ON THE JET FORMATION BY EXPLOSIVES WITH LINED HEMISPHERICAL CAVITIES

By Sampooran Singh, Defence Science Laboratory, Ministry of Defence, New Delhi.

The X-ray flash photography[1] of explosives with lined hemispherical cavities indicated that the liner progressively turns 'inside' 'out' during collapse and squirts out a jet along the axis. Kolsky[2] also studied the deformation of a hemispherical liner by interposing a water layer between the explosive and the metal and suggested that a rather different mechanism is operative here as compared to a conical liner. The present is an attempt to explain these observations by extending the conventional hydrodynamic theory of jet formation by lined conical cavities.[3,4].

Let us consider a hemispherical liner to be divided into small elements and each element subtends an angle α* with the axis of propagation of detonation. The angle varies from 90° to 0° as a plane detonation wave sweeps from apex to base respectively. As the pressure of detonation is very high compared to strength of metals, so each element could be treated as a perfect fluid. The individual zonal elements are considered independent of one another and all the relations needed are obtained by applying the laws of conservation of mass, momentum, energy and Bernoulli's theorem to individual zonal elements.

Shreffler and Deal[5] have shown that the velocity of a metal plate of thickness 0·06 in. to 0·1 in. driven by high explosives is approximately one half of the velocity of detonation. The usual thickness of liners met in shaped charges of 2 in. to 3 in. calibre are about 0·06 in. to 0·1 in. and it may be assumed that the collapse velocity of an element of liner, when $\alpha=90°$, is about one half the detonation velocity. The experiments on 'inverted' 80° conical liners indicate that the element near the apex of liner collapses with a velocity of 3100 m./sec. Eichelberger and Pugh[7] have shown that, in case of 44° steel conical liner, the element near the apex of liner collapse with a velocity of 2850 m./sec. Walsh, Shreffler and Willig[8], in their wedge shape collision experiments, showed that the plate (mild steel) velocity at 30° collision is 2520 m./sec. The above experiments lead us to assume an approximate expression connecting velocity of collapse V_o and α as follows

$$V_0=\tfrac{1}{4}U_d\,(1+\sin\alpha) \qquad \ldots \qquad \ldots \qquad \ldots \qquad (1)$$

where U_d is the velocity of detonation of the explosive. The U_d is constant as the detonation wave travels from the apex of the liner to the thin belt of explosive around the base of the liner. The V_0 decreases continuously from apex to base, but each V_0 is assumed to be independent of the velocities of the neighbouring elements. The masses of the ring-shaped elements of the hemispherical liner increases from apex to base and near the base the belt of explosive surrounding the liner is much thinner; and these result in further decrease of collapse velocities† from apex to base but, as a first approximation that is neglected in this paper.

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* The angle between the detonation wave front and normal to the small element is also α

† This is analogous to the collapse velocity gradient in case of conical liner, where the collapse velocities for an element near the apex and an element near the base of a conical liner are 2,850 m./sec. and 1,500 m./sec. respectively.

Consider a system of co-ordinates moving with constant velocity U_d sec α (a small element is considered where α is constant): in this system detonation wave is stationary but the given zonal element would enter the detonation wave front with a velocity U_d sec α Since the pressures resulting from the detonation wave are everywhere perpendicular to the motion of the element, it results in only changing the direction, and not the magnitude of the element velocity. The arguments given in the earlier paper[4] then are valid and it can be shown that

$$V_o = 2U_d \sec \alpha \sin \delta \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

where δ is the angle between the direction of V_o and the perpendicular to the original liner element. V_o remains constant until the axis is reached. Combining Eqs (1) and (2), we have

$$\delta = \sin^{-1} \left(\tfrac{1}{8} \cos \alpha + \tfrac{1}{16} \sin 2\alpha\right) \quad \ldots \quad \ldots \quad (3)$$

The position of P in the parent liner (Fig. 1) is fixed by a length M measured from the apex along the axis to the plane of the zonal element P. After collapsing the element P reaches the axis at Q and the position of Q is fixed by a length N measured from the apex along the axis. Let T represent the time ($T = M/U_d$) the detonation wave takes in travelling from the apex to the liner element P, and (t—T) the time element P takes to travel to Q. The distance travelled by P in the direction PQ at any time $\bar{t}$ is ($\bar{t}$—T) V_o. The distances travelled by different elements of the liner can be calculated at any time $\bar{t}$ and the shape of liner can be graphically traced.

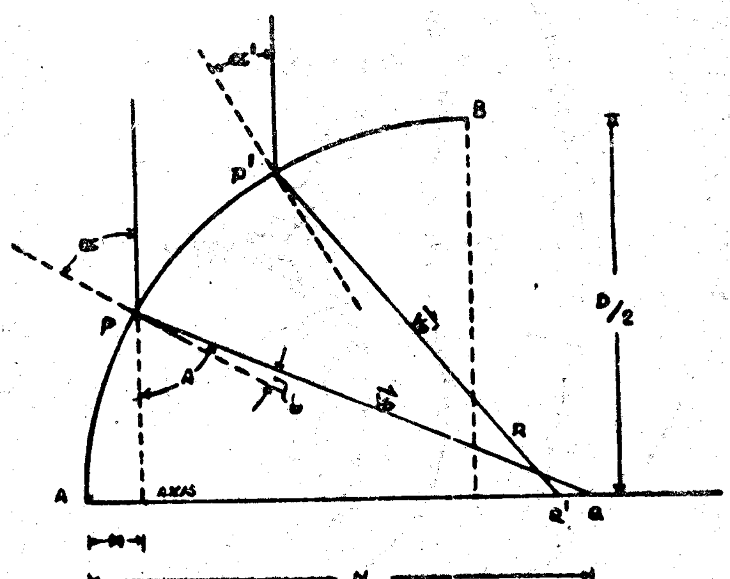

Fig. 1. APB is the upper half and AQ the axis of the original hemispherical liner. When a detonation wave sweeps from apex to base along the liner, the elements P and P′ move towards the axis with velocities V_b and V_o' and reach the axis at Q and Q′ respectively.

Fig. 2 shows the stages of hemispherical liner collapse (taking U_d =7,510 m./sec.) at different intervals of time. The shape E shows that the top (apex of the hemisphere is forced forwards until in F the liner is turned completely 'inside' out into the form of a cone with a rounded apex. In shape G, the liner can be seen to have folded up and the jet has started travelling to the right. The apex of the liner forms the head of the jet, while the sides fold back and follow behind. The predicted shapes shown in Fig. 2 are very similar to that observed by Kolsky and X-ray flash photography.

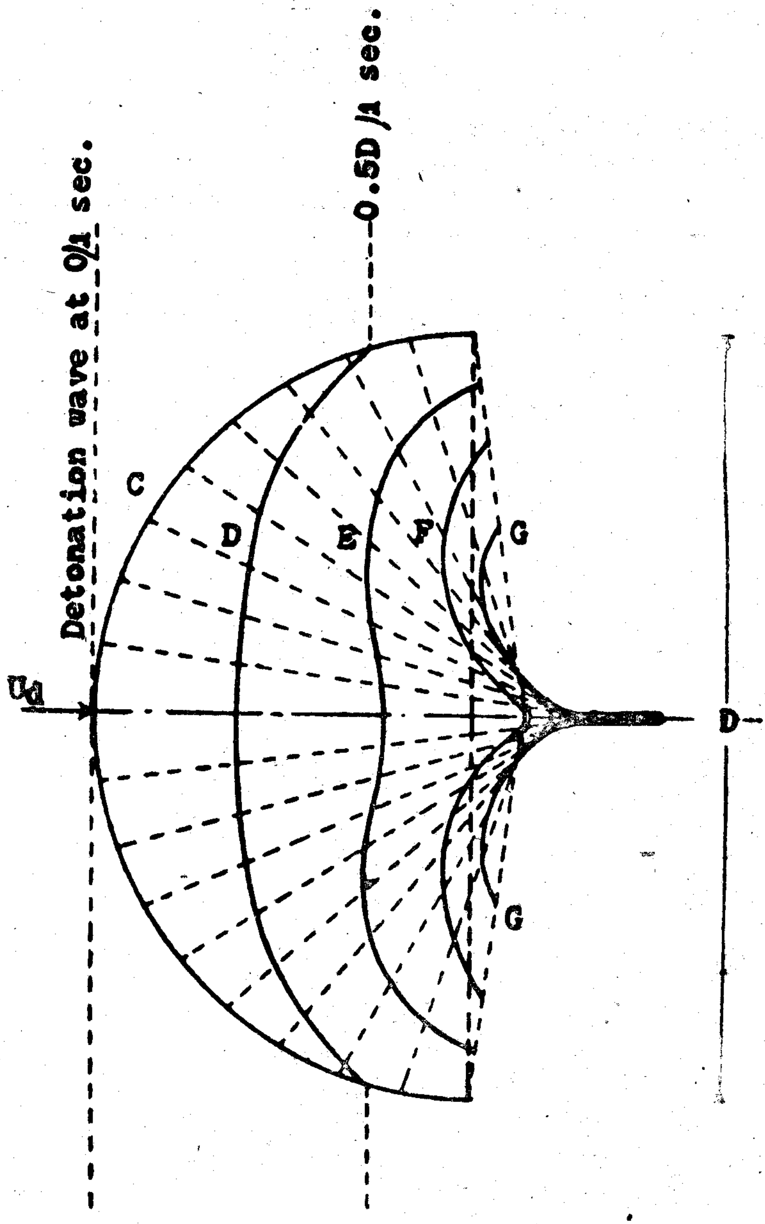

Fig. 2 Stages of hemispherical liner collapse at different intervals of time. The liner is divided into 18 equal segments, position of each element, is calculated at different intervals of time and the shapes of liner are graphically traced. The shapes C, D, E, F and G are at 0μ sec., 0·5 Dμ sec., 1·0 Dμ sec., 1·5 Dμ sec. and 2·0 Dμ sec. respectively.

When an element of mass dm reaches the axis, it splits up into two parts; one part of mass dm_j goes into the jet and the other part of mass dm_s goes into the slug, which proceed along the axis at the constant velocities V_j and V_o respectively. By arguments given in the earlier paper[4], the expressions for V_j and dm_s can be derived and are as follows:

$$V_j = V_o \operatorname{cosec} \frac{\beta}{2} \cos\left(\alpha+\delta-\frac{\beta}{2}\right) \quad \ldots \quad (4)$$

$$dm_s = dm \cos^2 \frac{\beta}{2} \quad \ldots \quad (5)$$

where β is the angle which the collapsing liner makes with the axis. The angle β depends upon the shape of the collapsing liner in the immediate nieghbourhood where it cuts the axis of the liner. In Fig. 1 at the instant t, P reaches the axis at Q, and P′ travelling in the direction P′Q′reaches at R. Let the cylindrical coordinates of R be (r, z) and the coordinates of the original position of P′ be $\left(\frac{D}{2}\cos\alpha, M.\right)$

Then

$$r = \frac{D}{2}\cos\alpha - V_o(t-T)\cos A \quad \ldots \quad (6)$$

$$z = \frac{D}{2}(1-\sin\alpha) + V_o(t-T)\sin A \quad \ldots \quad (7)$$

where $A=\alpha+\delta$. The slope of the contour of the collapsing element of the liner at any given time t can be obtained by differentiating r with respect to z $(\partial\gamma/\partial Z = (\partial\gamma/\partial\alpha)(\partial\alpha/\partial Z)$ while t is kept constant, and $(\partial\gamma/\partial Z)_{\gamma=0} = \tan\beta$. The time when a given element reaches the axis can be obtained by substituting $r=0$ in Eq (6), which gives

$$t - T = D\cos\alpha / (2V_o \cos A) \quad \ldots \quad (8)$$

β is given by the expression

$$\tan\beta = \frac{n\alpha + V_o \cos A/U_d + V'_o/V_o - A' \tan A}{1 - V_o \sin A/U_d - V'_o \tan A/V_o - A')} \quad \ldots \quad (9)$$

$$\text{where } V'_o = \frac{\partial V_o}{\partial\alpha}, \quad \frac{\partial T}{\partial a} = -\frac{D\cos\alpha}{2U_d} \text{ and } A' = \frac{\partial A}{} = 1 + \frac{\partial\delta}{\partial\alpha}$$

$$= 1 + \frac{V'_o \cos\alpha - V_o \sin\alpha}{2U_d \cos\delta}$$

To obtain the value of β for any element of the line, the values of α, A, V_o and U_d are substituted in Eq (9). The calculated β decreases continuously from apex to base of the liner, and for an element at the apex and at the base of the liner are 170° and 120° respectively. It was simpler, however, to substitute values of α, A, V_0 and $(t-T)$ into the parametric Eqs.(6) and (7) from which Eq. (9) was derived and to plot the resulting values of z and r for various instants of time. By drawing separate curves through the points determined for each instant of time, a series of contours of the collapsing the liner were obtained. The angle between a contour and the axis, where the contour intersects the axis, was the value of β for the particular element that intersects the axis.

The distribution of mass dm into dm_j and dm_s is determined by β only. The major part of the metal of the element goes into the jet and so the whole jet would appear to be wider as compared to fine (narrower) jet from a conical liner. The whole hemispherical liner is progressively used up in the formation of jet and there is no massive slug. The absence of massive slug was confirmed by us by firing the liners in a tank of water.

In Fig. 1, $N' < N$ and the slug element S_p from element P is ahead of the jet element J_p from element P′. Thus the slug from any element being ahead of the jet from a subsequent element, gains extra momentum at the expense of jet due to the subsequent element. The whole jet would present a 'non-uniform' structure, *i.e.*, the jet-elements and slug-elements would appear alternately.

Walsh, Shreffler and Willig[8] have shown that when the angle between a sedge is below a certain critical angle, the formation of jet does not take place. The 25° iron collision and 30° iron collision were found to be jetless and jet-forming respectively. This indicates that elements were near the base of a hemispherical liner (where α is less than 14°) do not contribute to jet formation.

The calculated velocities of the head and tail of the jet are of the order of 4×10^3m./sec. and 2×10^3m/sec. respectively. This gradient in velocity causes it to lengthen as it travels.

There are five variables V_0, δ, V_j , dm_s/dm and β. The absence of slug rules out experimental determination of β At this writing only V_j can be determined accurately and as such it is not possible to advance experimental evidence* to check the theory.

Thus the successive stages of collapse of hemispherical liners observed by Kolsky by X-ray flash photography, and absence of massive slug are explained by a simple extension of the hydrodynamic theory of jet formation by conical liners. The jets from hemispherical liners are wider as compared to fine (narrower) jets from conical liners.

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* Eichelberger and Pugh presented experimental verification of the theory of jet formation by conical liners by determining V_j and dm_s/dm for each element of the liner.

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