STRIDE LENGTH AND FREQUENCY OF STEPPING IN NORMAL WALKING

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Introduction

It is a matter of common experience that both frequency of stepping and stride length increase with increasing speed of walking. It is also a common observation that taller men walk with longer strides for the same speed of walking[1]. No attempt seems to have been made so far to connect these variables in a quantitative way. The present work was undertaken to arrive at a quantitative relation between speed of walking, stride length and frequency of stepping for normal individuals.

Theoretical

Speed of walking V, being the product of frequency of stepping n, and stride length S, an increment in the speed is caused by increasing both the frequency of stepping and the stride length. This may be expressed in mathematical symbols as,

$$V = Sn \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (i)$$

$$\text{and } \delta V = S\,\delta n + n\delta S \quad \ldots \quad \ldots \quad \ldots \quad (ii)$$

For a normal unfatigued man, his walking may be assumed to be unbiased with respect to the two factors S and n at ordinary speeds. That is to ay, he increases his speed of walking by increasing S and n in the same ratio. In other words, an increase of δV in speed is caused by equal fractional increments in S and n, so that

$$\frac{\delta S}{S} = \frac{\delta n}{n} \quad \ldots \quad \ldots \quad (iii)$$

Integrating the above we get,

$$\frac{S}{n} = a \quad \ldots \quad \ldots \quad \ldots \quad (iv)$$

where 'a' is a constant likely to depend on the stature of the individuals.

Combining (*i*) and (*iv*) we have,

$$S = \sqrt{aV} \quad \ldots \quad \ldots \quad \ldots \quad (v)$$

and

$$n = \sqrt{V/a} \quad \ldots \quad \ldots \quad \ldots \quad (vi)$$

so that both stride length and frequency of stepping should vary directly as the square root of speed of walking.

At very low and high speeds, walking ceases to be normal, becau e the frequency of stepping must be limited between a maximum and a minimum, and the frequency should, therefore change more and more slowly with changing velocity as the limiting values of the former are approached. Thus, near the extremes, the stride length and not the frequency becomes the predominants factor controlling speed. In other words beyond a certain range of speeds which may be termed as the 'normal range', relation (*iii*) will not be obeyed. As such, it is expected that observed values at such extremes should deviate from the general trend indicated in (*v*) and (*vi*).

It may be noted that the effect of height has been ignored in deducing the above relations and can be allowed for on the basis of experimental observations.

Experimental Results and Discussions

Stride and Frequency of Stepping in Walking—Two normal subjects were studied over a very wide range of speeds. They were asked to walk a measured distance. The total time taken was noted correct to 0·2 second, and the total number of steps were counted to the nearest integer. The stride length S, and frequency of stepping per minute N, were calculated from these values. The results are given in Table I.

TABLE 1

Subject No.	Height (inches)	Speed of walking (m.p.h.)	Frequency No. of steps per minute (N)	Stride length S (in feet)
		0.36*	58	0.55
		0.54**	60	0.81
		0.79***	69	1.01
		1.28**	77	1.47
1	60.0	1.48	81	1.61
		1.89	93	1.80
		2.40	102	2.06
		2.87	114	2.22
		3.28	119	2.43
		3.38	121	2.47
		3.93	128	2.70
		4.02	129	2.74
		4.12	128	2.84
		4.21	129	2.86
		4.35	129	2.97
		0.48*	55	0.77
		0.91**	64	1.25
		1.34**	77	1.54
		1.65**	86	1.70
		1.74	85	1.81
		1.85	89	1.84
		2.33	101	2.02
		2.72	108	2.21
		2.91	109	2.35
2	63.5	3.03**	115	2.32
		3.58	118	2.67
		3.82**	127	2.64
		3.93	128	2.71
		4.30	133	2.85
		4.55	139	2.89
		4.86	142	3.00
		5.02**	143	3.09
		5.13	141	3.20
		5.52*	146	3.32

* Average for 3 values.

** Average for 2 values.

*** Average for 4 values.

Fig. 1 shows S plotted against N for both these subjects.

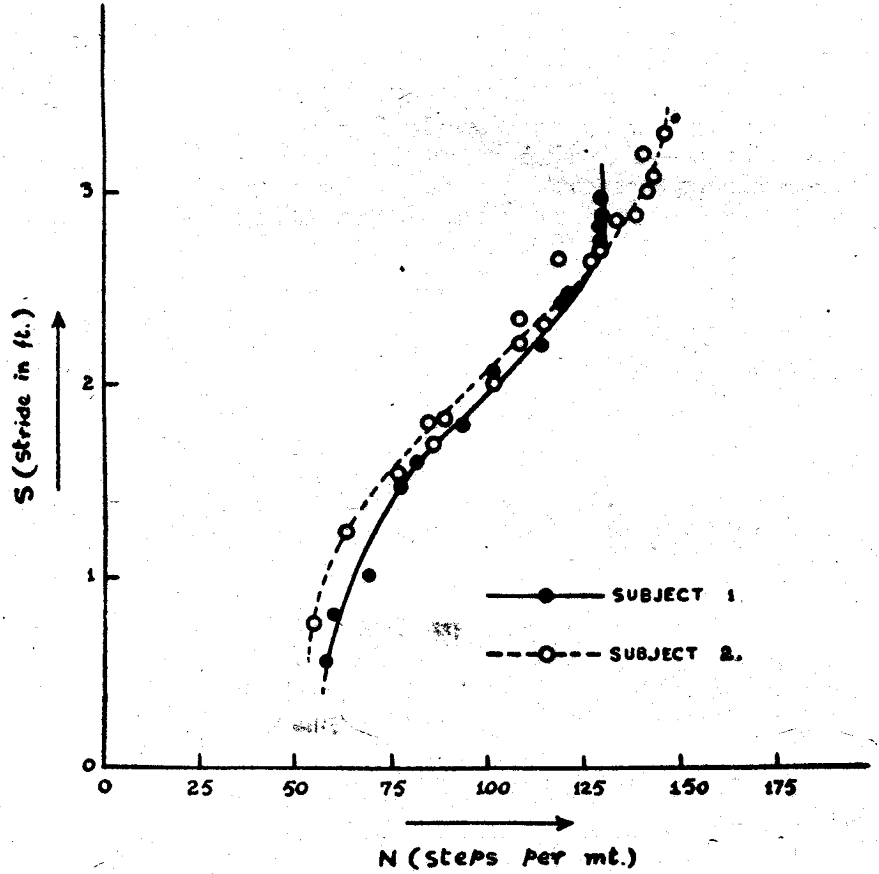

Fig. 1—Relationship between Stride Length and Frequency of Stepping in normal unfatigued individuals.

It will be seen from the figure that except for very low and very high values of N, points for each subject lie on a straight line passing through the origin. The slope for subject 1, is slightly less than that for subject 2, which is obviously due to the height factor, the former subject being shorter than the latter by about 3″. Taking the linear portion to represent the normal range, it will be seen that for subject 1, the range is from about 1·2 to 3·7 i.p.h. whereas for subject 2, it is longer on both sides. The main conclusion to be drawn from the above, is that within the normal range the relation (*iv*) i.e. S/n=a, is satisfied.

Dependence of the value of 'a' on Stature—The stride length being primarily related to leg length, it is expected that value of the constant 'a' should be determined by the individual leg lengths. In order to find the relation, observations were taken on eleven more subjects. Five of them were studied at different speeds within the normal range, while the rest were studied only at their natural speeds of walking. The leg length for each subject (from superior border of great Trochanter to the sole of foot) was measured with an ordinary tape. Values of S/n with the corresponding leg lengths for all the subjects are given in Table II and shown graphically in Fig. 2.

TABLE II

Subject No.	Height (inches)	Leg length (inches)	S/n
1	60.0	30.3	1.20
2	63.5	32.9	1.30
3	63.5	33.0	1.30
4	64.5	33.7	1.45
5	68.5	38.0	1.42
6	70.5	36.5	1.43
7	63.0	32.8	1.31
8	65.5	34.7	1.37
9	66.0	34.7	1.42
10	66.5	35.0	1.41
11	67.5	37.0	1.52
12	69.8	38.0	1.58
13	71.6	37.7	1.48

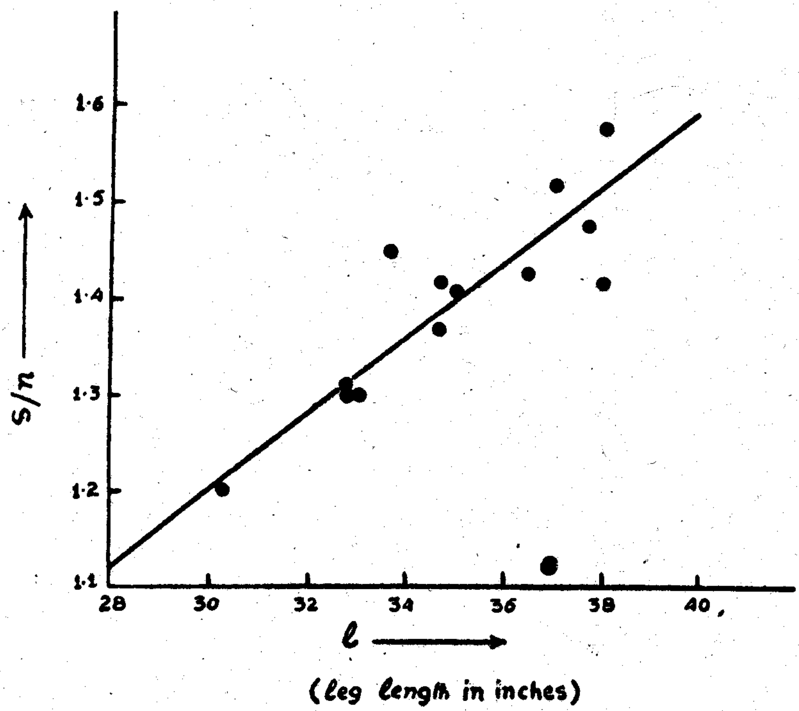

Fig. 2—Relationship between the ratio, Stride Length/Frequency of Stepping and Leg Length.

A straight line passing through the origin reasonably fits the points. The equation of the straight line can be given as

$$S/n = 0 \cdot 04 l \ldots\ldots\ldots\ldots (vii)$$

Where S is in feet, n is the number of steps per second, and 'l' is the leg length in inches.

Relation Between Leg Length and Body Height—In view of the practical difficulties associated with measurement of leg length, it is desirable that the constant 'a' should be expressed in terms of body height of individuals.

In order to study the nature of dependence of leg length on body height, observations were taken on 31 normal male subjects (height ranging from 60″ to about 71″) with anthropometer beam calipers. The values are represented in Fig. 3.

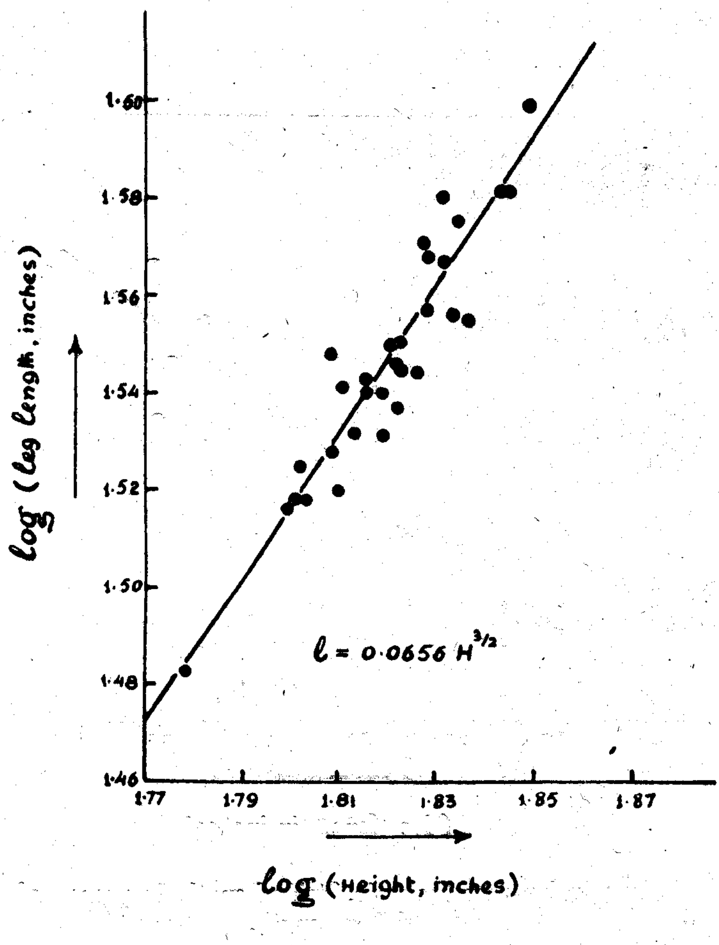

Fig. 3—Relationship between Leg Length and Body Height.

Statistical analysis of the data shows, that $l \propto H^{1.4913}$ Within the practical range, we are therefore justified in taking the power of H as 3/2, which leads to the expression

$$l=0\cdot0656\ H^{3/2} \qquad \ldots \qquad \ldots \qquad (viii)$$

where l and H are in inches.

Stride Length and Frequency of Stepping with Speed of Walking and Body Height—From the above considerations, it is possible to arrive at quantitative relationships connecting the different variables.

From equations (*vii*) and (*viii*) we get

$$a=2\cdot624\times10^{-3}\ H_{in}^{\ 3/2} \qquad \ldots \qquad \ldots \qquad (ix)$$

If 'a_0' is the value of 'a' for a standard man of height H_0=66 inches we have from (*ix*)

$$a_o=1\cdot41,$$

so that for any height H″, 'a' can be given as

$$a=1\cdot41\left(H/_{H_0}\right)^{3/2} \qquad \ldots \qquad \ldots \qquad (x)$$

Thus finally from (*v*), (*vi*) and (*x*) we get

$$S = \sqrt{1\cdot41}\left(H/_{H_0}\right)\ \sqrt{V}\ ,$$ where S is in feet, and V, in ft/sec. This may be rewritten as

$$S = \left(H/_{H_0}\right)^{3/4} \sqrt{2.V_{m.p.h.}} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (xi)$$

And the expression for frequency of stepping can be written as

$$n=\left(H_0/_H\right)^{3/4}\sqrt{V_{m.p.h.}} \qquad \ldots \qquad \ldots \qquad (xii)$$

Actual calculations show that within the range of heihtsstudied in the present series, very little error is introduced by replacing $\left(H/_{H_0}\right)^{3/4}$ by $H/_{H_0}$ itself in equations (*xi*) and (*xii*).

In Table III, the experimental values for height, stride length, frequency of stepping and speed of walking are given for five subjects. In Fig. 4, $S\times {H_0}_H$ has been plotted against $\sqrt{V_{m.p.h.}}$ for these five subjects together with the values within the 'normal range' for the first two subjects from Table I. A straight line passing through the origin and having a slope of 1·41, gives a reasonable fit over the entire range as should be expected from equation (*xi*).

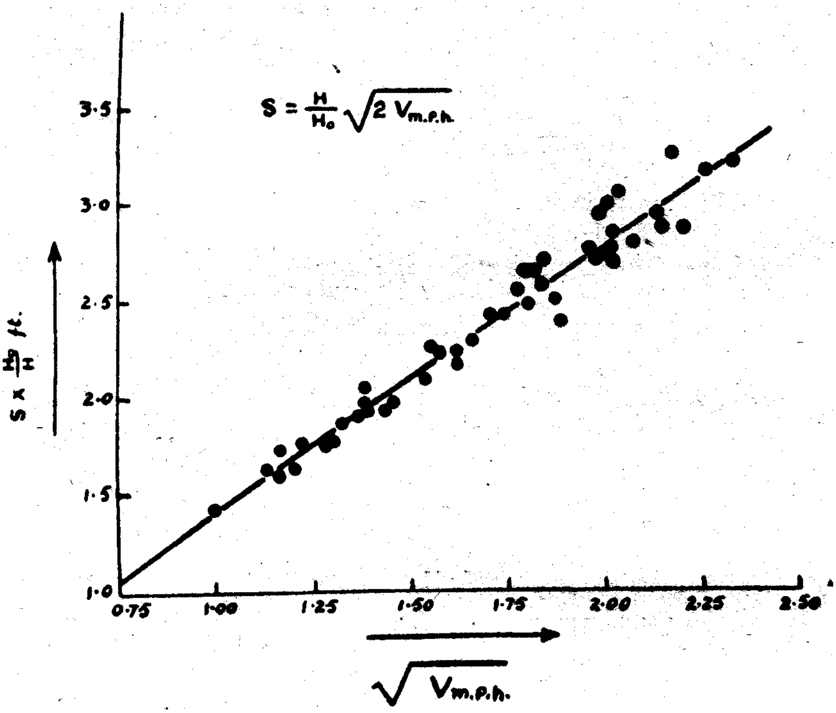

Fig. 4—Relationship between Stride Length and Speed of Walking for a Standard man (H_o=66″).

TABLE III

Subject No.	Height (inches)	Speed of walking (m.p.h.)	Frequency of stepping, N (No. of steps per minute)	Stride length S (in feet)
		1.64	86	1.69
		2.45	101	2.14
3	63.5	3.34	118	2.50
		3.95	130	2.68
		5.37	152	3.11
		1.35	71	1.68
		1.89	83	2.01
4	64.5	3.12	109	2.52
		3.19	108	2.60
		4.08	120	3.00
		4.69	129	3.19

TABLE III—*contd.*

Subject No.	Height (inches)	Speed of walking (m.p.h.)	Frequency of stepping, N (No. of steps per minute)	Stride length S (in feet)
5	68.5	2.07	89	2.06
		2.63	100	2.32
		3.24	111	2.58
		3.92	120	2.87
		5.04	134	3.31
6	70.5	1.00	58	1.52
		1.89	80	2.07
		3.48	114	2.68
		4.07	124	2.90
		4.83	137	3.10
14	63.0	1.44	81	1.56
		2.08	97	1.89
		2.60	109	2.09
		4.03	130	2.74
		4.52	143	2.77

It may be mentioned, in passing, that observations taken on a few more individuals at their natural speeds of walking, seem to indicate that the natural frequency of stepping is a constant, independent of individual height and has a value of about 110/min. This, however, needs confirmation as it is difficult for the subjects to walk consciously at their natural speeds.

Summary of Conclusions

1. Within 'normal range' of speeds of walking, it has been observed that

   (*a*) For a norma unfatigued man, his stride length varies directly as the frequency of stepping.

   (*b*) Both stride length and frequency of stepping vary directly as the square root of speed.

   (*c*) The rat o stride length/frequency varies directly as the leg length.

2. Leg length l, is related to body height H, as given by $l = 0{\cdot}0656H^{3/2}$ both l and H being in inches.

3. Within the range of heights studied (60″ to 71″) experimental observations within the normal range of speeds, are reasonably represented by the following equations

$$S = \left({}^{H}/_{H_o} \right)^{3/4} \sqrt{2\, V_{m.p.h.}}$$

and $n = \left({}^{H_o}/_{H} \right)^{3/4} \sqrt{V_{m.p.h.}}$ where S is the stride length in feet, in the number of steps per second, H the height of the individual and H_o that of a standard man (66″).

For practical purposes, $\left({}^{H}/_{H_o} \right)^{3/4}$ for the range of heights studied, may be replaced by ${}^{H}/_{H_o}$ without introducing any appreciable error.

4. The frequency of stepping at the natural speed of walking is likely to be a constant independent of body height, and of the order of 110 steps per minute.

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1. Lister Erickson et al—'The Energy Cost of Horizontal and Grade Walking on the Motor Driven Treadmill', The Amer. J. Physiol, *145*, 393, 1946.