C_1+C_2}{2}\right)}\frac{df_2}{dx} = -\frac{(C_1+C_2Z_2)}{(x+l)}\frac{\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)}$$

This can be written as

$$\frac{dx}{x+l} = -M_2 \frac{(1-f_2)df_2}{\left(1+\frac{C_1}{C_2}\right)-\left(1-\theta_2\right)f_2-\theta_2 f_2^2} \quad \ldots (26)$$

where

$$M_2 = \frac{A^2D_2^2\left(1+\frac{C_1+C_2}{3W_1}\right)}{\lambda C_2 W_1 \beta_2^2\left(1+\frac{C_1+C_2}{2W_1}\right)^2}$$

Equation (26) can also be put in the form

$$\frac{dx}{x+l} = -\frac{M_2C_2}{(C_1+C_2)} \frac{(1-f_2)\,df_2}{(1-af_2)(1+bf_2)} \quad .. (27)$$

where

$$a - b = \frac{C_2(1-\theta_2)}{(C_1+C_2)} \qquad a > b.$$

$$a\,b = \frac{C_2\theta_2}{(C_1+C_2)}$$

Therefore $$\frac{dx}{x+l} = -\frac{M_2C_2}{(C_1+C_1)}\left[\frac{(a-1)}{(a+b)}\frac{1}{(1-af_2)} + \frac{1+b}{a+b}\,\frac{1}{1+bf_2}\right]df_2 \quad .. (28)$$

Integrating equation (28), we get

$$\log(x+l) = -\frac{M_2C_2(1-a)}{a(a+b)(C_1+C_2)}\log(1-af_2) - \frac{M_2C_2(1+b)}{b(a+b)(C_1+C_2)}\log(1+bf_2) + \text{const.}$$

The constant of integration is obtained from the conditions that when the first charge is 'all-burnt' i.e. at $f_{2b} = 1-\frac{1}{K}$, $x = x_b$ Thus

$$\log\left(\frac{x+l}{x_b+l}\right) = \frac{M_2C_2(1-a)}{a(a+b)(C_1+C_2)}\log\left\{\frac{1-a\left(1-\frac{1}{K}\right)}{1-af_2}\right\} +$$

$$\frac{M_2C_2(1+b)}{b(a+b)(C_1+C_2)} \times \log\left\{\frac{1+b\left(\frac{1}{1-K}\right)}{1+bf_2}\right\}$$

$$\text{or} \left(\frac{x+l}{x_b+l}\right)$$

$$= \left[\frac{1-a\left(1-\frac{1}{K}\right)}{1-af_2}\right]^{\frac{M_2C_2(1-a)}{a(a+b)(C_1+C_2)}} \left[\frac{1+b\left(1-\frac{1}{K}\right)}{1+bf_2}\right]^{\frac{M_2C_2(1+b)}{b(a+b)(C_1+C_2)}} \quad .. (29)$$

This relates the shot travel to the parameter f_2.

Also the pressure P while the charge C_2 is burning is given by

$$P = \frac{\lambda\left[(C_1+C_2)(1-f_2)(1+\theta_2 f_2)\right]}{A(x+l)} \frac{\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \quad .. (30)$$

where $(x + l)$ is given by (29)

Now the shot-travel, pressure and velocity at "all-burnt" are given by

$$\left(\frac{x_B + l}{x_b + l}\right) = \left[1 - a\left(1 - \frac{1}{K}\right)\right]^{\frac{M_2 C_2 (1-a)}{a(a+b)(C_1+C_2)}} \times$$

$$\left[1 + b\left(1 - \frac{1}{K}\right)\right]^{\frac{M_2 C_2 (1+b)}{b(a+b)(C_1+C_2)}} \quad \ldots \quad \ldots (31)$$

$$P_B = \frac{\lambda (C_1 + C_2)}{A (x_B + l)} \frac{\left(1 + \frac{C_1+C_2}{2W_1}\right)}{\left(1 + \frac{C_1+C_2}{3W_1}\right)} \quad \ldots \quad \ldots (32)$$

where $(x_B + l)$ is given by (31)

and

$$V_B = \frac{A D_2}{\beta_2 \left(W_1 + \frac{C_1+C_2}{2}\right)} \quad \ldots \quad \ldots (33)$$

Maximum Pressure

Quantities here will be denoted by suffix 'm'. The maximum pressure may occur when

(*a*) both the charges are burning.

(*b*) charge C_1 is burnt out and charge C_2 is burning

and (*c*) at the position of "all-burnt".

Case(*a*):—When both the charges are burning, pressure P is given by

$$P = \frac{\left[C_1(1-f_1)(1+\theta_1 f_1) + \frac{C_2}{K}(1-f_1)\left\{1+\theta_2\left(1-\frac{1}{K}\right) + \frac{\theta_2}{K}f_1\right\}\right]\lambda}{A(x+l)} \frac{\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \quad \ldots \quad \ldots (34)$$

For pressure to be maximum dP=O. Therefore differentiating (34) and simplifying, we have

$$f_{1m} = 1 - \frac{(1+\theta_1) + \frac{C_2}{C_1 K}(1+\theta_2)}{2\theta_1 + \frac{2\theta_2}{K^2}\frac{C_2}{C_1} + M_1} \quad \ldots \quad \ldots (35)$$

Hence

$$P_m = \frac{C_1(1-f_{1m})(1+\theta_1 f_{1m}) + \frac{C_2}{K}(1-f_{1m})\left\{1+\theta_2\left(1-\frac{1}{K}\right)+\frac{\theta_2}{K}f_{1m}\right\}}{A(x_m+l)} \times \frac{\lambda\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \quad \ldots \quad \ldots (36)$$

where

$$(x_m+l)=l\left[\frac{1+\frac{C_2}{C_1K}\left\{1+\theta_2\left(1-\frac{1}{K}\right)\right\}+\left\{\theta_1+\frac{C_2\theta_2}{C_1K^2}\right\}}{1+\frac{C_2}{C_1K}\left\{1+\theta_2\left(1-\frac{1}{K}\right)\right\}+f_{1m}\left\{\theta_1+\frac{C_2\theta_2}{C_1K^2}\right\}}\right]^{\frac{M_1C_1}{C_1\theta_1+\frac{C_2\theta_2}{K^2}}} \quad \ldots \quad \ldots (37)$$

The above is true if

$$f_{1m} \geqslant 0$$

$$i.e. \quad 1 \geqslant \frac{\left(1+\theta_1\right)+\frac{C_2}{C_1K}\left(1+\theta_2\right)}{2\theta_1+\frac{2\theta_2}{K^2}\frac{C_2}{C_1}+M_1} \quad \ldots \quad \ldots (38)$$

Case (*b*):—When charge C_1 is burnt out and charge C_2 is burning, pressure is given by

$$P=\frac{C_1+C_2(1-f_2)(1+\theta_2 f_2)}{A(x+l)} \; \frac{\lambda\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \quad \ldots (39)$$

For pressure to be maximum d P=O. Therefore differentiating (39) and simplifying, we have

$$f_{2m}=1-\frac{1+\theta_2}{M_2+2\theta_2} \quad \ldots \quad \ldots (40)$$

Hence

$$P_m=\frac{C_1+C_2(1-f_{2m})(1+\theta_2 f_{2m})}{A(x_m+l)} \; \frac{\lambda\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \quad \ldots (41)$$

where

$$(x_m + l) = (x_b + l)\left[\frac{1-a\left(1-\frac{1}{K}\right)}{1-a\,f_{2m}}\right]^{\frac{M_2 C_2 (1-a)}{a(a+b)(C_1+C_2)}} \times$$

$$\left[\frac{1+b\left(1-\frac{1}{K}\right)}{1+b\,f_{2m}}\right]^{\frac{M_2 C_2 (1+b)}{b(a+b)(C_1+C_2)}} \quad \cdot\cdot \quad \cdot\cdot (42)$$

The above is true only if

$$1-\frac{1}{K} > f_{2m} > 0$$

$$i.\,e. \quad 1-\frac{1}{K} > 1-\frac{1+\theta_2}{M_2+2\theta_2} > 0$$

$$i.\,e. \quad \frac{1}{K} < \frac{1+\theta_2}{M_2+2\theta_2} < 1 \quad \cdot\cdot \quad \cdot\cdot (43)$$

Case (e):—Since at "all-burnt" $Z_1 = Z_2 = 1$, we have

$$P_m = \left(\frac{\lambda\,(C_1+C_2)}{A\,(x_B+l)}\right)\frac{\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \quad \cdot\cdot \quad \cdot\cdot (44)$$

where

$$(x_B + l) = l\left[\frac{C_1\,(1+\theta_1)+C_2\,(1+\theta_2)}{(C_1+C_2)}\right]^{\frac{M_1 C_1}{C_1\theta_1+C_2\theta_2}} \quad \cdot\cdot (45)$$

if both the charges burn out simultaneously, and

$$(x_B + l) = (x_b + l)\left[1-a\left(1-\frac{1}{K}\right)\right]^{\frac{M_2 C_2 (1-a)}{a(a+b)(C_1+C_2)}} \times$$

$$\left[1+b\left(1-\frac{1}{K}\right)\right]^{\frac{M_2 C_2 (1+b)}{b(a+b)(C_1+C_2)}} \quad \cdot\cdot \quad \cdot\cdot (46)$$

if they burn out at different times.

Muzzle Velocity

When both the charges have been burnt out, the equation of state and the dynamical equation of the motion of the shot are

$$P(x + l) = \frac{\lambda (C_1+C_2)}{A} \frac{\left(1 + \frac{C_1+C_2}{2W_1}\right)}{\left(1 + \frac{C_1+C_2}{3W_1}\right)} \quad .. \quad .. (47)$$

$$\left(W_1 + \frac{C_1+C_2}{2}\right)\frac{dv}{dt} = AP \quad .. \quad .. (48)$$

Eliminating P from (47) and (48), we get

$$V\frac{dv}{dx} = \frac{\lambda (C_1+C_2)}{(x+l)} \frac{1}{\left(W_1+\frac{C_1+C_2}{3}\right)} \quad .. \quad .. (49)$$

Integrating and applying the initial conditions that when $x=x_B$, $V=V_B$, we have

$$V^2 = V_B^2 + \frac{2\lambda(C_1+C_2)}{\left(W_1+\frac{C_1+C_2}{3}\right)} \log\left(\frac{x+l}{x_B+l}\right) \quad .. (50)$$

which gives the velocity at any $x > x_B$. Hence the muzzle velocity is given by

$$V_3^2 = V_B^2 + \frac{2\lambda(C_1+C_2)}{\left(W_1+\frac{C_1+C_2}{3}\right)} \log\left(\frac{x_3+l}{x_B+l}\right) \quad .. (51)$$

where x_3 is the length of the bore of the gun.

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