MATCHING THEORY IN PSYCHOLOGICAL TEST

By P. V. Krishna Iyer and M. N. Bhattacharyya.

Introduction

Simple matching is said to occur when elements of the same kind occur in the same order in two sets, each of n elements of k types, arranged at random, with fixed or varying probabilities. When two sets of n cards containing n_{11}, n_{12}; n_{21}, n_{22}; n_{31}, n_{32}...............n_{k_1} n_{k_2} cards of black, white, red etc. colours are arranged at random in two rows and if we compare the cards of the two rows in order, we may find a number of instances where cards of the same kind occur together in the same order in both the rows. Such occurrences are usually termed 'matchings'.

Iyer in a recent paper considered the probability distribution of (*i*) the number of matched pairs of a given colour and (*ii*) the total number of matched pairs for all kinds of colours, when two sets of cards, D_1 and D_2, each consisting of n cards of k colours, are arranged at random in a sequence. He considered these distributions both for the infinite and finite sampling. In infinite sampling the probabilities for black, white, red etc. cards in the two sets, namely, p_{11}, p_{21}, p_{31}.............p_{k1} and p_{12}, p_{22}, p_{32},..........p_{k2} remain the same for all draws whereas for finite sampling these probabilities vary from draw to draw. In this case we are concerned with the distributions arising from n_{11}, n_{21}, n_{31}.....n_{k_1} and n_{12}, n_{22}, n_{32}.....n_{k_2} cards.

Matchings may be considered both in the qualitative and quantitative sense. In the quantitative sense the cards of the two sets may be assigned the scores θ_1, θ_2, θ_3..................θ_k for the different colours. Then the probabilities for θ_1, θ_2..........θ_k for the two sets are p_{11}, p_{21}, p_{31},....p_{k1} and p_{12}, p_{22}, p_{32}........p_{k2} respectively in the case of infinite sampling. We can consider the distribution of $\Sigma(x_r - y_r)$, $\Sigma|x_r - y_r|$ and $\Sigma(x - y_r)^2$, where x_r and y are the scores of the rth cards in the two sets. The first two moments of the above statistics in three different cases, namely, when both sets refer to infinite sampling, first set finite and the second set infinite, and both sets finite sampling, are given below.

Statistic—$\Sigma\,|x_r - y_r|$

1st set infinite, 2nd set ,,

$$\mu'_1 = n\sum_{r,s=1}^{k} |\theta_r - \theta_s| p_{r1}\, p_{s2} \quad \text{.........................(1·1)}$$

$$\mu_2 = n\left[\Sigma(\theta_r - \theta_s)^2 p_{r_1} p_{s_2} - \left\{ \Sigma|\theta_r - \theta_s| p_{r_1} p_{s_2} \right\}^2\right] \quad \text{........(1·2)}$$

1st set finite
2nd set infinite

$$\mu'_1=\Sigma|\theta_r-\theta_s|n_{r_1}p_{s_2} \quad \ldots\ldots(2\cdot1)$$

$$\mu_2=[\Sigma(\theta_r-\theta_s)^2 n_{r_1}p_{s_2}(1-p_{s_2}) -2\Sigma|\theta_r-\theta_s||\theta_r-\theta_t|n_{r_1}p_{s_2}p_{t_2}] \quad \ldots\ldots(2\cdot2)$$

1st set finite
2nd set finite

$$\mu'_1=\frac{1}{n}\left[\Sigma\,|\theta_r-\theta_s|n_{r_1}n_{s_2}\right] \quad \ldots\ldots(3\cdot1)$$

$$\mu_2=\Sigma\frac{n_{r_1}n_{s_2}}{n}\left[\frac{(n_{r_1}-1)(n_{s_2}-1)}{n-1}-\frac{n_{r_1}n_{s_2}}{n}+1\right](\theta_r-\theta_s)^2$$

$$+2\Sigma\frac{n_{r_1}n_{s_2}n_{t_2}}{n}\left[\frac{n_{r_1}-1}{n-1}-\frac{n_{r_1}}{n}\right]|\theta_r-\theta_s||\theta_r-\theta_t|$$

$$+2\Sigma\frac{n_{r_1}n_{t_1}n_{s_2}}{n}\left[\frac{n_{s_2}-1}{n-1}-\frac{n_{s_2}}{n}\right]|\theta_r-\theta_s||\theta_t-\theta_s|$$

$$+2\Sigma\frac{n_{r_1}n_{s_2}n_{t_1}n_{u_2}}{n}\left[\frac{1}{n-1}-\frac{1}{n}\right]|\theta_r-\theta_s||\theta_t-\theta_u| \quad \ldots\ldots(3\cdot2)$$

For $\Sigma(x_r-y_r)$,

$\mu_1' = 0$ in all cases.

μ_2=(1·2) or (2·2) or (3·2) with the moduli replaced by the algebraic values.

For the distribution of $\Sigma(x_r-y_r)^2$, μ_1 and μ_2 are given by the same expressions as for $\Sigma|x_r-y_r|$ with the change that $|\theta_r-\theta_s|$ and $(\theta_r-\theta_s)^2$ are replaced by $(\theta_r-\theta_s)^2$ and $(\theta_r-\theta_s)^4$ respectively.

It has been shown that the distributions of $\Sigma(x_r-y_r)$, $\Sigma|x_r-y_r|$ and $\Sigma(x_r-y_r)^2$ tend to the normal form fo: moderately large values of n. Therefore any of these statistics can be used as a test of deviation from randomness between two sequences. However, $\Sigma(x_r-y_r)$ is not so powerful as $\Sigma|x_r-y_r|$ or $\Sigma(x_r-y_r)^2$ (*vide* reference quoted). It may be further seen from Table I that the expected values and variances for different situations for cards of 4, 5 and 6 sets are the same or practically the same and hence tests of randomness between two sequences can be done by taking any of the three situations mentioned above.

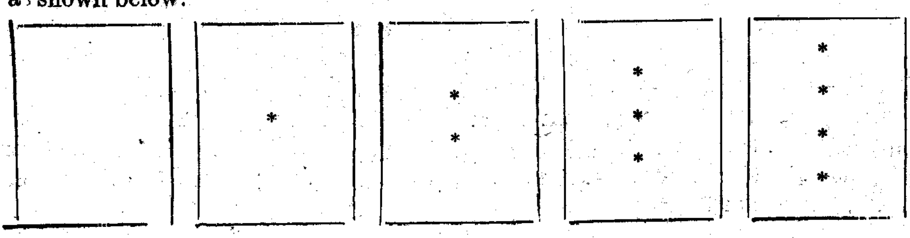
TABLE I

Mean and variance for finite and infinite sampling

Type of sampling	No. of groups	No. of cards in a group	$\Sigma(x_r - y_r)$		$\Sigma\lvert x_r - y_r\rvert$		Coef. of vari.%	$\Sigma(x_r - y_r)^2$		Coef. of vari. %
			μ_1'	μ_2	μ_1'	μ_2		μ_1'	μ_2	
Both sets infinite ..	4	4	0	40	20	15	19·37	40	132	28·72
	5	5	0	100	40	36	15·00	100	540	23·24
	6	6	0	210	70	$73\frac{8}{9}$	12·28	210	1,673	19·48
1st set finite 2nd set infinite ..	4	4	0	20	20	14	18·71	40	116	26·93
	5	5	0	50	40	33·2	14·41	100	470	21·68
	6	6	0	105	70	$67\frac{2}{3}$	11·75	210	1,449	18·13
1st set finite 2nd set finite ..	4	4	0	0	20	$13\frac{13}{15}$	18·62	40	$106\frac{2}{3}$	25·82
	5	5	0	0	40	$31\frac{2}{3}$	14·07	100	$416\frac{2}{3}$	20·41
	6	6	0	0	70	63·2	11·36	210	1,260	16·90

Applications

It can be suggested that the above distributions may be useful in psychological test for testing the extra sensory power, if any, of a subject for making correct prediction. For this purpose a set of twenty five cards of size $3\frac{1}{2}'' \times 2\frac{1}{2}''$ was made. One side of the cards was completely dark coloured without any decoration so as to avoid any psychological disturbance in the subject, while the other side was white containing one, two, three, four black dots or blank as shown below.



There were five blank cards, five with one, five with two, five with three and five with four black dots. Thus for the present set of cards we see that

$$\theta_1=0, \theta_2=1, \theta_3=2, \theta_4=3, \theta_5=4 \text{ and } n_{11}=n_{21}=n_{31}=n_{41}=n_{51}=5.$$

A subject was tested by two methods. In method *A* he was told about the content of the pack of cards and was asked to guess the number of dots one by one for a thoroughly shuffled pack of cards. The actual and the predicted number of dots for each card were recorded. In method B each card, after prediction, was shown to the subject and he was asked to make the next prediction. The actual and predicted number of dots for each card were recorded as in method *A*. Data were collected for eleven scientific workers of the laboratory, the age of the subjects varying between 22 and 30 years, repeating the experiment thrice for each subject on different days. A typical set of data is shown below.

Method A		Method B	
Actual	Predicted	Actual	Predicted
1	3	1	3
4	2	1	4
0	0	1	0
2	3	0	1
1	3	3	3
1	4	4	2
0	3	3	0
3	3	4	4
3	0	4	2
2	4	2	0
1	3	2	1
0	2	4	2
0	3	2	3
0	3	3	1
4	2	0	3
2	3	1	2
2	3	0	2
4	4	0	4
2	3	2	0
4	3	1	1
1	3	2	3
3	3	0	2
3	3	3	1
3	3	3	0
4	3	4	1

The hypothesis behind the test is that if the subject is devoid of any extra sensory powers he will be predicting at random and hence his scores in both the methods are expected to be the same. In the light of this hypothesis the collected data have been analysed and the analyses of variance are given in the following tables.

TABLE II

Analysis of variance based on $\Sigma(x_r - y_r)$

Variation due to	D.F.	Sum of squares	Mean Sum of squares	F—Test
Subjects	10	1,014·33	101·43	2·56*
Methods A & B	1	19·64	19·64	..
Interaction Subjects X methods ..	10	637·36	63·74	1·61
Error	44	1,739·34	39·53	..
Total ..	65	3,410·67	..	..

TABLE III

Analysis of variance based on $\Sigma|x_r - y_r|$

Variation due to	D.F.	Sum of squares	Mean sum of squares	F—test
Subjects	10	205·36	20·54	Not significant.
Methods A & B	1	19·63	19·63	
Interaction Subject X methods ..	10	289·37	28·94	
Error	44	1,382·00	31·41	
Total ..	65	1,896·36		

TABLE IV

Analysis of variance based on $\Sigma(x_r - y_r)^2$

Variation due to	D.F.	Sum of squares	Mean sum of sqaures	F—test
Subjects	10	3,586	358·60	Not significant.
Methods A & B	1	332	332·00	
Interaction Subject X methods ..	10	4,543	454·30	..
Error	44	19,078	433·59	..
Total ..	65	27,539	..	..

The above analyses of variance tables reveal no significant difference either between the subjects or between the two methods A & B of testing.

The data have been further analysed following an entirely different procedure, with a view to coming to a sound conclusion. We assume that the subject has no extra sensory powers. So in both the cases *i.e.*, when the preceding card is shown to him or not the subject will be predicting at random. The randomness of the two sequences can be tested either considering both of them referring to infinite-infinite or finite-infinite, or finite-finite. Because of the equality of means and variances the test is the same in three situations.

On the assumption of randomness

$$\left[\frac{\left\{\Sigma(x_r - y_r) - E\Sigma(x_r - y_r)\right\}^2}{v[\Sigma(x_r - y_r)]}\right]$$

follows χ^2 with 1 d. f. This is true for two other statistics, namely, $\Sigma|x_r - y_r|$ and $\Sigma(x_r - y_r)^2$.

The value of χ^2 has been calculated for each score of the subjects and for all the three statistics both for methods A & B by treating the sequences as finite-infinite.

TABLE IV

χ^2—tests

Subject	occasions	The calculated value of χ^2					
		$\Sigma(x_r - y_r)$		$\Sigma\|x_r - y_r\|$		$\Sigma(x_r - y_r)^2$	
		A	B	A	B	A	B
1	(I)	2·42	·08	·27	0	·05	·03
	(II)	·18	0	·27	·48	·17	1·44
	(III)	·98	0·2	·03	·03	·05	·61
2	(I)	·98	·18	1·48	·03	1·33	·17
	(II)	3·38	0	·27	·12	·26	·21
	(III)	·18	·72	·27	·48	1·33	·54
3	(I)	5·12*	0	1·08	·12	1·23	·31
	(II)	7·22**	·50	·27	·75	·61	·10
	(III)	·08	·72	·12	0	0	0·3

TABLE IV—*contd.*

χ^2 —*tests*—contd.

Sub-ject	occasions	The calculated value of χ^2					
		$\Sigma(x_r - y_r)$		$\Sigma\lvert x_r - y_r \rvert$		$\Sigma(x_r - y_r)^2$	
		A	B	A	B	A	B
4	(I)	·08	·02	7·71**	2·44	6·20*	1·33
	(II)	·32	·08	1·93	1·93	1·67	·99
	(III)	·02	·72	·03	0	·05	·01
5	(I)	·08	0	3·01	4·34*	3·07	3·07
	(II)	·08	7·22**	·48	·75	·54	1·13
	(III)	2·88	0	0	·12	·69	·42
6	(I)	·98	·08	·27	·12	·36	·01
	(II)	·32	·72	1·08	·48	·99	·13
	(III)	1·28	0	3·01	·12	1·44	·13
7	(I)	·32	·08	0	1·08	·31	·31
	(II)	·08	4·50*	1·93	·27	·99	·17
	(III)	1·28	0	·48	0	6·67**	·03
8	(I)	1·28	·02	·12	1·48	·69	2·61
	(II)	5·78*	0	·75	·12	·48	·08
	(III)	·32	0	·12	·48	·31	·21
9	(I)	·32	·08	1·93	0	1·67	·03
	(II)	·98	·32	·75	0	1·13	0
	(III)	·02	·02	2·44	·27	1·79	·05
10	(I)	·02	0	3·64	1·08	2·04	1·44
	(II)	·32	·18	·48	·03	·69	·26
	(III)	·08	·18	·48	·27	·13	·17
11	(I)	·98	·08	2·44	·12	2·94	·03
	(II)	10·58**	·02	3·64	·27	3·93*	1·33
	(III)	2·88	0	·12	1·93	·03	1·44
	Total	51·82	16·54	40·90	19·71	42·94	18·82

The values of χ^2 do not on the whole show any significant departure from randomness. Thus our experiment shows that the subjects have not shown any extra sensory powers in guessing the cards. They were predicting the cards at random.

These distributions can obviously be used for testing the extra sensory powers of an individual. The use of modified form of these distributions for testing the memory power of an individual is under investigation and it has been found that a fairly satisfactory test bringing the principle of matching can be evolved. This will be dealt with in another paper to be published shortly.

Summary

Matching theory was utilised for testing the extra sensory powers of subjects. The subjects were asked to guess one by one a pack of twenty five cards, consisting of five blank cards, five with one, five with two, five with three and five with four black dots. Two methods of testing, A & B were followed. In method B the preceding card was shown to the subject before making the next prediction while in method A no such help was given. The two sequences, namely, the actual and predicted number of dots were compared for randomness with the help of the statistics $\Sigma(x-y_r)$, $\Sigma|x_r-y_r|$ and $\Sigma(x_r-y_r)^2$, where x_r and y_r represent the rth score of the actual and predicted sequences respectively. The tests revealed that the subjects, for which the data were collected, have no extra sensory powers. They were guessing at random both in method A & B.

Our thanks are due to Dr. Kothari, Scientific Adviser to the Ministry of Defence, for suggesting to take up the investigation and Dr. Sohan Lal for his valuable advice regarding the procedure for this investigation.

Reference

Krishna Iyer, P.V.—"Some distributions arising in matching problems". *J. Ind.* soc. Agri. Stat., 1954, 6, *5*.