GENERALISED STUDY OF THE FORMFUNCTION OF A MODIFIED MULTI-TUBULAR CHARGE WITH HOLES SYMMETRICALLY DISTRIBUTED IN A 2-DIMENSIONAL SPACE

GIRRAJ PRASAD

Department of Mathematics, University of Rajasthan, Jaipur

(*Received* 20 *February* 1968; *revised* 21 *September* 1968)

The formfunction of a modified multi-tubular charge with holes symmetrically distributed in a 2-dimensional space has been studied. A general expression for the formfunction and the ratio S/S_0 has been obtained from which formfunctions for distributions of holes about 3-fold, 4-fold and 6-fold axes have been derived. The charge has been so modified that the burning is complete at the end of the first phase of combustion.

The formfunctions of various types of multi-tubular charges have been discussed by a number of authors. The formfunction of a modified tri-tubular and hepta-tubular charges has been studied by Kapur & Jain[1], of 19-tubular charge by Kothari[2] and of N-tubular charge by Jain[3]. The arrangements of holes in these charges may be divided into two groups : (*i*) where at the end of the first phase of combustion, every hole touches not only its neighbouring holes in the same ring but also those in the other rings (when the number of rings is more than one), as in the case of 19-tubular charge, Type II, studied by Kothari[4] ; (*ii*) where at the end of the first phase of combustion, every hole touches its neighbouring holes on the same ring but not those situated on other rings.

In the second group, some portion of the charge is left between two rings which is burnt in several stages. In order that this portion may be minimised as much as possible, the holes should be so arranged that every hole touches all its neighbouring holes on the same ring as well as those on other adjacent rings. This can be done by distributing the centres of the holes symmetrically in a 2-dimensional space.

The general theory of such a distribution has been discussed by Patni, *et al*[5]. It has been shown that for this purpose holes have to be distributed about 3-fold, 4-fold, and 6-fold axes of symmetry only. Any number, say n, of rings can be taken. These rings are n concentric similar and similarly situated equilateral triangles in the case of 3-fold axis, similar and similarly situated squares in the case of 4-fold axis[6] and similar and similarly situated hexagon in the case of 6-fold axis of symmetry[7]. The centres of the holes are distributed along the sides of these polygons. In the present case the charges are totally modified, i.e. the entire part of the charge remaining at the end of the first phase of combustion is inhibited so that burning is complete in one stage. The modified charges belonging to group (*i*) and studied by the various authors so far are particular cases of the charges discussed in this paper.

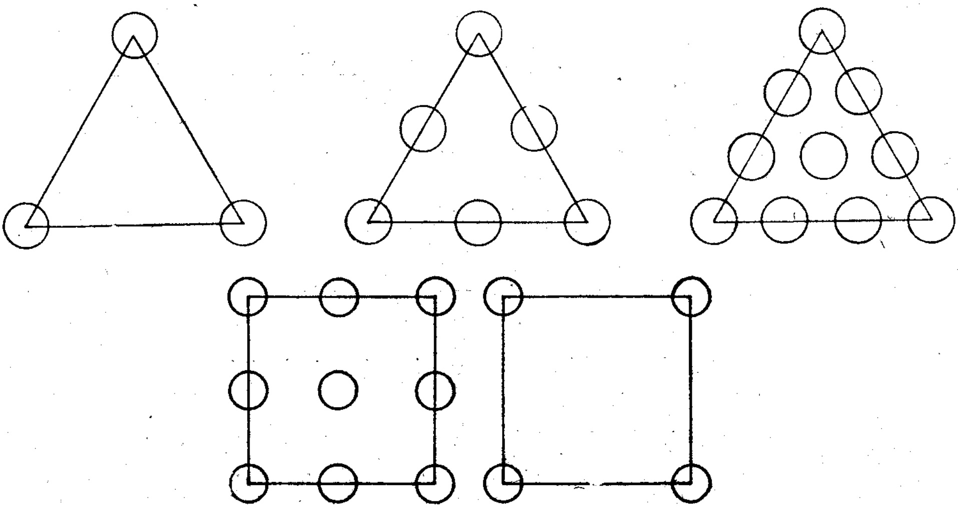


Fig. 1—Arrangements of holes in the first ring in different categories of various symmetrical distributions.

NOTATION

D = external diameter of the charge.

d = diameter of the holes of the charge.

L = length of the charge.

e_r = web size, i.e. the distance between any two adjacent holes or between any exterior hole and the curved surface of the grain.

m = ratio of the diameter of the charge to the diameter of a hole $= \frac{D}{d}$

ρ = ratio of the length of the charge to its diameter $= \frac{L}{D}$

V_0 = initial volume of the charge.

V = volume of the charge at any instant t.

S_0 = initial surface of the charge.

S = surface of the charge at any instant t.

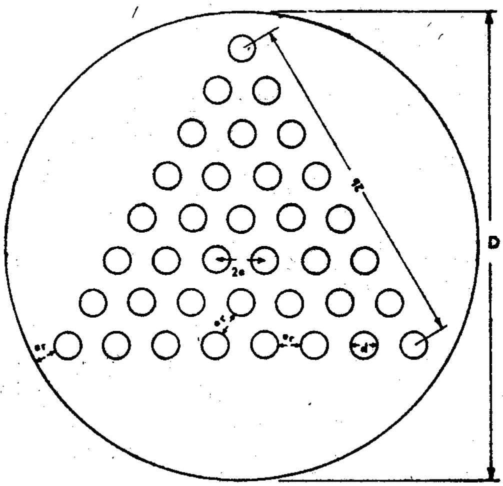
Categories of arrangements : In the case of distribution of holes about the 3-fold axis of symmetry, the centres of the holes are situated on the vertices of exactly alike equilateral triangles and are distributed along the sides of similar and similarly situated equilateral trianlges, called rings. The minimum number of holes on a side of the innermost triangle (i.e. the 1st ring) can be 2, 3 or 4 and these arrangements will be called category I, II and III respectively. In category III, there is an extra hole in the centre of the ring.

In case of distribution about 4-fold axis, the centres of the holes are situated on the vertices of exactly alike squares, and are distributed along the sides of similar and similarly situated squares, called rings. The minimum number of holes on a side of the innermost square can be 2 or 3 and these arrangements will be called category I and II respectively. Here also, there is an additional hole at the centre of the first ring in case of category II.

In case of distribution about 6-fold axis of symmetry, there is only one category. The centres of the holes are situated on the vertices of exactly alike hexagons, and are distributed along the sides of similar and similarly situated hexagons called rings. On a side of the first ring there are 2 holes and there is an additional hole at the centre of this ring.

The different categories are shown in Fig. 1.

Let there be n rings in the charge, and r the number of the category in any distribution. Thus



$$\left.\begin{array}{ll} r = 1, 2, 3 & \text{for distribution about 3-fold axis} \\ r = 1, 2 & \text{for distribution about 4-fold axis} \\ r = 1 & \text{for distribution about 6-fold axis} \end{array}\right\} \quad (1)$$

We define

$$\mu_r = N' \, (n - 1) + r \qquad (2)$$

where

$N' = 3$ for distribution about 3-fold axis

$N' = 2$ for distribution about 4-fold axis

$N' = 1$ for distribution about 6-fold axis

The number of holes on a side of the

$$n\text{th ring} = \mu_r + 1 \qquad (3)$$

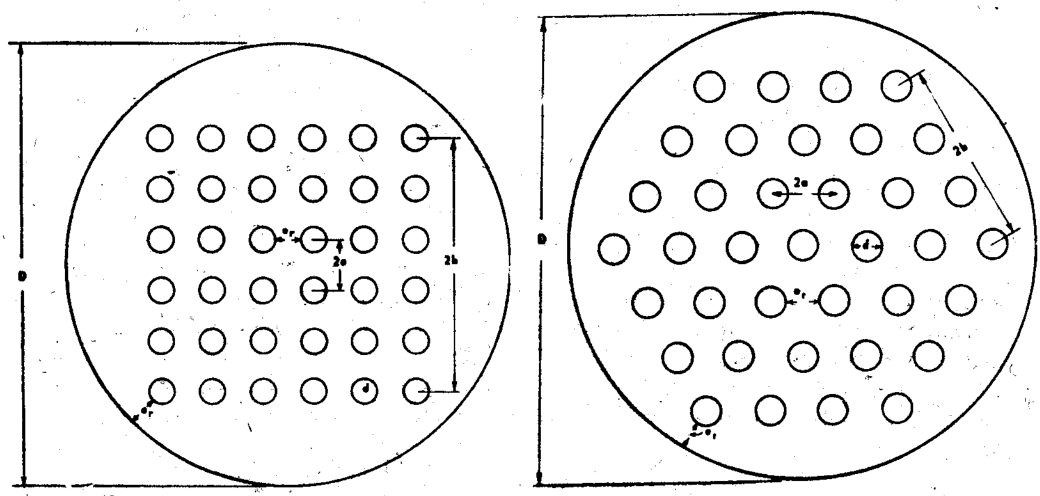


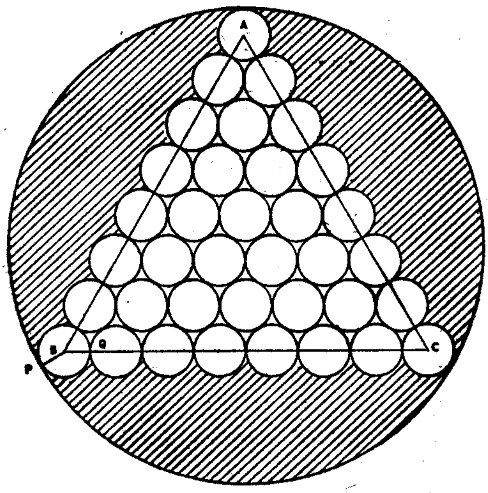
Fig. 2—Section of the charge in different cases at the beginning of combustion for $n = 3$, $r = 1$.

and the total number of holes in the charge is given by

$$\left.\begin{aligned} N_r &= \tfrac{1}{2}(\mu_r + 1)(\mu_r + 2) && \text{for distribution of holes about 3-fold axis} \\ N_r &= (\mu_r + 1)^2 && \text{for distribution of holes about 4-fold axis} \\ N_r &= 3\mu_r(\mu_r + 1) + 1 && \text{for distribution of holes about 6-fold axis} \end{aligned}\right\} \quad (4)$$

Section of the charge in different cases at the beginning of combustion, for $n = 3$, $r = 1$ are shown in Fig. 2.

Let $2a$ be the side of the innermost polygon (i.e. the first ring) and $2b$ the side of the outermost polygon (i.e. the nth ring). Then



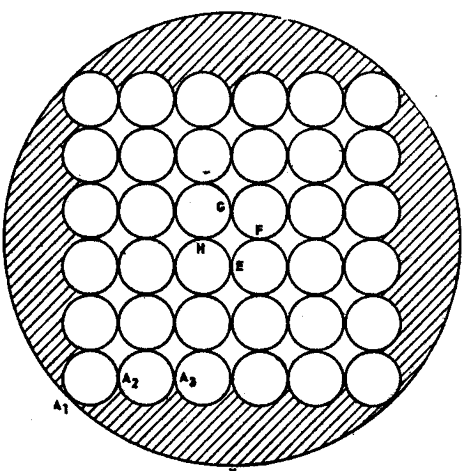
$$b = \frac{\mu_r}{r} a, \text{ as shown in Fig. 2.} \quad (5)$$

$$\text{Now } D = \frac{2a(\mu_r + 2\cos\theta)}{r\cos\theta} - d = md$$

$$\text{Hence } a = \frac{r(m+1)\cos\theta}{2(\mu_r + 2\cos\theta)} d \quad (6)$$

Where

$$\left.\begin{aligned} \theta &= \frac{\pi}{6} \text{ for distribution about 3-fold axis} \\ \theta &= \frac{\pi}{4} \text{ for distribution about 4-fold axis} \\ \theta &= \frac{\pi}{3} \text{ for distribution about 6-fold axis} \end{aligned}\right\} (7)$$



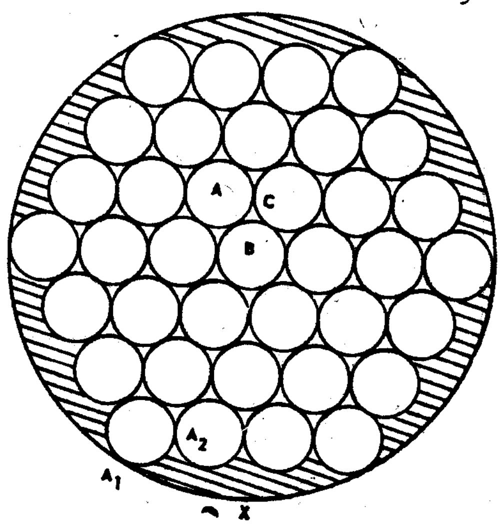


Fig. 3—Section of the charge at the end of the first phase of combustion for different cases when $n = 2$, $r = 1$.

and
$$e_r = \frac{(2a - rd)}{r}$$
$$= \frac{(m-1)\cos\theta - \mu_r}{\mu_r + 2\cos\theta}\, d = K'd \qquad (8)$$

where
$$K' = \frac{(m-1)\cos\theta - \mu_r}{\mu_r + 2\cos\theta} \qquad (9)$$

At the end of the first phase of combustion, when every hole touches all holes adjacent to it, the charge consists of :

(*i*) μ_r^2 exactly alike prisms whose bases are curvilinear triangles in the distributions about 3-fold and 6-fold axes and curvilinear squares in case of the distribution about 4-fold axis,

(*ii*) exactly alike prisms lying outside the *n*th ring, numbering 3, 4 or 6 in the three cases. The bases of these prisms are curvilinear in form as shown in Fig. 3.

The diameter of the charge at this instant

$$= 2\left(b \sec\theta + \frac{a}{r}\right)$$
$$= \frac{(m+1)(\mu_r + \cos\theta)}{(\mu_r + 2\cos\theta)}\, d = D' \text{ (say)} \qquad (10)$$

and the diameter of a hole at this instant

$$= \frac{2a}{r}$$
$$= \frac{(m+1)\cos\theta}{\mu_r + 2\cos\theta}\, d = d' \text{ (say)} \qquad (11)$$

The area of the section of the charge at this instant which is to be inhibited

$$= \frac{\pi D'^2}{4} - N_r \frac{\pi d'^2}{4}$$
$$= \frac{\pi (m+1)^2 \{(\mu_r + \cos\theta)^2 - N_r \cos^2\theta\} d^2}{4(\mu_r + 2\cos\theta)^2} \qquad (12)$$
$$= \frac{\pi d^2}{4} \triangle$$

where
$$\triangle = \frac{(m+1)^2 \{(\mu_r + \cos\theta)^2 - N_r \cos^2\theta\}}{(\mu_r + 2\cos\theta)^2} \qquad (13)$$

Inhibiting this portion from burning, we have

$$V_0 = \left[\frac{\pi D^2}{4} - N_r \frac{\pi d^2}{4} - \frac{\pi d^2}{4}\triangle\right] \times L$$
$$= \frac{\pi \rho m d^3}{4}(m^2 - N_r - \triangle) \qquad (14)$$

and

$$V = \left[\pi\left\{\frac{D}{2}-\frac{e_r(1-f)}{2}\right\}^2 - N_r\pi\left\{\frac{d}{2}+\frac{e_r(1-f)}{2}\right\}^2-\frac{\pi d^2}{4}\triangle\right]\left[L-e_r(1-f)\right]$$

$$= \frac{\pi\rho m d^3}{4}(m^2-N_r-\triangle)-\frac{\pi K'(1-f)d^3}{4}\Big[\rho m\left\{2m+2N_r+K'(N_r-1)\right\}$$

$$+m^2-N_r-\triangle-K'^2(N_r-1)-2K'(m+N_r)-fK'\Big\{\rho m\ (N_r-1)$$

$$-2(m+N_r)-2K'(N_r-1)\Big\}-f^2K'^2(N_r-1)\Big] \quad (15)$$

Hence

$$z = \frac{V_0\delta - V\delta}{V_0\delta}$$

$$=(1-f)(1-B'f-C'f^2) \quad (16)$$

where

$$B' = \frac{K'^2\{\rho m(N_r-1)-2(m+N_r)-2K'(N_r-1)\}}{\rho m(m^2-N_r-\triangle)} \quad (17)$$

$$C' = \frac{K'^3(N_r-1)}{\rho m(m^2-N_r-\triangle)} \quad (18)$$

Now $\dfrac{S}{S_0} = \dfrac{\frac{dz}{df}}{\left(\frac{dz}{df}\right)_{f=1}}$, which from (16), (17) and (18) gives

$$\frac{S}{S_0} = \alpha'-\beta'f-\gamma'f^2 \quad (19)$$

where

$$\left.\begin{aligned}\alpha' &= \frac{1+B'}{1-B'-C'}\\ &= \frac{\rho m(m^2-N_r-\triangle)+K'^2\{\rho m(N_r-1)-2(m+N_r)-2K'(N_r-1)\}}{\rho m(m^2-N_r-\triangle)-K'^2\{\rho m(N_r-1)-2(m+N_r)-K'(N_r-1)\}}\end{aligned}\right\} \quad (20)$$

$$\left.\begin{aligned}\beta' &= \frac{2(B'-C')}{1-B'-C'}\\ &= \frac{2K'^2\{\rho m(N_r-1)-2(m+N_r)-3K'(N_r-1)\}}{\rho m(m^2-N_r-\triangle)-K'^2\{\rho m(N_r-1)-2(m+N_r)-K'(N_r-1)\}}\end{aligned}\right\} \quad (21)$$

$$\left.\begin{aligned}\gamma' &= \frac{3C'}{1-B'-C'}\\ &= \frac{3K'^3(N_r-1)}{\rho m(m^2-N_r-\triangle)-K'^2\{\rho m(N_r-1)-2(m+N_r)-K'(N_r-1)\}}\end{aligned}\right\} \quad (22)$$

In order that the all burnt position of the charge may not occur before the rupture of the grain

$$L > e_r$$

or

$$\rho > \frac{(m-1)\cos\theta - \mu_r}{m(\mu_r + \cos\theta)} = \rho_{min} \tag{23}$$

where

$$\rho_{min} = \frac{(m-1)\cos\theta - \mu_r}{m(\mu_r + \cos\theta)} \tag{24}$$

Differentiating (19) we get

$$\frac{d^2}{df^2}\left(\frac{S}{S_0}\right) = -\beta' - 2\gamma' f \tag{25}$$

and

$$\frac{d^2}{df^2}\left(\frac{S}{S_0}\right) = -2\gamma' \tag{26}$$

Since γ' is always positive, $\frac{d^2}{df^2}\left(\frac{S}{S_0}\right)$ is always negative. Therefore, $\frac{S}{S_0}$ can have only a maximum value for some value of f.

For a maximum $\frac{d}{df}\left(\frac{S}{S_0}\right) = -\beta' - 2\gamma' f = 0$

so that

$$f = \frac{-\beta'}{2\gamma'}$$

Since

$$1 \geqslant f \geqslant 0,$$

we have

$$-\frac{\beta'}{2\gamma'} \leqslant 1 \tag{27}$$

and

$$-\frac{\beta'}{2\gamma'} \geqslant 0 \tag{28}$$

From (21) and (22)

$$-\frac{\beta'}{2\gamma'} = \frac{\{3K'(N_r - 1) + 2(m + N_r) - \rho m(N_r - 1)\}}{3K'(N_r - 1)} \tag{29}$$

so that from (27) and (29)

$$\rho \geqslant \frac{2(m + N_r)}{(N_r - 1)m} = \rho_1 \text{ (say)} \tag{30}$$

Similarly, from (28) and (29)

$$\rho \leqslant \frac{3K'(N_r - 1) + 2(m + N_r)}{m(N_r - 1)} = \rho_2 \text{ (says)} \tag{31}$$

(30) and (31) give the value of ρ for which a maximum of $\frac{S}{S_0}$ occurs at the beginning of combustion and at the rupture of the grain respectively. In order that the maximum may occur between these two stages, ρ should lie between ρ_1, and ρ_2.

With the help of (24), (29), (30) and (31) we get for a maximum of $\frac{S}{S_0}$

$$f = -\frac{\beta'}{2\gamma'} = 1 - \frac{\rho - \rho_1}{3\rho_{min}} = \frac{\rho_2 - \rho}{3\rho_{min}} \tag{32}$$

Also

$$\frac{d}{df}\left(\frac{S}{S_0}\right)_{f=1} = -\beta' - 2\gamma'$$

$$= \frac{2K'^2 (N_r - 1)(\rho_1 - \rho)}{\rho m (m^2 - N_r - \triangle) - K'^2 \{\rho m (N_r - 1) - 2(m + N_r) - K'(N_r - 1)\}} \tag{33}$$

and

$$\frac{d}{df}\left(\frac{S}{S_0}\right)_{f=0} = -\beta'$$

$$= \frac{2K'^2 (N_r - 1)(\rho_2 - \rho)}{\rho m (m^2 - N_r - \triangle) - K'^2 \{\rho m (N_r - 1) - 2(m + N_r) - K'(N_r - 1)\}} \tag{34}$$

Hence for any value of n,

(*i*) if $\rho_1 < \rho \leqslant \rho_2$, $\frac{d}{df}\left(\frac{S}{S_0}\right)$ is negative in the beginning and then positive.

In such a case the charge is first progressive and then degressive ;

(*ii*) if ρ_{min} i.e. $\frac{(m-1)\cos\theta - \mu_r}{m(\mu_r + \cos\theta)} \leqslant \rho \leqslant \rho_1$, $\frac{d}{df}\left(\frac{S}{S_0}\right)$ is always positive and the charge is throughout degressive ;

(*iii*) if $\rho > \rho_2$, $\frac{d}{df}\left(\frac{S}{S_0}\right)$ is always negative and the charge is throughout progressive.

Given below are values of B', C', α', β', γ', ρ_{min} , ρ_1 and ρ_2 for different distributions as obtained from (17), (18), (20), (21), (22), (24), (30) and (31) respectively by putting appropriate values of K' and $\triangle$.

I. Distribution about 3-fold axis

$$B' = \frac{\{\sqrt{3}(m-1) - 2\mu_r\}[2(\mu_r + \sqrt{3})(N_r - 1)m\rho - 2(m+1)(2\mu_r + \sqrt{3}N_r + \sqrt{3})]}{2m\rho(\mu_r + \sqrt{3})[m(4\mu_r + 3\sqrt{3} + \sqrt{3}N_r) + \{(2\mu_r + 3\sqrt{3})N_r + (2\mu_r + \sqrt{3})\}]} \tag{35}$$

$$C' = \frac{\{\sqrt{3}(m-1) - 2\mu_r\}^2 (N_r - 1)}{2m\rho(\mu_r + \sqrt{3})[m(4\mu_r + 3\sqrt{3} + \sqrt{3}N_r) + \{(2\mu_r + 3\sqrt{3})N_r + (2\mu_r + \sqrt{3})\}]} \tag{36}$$

$$\alpha' = \frac{2(m+1)(2\mu_r + \sqrt{3}N_r + \sqrt{3})[2(\mu_r + \sqrt{3})m\rho - (\sqrt{3}m - \sqrt{3} - 2\mu_r)]}{8m\rho(\mu_r + \sqrt{3})^2(m + N_r) + \{\sqrt{3}(m-1) - 2\mu_r\}[m(4\mu_r + 3\sqrt{3} + \sqrt{3}N_r) + \{(2\mu_r + 3\sqrt{3})N_r + (2\mu_r + \sqrt{3})\}]} \tag{37}$$

$$\beta' = \frac{2\{\sqrt{3}(m-1) - 2\mu_r\}[2(\mu_r + \sqrt{3})(N_r - 1)m\rho - (4\mu_r + 3\sqrt{3}N_r + \sqrt{3})m - \{3(2\mu_r + \sqrt{3}) - 2(2\mu_r - \sqrt{3})N_r\}]}{8m\rho(\mu_r + \sqrt{3})^2(m + N_r) + \{\sqrt{3}(m-1) - 2\mu_r\}[m(4\mu_r + 3\sqrt{3} + \sqrt{3}N_r) + \{(2\mu_r + 3\sqrt{3})N_r + (2\mu_r + \sqrt{3})\}]} \tag{38}$$

$$\gamma' = \frac{3\{\sqrt{3}(m-1)-2\mu_r\}^2(N_r-1)}{8m\rho(\mu_r+\sqrt{3})^2(m+N_r)+\{\sqrt{3}(m-1)-2\mu_r\}[m(4\mu_r+3\sqrt{3}+\sqrt{3}N_r)+\{(2\mu_r+3\sqrt{3})N_r+(2\mu_r+\sqrt{3})\}]} \quad (39)$$

$$\rho_{min} = \frac{\sqrt{3}(m-1)-2\mu_r}{2(\mu_r+\sqrt{3})m} \quad (40)$$

$$\rho_1 = \frac{2(m+N_r)}{(N_r-1)m} \quad (41)$$

$$\rho_2 = \frac{(\sqrt{3}+3\sqrt{3}N_r+4\mu_r)m+\sqrt{3}(N_r+3)-2\mu_r(N_r-3)}{2(\mu_r+\sqrt{3})(N_r-1)m} \quad (42)$$

If we put $n = 1$, $r = 1$, we obtain the results for a modified tri-tubular charge studied by Kapur & Jain[1].

II. Distribution about 4-fold axis

$$B' = \frac{(\sqrt{2}m-\sqrt{2}-2\mu_r)\{2m\rho(\mu_r+\sqrt{2})(N_r-1)-2(m+1)(2\mu_r+\sqrt{2}N_r+\sqrt{2})\}}{2m\rho(\mu_r+\sqrt{2})\{m(4\mu_r+3\sqrt{2}+\sqrt{2}N_r)+(2\mu_rN_r+3\sqrt{2}N_r+2\mu_r+\sqrt{2})\}} \quad (43)$$

$$C' = \frac{(\sqrt{2}m-\sqrt{2}-2\mu_r)^2(N_r-1)}{2m\rho(\mu_r+\sqrt{2})\{m(4\mu_r+3\sqrt{2}+\sqrt{2}N_r)+(2\mu_rN_r+3\sqrt{2}N_r+2\mu_r+\sqrt{2})\}} \quad (44)$$

$$\alpha' = \frac{2(m+1)(2\mu_r+\sqrt{2}N_r+\sqrt{2})\{2m\rho(\mu_r+\sqrt{2})-(\sqrt{2}m-\sqrt{2}-2\mu_r)\}}{8m\rho(\mu_r+\sqrt{2})^2(m+N_r)+(\sqrt{2}m-\sqrt{2}-2\mu_r)[m(4\mu_r+\sqrt{2}N_r+3\sqrt{2})+\{N_r(2\mu_r+3\sqrt{2})+(2\mu_r+\sqrt{2})\}]} \quad (45)$$

$$\beta' = \frac{2(\sqrt{2}m-\sqrt{2}-2\mu_r)[2(\mu_r+\sqrt{2})(N_r-1)-m\rho-(4\mu_r+3\sqrt{2}N_r+\sqrt{2})-\{3(2\mu_r+\sqrt{2})-N_r(2\mu_r-\sqrt{2})\}]}{8m\rho(\mu_r+\sqrt{2})^2(m+N_r)+(\sqrt{2}m\sqrt{2}-2\mu_r)[m(4\mu_r+3\sqrt{2}+\sqrt{2}N_r)+\{(2\mu_r+3\sqrt{2})N_r+(2\mu_r+\sqrt{2})\}]} \quad (46)$$

$$\gamma' = \frac{3(N_r-1)(\sqrt{2}m-\sqrt{2}-2\mu_r)^2}{8m\rho(\mu_r+\sqrt{2})^2(m+N_r)+(\sqrt{2}m-\sqrt{2}-2\mu_r)[m(4\mu_r+\sqrt{2}N_r+3\sqrt{2})+\{N_r(2\mu_r+3\sqrt{2})+(2\mu_r+\sqrt{2})\}]} \quad (47)$$

$$\rho_{min} = \frac{\sqrt{2}(m-1)-2\mu_r}{2(\mu_r+\sqrt{2})m} \quad (48)$$

$$\rho_1 = \frac{2(m+N_r)}{(N_r-1)m} \quad (49)$$

$$\rho_2 = \frac{m(4\mu_r+3\sqrt{2}N_r+\sqrt{2})+3(2\mu_r+\sqrt{2})-N_r(2\mu_r-\sqrt{2})}{2(\mu_r+\sqrt{2})(N_r-1)m} \quad (50)$$

III. Distribution about 6-fold axis

$$B' = \frac{2\,(m - 2n - 1)\,\{3n\,(n + 1)\,\rho m - (m + 1)\,(3n + 2)\}}{2\rho m\,(n + 1)\,[m\,(3n + 4) + (6n^2 + 9n + 4)]} \quad (51)$$

$$C' = \frac{3n\,(m - 2n - 1)^2}{2\rho m\,(n + 1)\,[m\,(3n + 4) + (6n^2 + 9n + 4)]} \quad (52)$$

$$\alpha' = \frac{2\,(m+1)\,(3n+2)\,\{2\,(n+1)\,m\rho - (m-2n-1)\}}{8m\rho\,(n + 1)\,(m + N_r) + (m - 2n - 1)\,[m\,(3n+4) + (6n^2 + 9n+4)]} \quad (53)$$

$$\beta' = \frac{2\,(m - 2n - 1)\,[6n\,(n + 1)\,m\rho - m\,(9n + 4) + (6n^2 - 3n - 4)]}{8m\rho\,(n + 1)\,(m + N_r) + (m - 2n - 1)\,[m\,(3n + 4) + (6n^2 + 9n + 4)]} \quad (54)$$

$$\gamma' = \frac{9n\,(m - 2n - 1)^2}{8m\rho\,(n + 1)\,(m + N_r) + (m - 2n - 1)\,[m\,(3n + 4) + (6n^2 + 9n + 4)]} \quad (55)$$

$$\rho_{min} = \frac{m - 2n - 1}{2\,(n + 1)\,m} \quad (56)$$

$$\rho_1 = \frac{2\,(m + N_r)}{m\,(N_r - 1)} \quad (57)$$

$$\rho_2 = \frac{(m + 1)\,(9n + 4) - 2\,(N_r - 1)}{2m\,(N_r - 1)} \quad (58)$$

Results for particular cases are at once obtained as follows :—

(*i*) by putting $n = 1$, for hepta-tubular charge as discussed by Kapur & Jain[1],

(*ii*) by putting $n = 2$, for nineteen tubular charge as discussed by Kothari[2].

ACKNOWLEDGEMENTS

The author is extremely thankful to Dr. G. C. Patni for his interest and guidance in the preparation of this paper and to Dr. T. C. Jain for valuable discussions.

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