# PROPAGATION OF PLANE SHOCK WAVES IN A LIQUID CONTAINING BUBBLES OF A CONDUCTING GAS

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(*Received* 19 *November* 1968)

This paper considers the propagation of plane shock waves in a mixture of conducting gas and liquid by taking appropriate equations of continuity, momentum, energy and an equation of state. When the temperature rise, which is small for a very wide range of problems, is neglected, across a shock, it is shown that the shock wave relations assume a simple form. It is also suggested that the relations can be applied to the collision between two normal shock waves.

## THE SPEED OF SOUND

Mallock[1] evaluated the speed of sound in a compressible liquid containing ordinary gas bubbles, assuming that the gas obeys Boyles' law and the mixture behaves like a homogeneous medium. Since the consideration of compressibility, however, has a very small effect, we take the liquid to be incompressible and the gas to be perfect. In a sample of mixture, if $\mu$ denotes the ratio between the masses of the gas and liquid, then

$$\frac{1+\mu}{\rho} = \frac{\mu}{\rho_g} + \frac{1}{\rho_0} \tag{1}$$

where $\rho$ is the mean density of the mixture, $\rho_g$ the density of the gas and $\rho_0$ the density of the liquid. Assuming that the gas obeys Boyles' law, we have

$$\frac{p}{\rho}\left[1 - \frac{\rho}{(1+\mu)\,\rho_0}\right] = k \tag{2}$$

where $k$ is a constant. The speed of sound $c \overset{def.}{=} \left(\frac{dp}{d\rho}\right)^{\frac{1}{2}}$ is therefore given by

$$c = \left[\frac{p}{\rho\left\{1 - \frac{\rho}{(1+\mu)\rho_0}\right\}}\right]^{\frac{1}{2}} \tag{3}$$

The equation (3) has been derived on the assumptions that the gas obeys Boyles' law and the mixture of the gas and liquid behaves like an equivalent homogeneous medium. It is important, however, to note that we are justified to use these assumptions only when the bubbles are small and the sound frequency is low. As $\mu \rightarrow \infty$ it is evident that equation (3) in an isothermal perfect gas, gives the local speed of sound but, since the expression for $c$ in equation (3) plays an important part in the shock theory, it is convenient to call it as the speed of sound although it is not so, in the strict sense of the term.

## SHOCK RELATIONS

In the system of coordinates, associated with the shock discontinuity and restricting ourselves to the case of infinite conductivity, the following conditions must be fulfilled[3]

$$< \rho\, u > = 0 \tag{4}$$

$$< p + \frac{h^2}{2} + \rho\, u^2 > = 0 \tag{5}$$

$$< \frac{p}{\rho} + \frac{C + \mu\, C_{vg}}{1 + \mu}\, T + \frac{1}{2}\, u^2 + \frac{h^2}{\rho} > = 0 \tag{6}$$

$$< h\, u > = 0 \tag{7}$$

where $h(4\pi)^{\frac{1}{2}}$ is the intensity of the magnetic field; the front of the discontinuity is parallel to the magnetic field and is perpendicular to the velocity of the flow. The bracket $< \;>$ denotes the jump in the quantity enclosed, viz $< f > = f_2 - f_1$, the subscripts 1 and 2 referring to the two sides of the discontinuity; $C$ is the specific heat of the liquid and $C_{vg}$ is the specific heat of the gas at constant volume. The equation of state for the mixture, obtained from that of the perfect gas, takes the form

$$\frac{p_1}{T_1}\left(\frac{1+\mu}{\rho_1} - \frac{1}{\rho_0}\right) = \frac{p_2}{T_2}\left(\frac{1+\mu}{\rho_2} - \frac{1}{\rho_0}\right) \tag{8}$$

It can be easily seen that when $\mu \to \infty$, $\dfrac{C + \mu C_{vg}}{1 + \mu} \to C_{vg}$ in the equation (6) and the equation (8) turns out to be the equation of state for a perfect gas. Further, when $h \to 0$, equations (4) to (8) are all identical with those which govern the propagation of shock waves in a perfect non-conducting gas. In what follows, we write $T_2$ as $T_1 + \triangle T$ and assume $\triangle T$ to be small. Then, from (4) and (5) we get

$$\rho_1 \frac{u_1^2}{\rho_2} - \frac{h_1^2}{2\rho_1^2}\left(\rho_2 + \rho_1\right) = \frac{< p >}{< \rho >} \tag{9}$$

From (8) we have

$$\frac{< \rho >}{\rho_2} = \frac{1}{p_2}\left\{1 - \frac{\rho_1}{(1+\mu)\,\rho_0}\right\}\left\{< p > - \frac{p_1 \triangle T}{T_1}\right\} \tag{10}$$

As a consequence of (9) and (10) we get,

$$\left\{1 - \frac{\rho_1}{(1+\mu)\rho_0}\right\} u_1^2 = \frac{p_2}{\rho_1}\left\{1 + \frac{p_1 \triangle T}{< p > T_1}\right\}\left[1 - \frac{h_1^2}{2 < p >}\left\{1 - (1+\mu)^2 \rho_0^2 \left[1 + \frac{p_1}{p_2}\left\{(1+\mu)\,\rho_0 - \rho_1\right\}\left\{1 + \frac{\triangle T}{T_1}\right\}\right]^{-2}\right\}\right] \tag{11}$$

From equations (4), (10) and (11) we deduce,

$$\left\{1-\frac{\rho_1}{(1+\mu)\rho_0}\right\}\left(u_1^2-u_2^2\right)=\frac{p_2}{\rho_1}\left(1+\frac{p_1\triangle T}{\langle p\rangle T_1}\right)\left[1-\frac{p_1^2}{p_2^2}\left\{1+\frac{\langle p\rangle\rho_1}{p_1(1+\mu)\rho_0}+\frac{\triangle T}{T_1}\left(1-\frac{\rho_1}{(1+\mu)\rho_0}\right)\right\}^2\right]$$
$$\left[1-\frac{h_1^2}{2\langle p\rangle}\left\{1-(1+\mu)^2\rho_0^2\left[1+\frac{p_1}{p_2}\left\{(1+\mu)\rho_0-\rho_1\right\}\left(1+\frac{\triangle T}{T_1}\right)\right]^{-2}\right\}\right] \tag{12}$$

and, from (10),

$$\left\langle\frac{p}{\rho}\right\rangle=\frac{\langle p\rangle}{(1+\mu)\rho_0}+\left\{1-\frac{\rho_1}{(1+\mu)\rho_0}\right\}\frac{p_1}{\rho_1}\,\frac{\triangle T}{T_1} \tag{13}$$

After a little arrangement between (6), (12) and (13), we have,

$$\frac{\langle p\rangle}{\rho_0(1+\mu)}+\left\{\left(1-\frac{\rho_1}{(1+\mu)\rho_0}\right)\frac{p_1}{\rho_1T_1}+\left(\frac{C+\mu C_{vg}}{1+\mu}\right)\right\}\triangle T=$$
$$\frac{1}{2}\left[1-\frac{p_1^2}{p_2^2}\left\{1+\frac{\langle p\rangle\rho_1}{p_1(1+\mu)\rho_0}+\frac{\triangle T}{T_1}\left(1-\frac{\rho_1}{(1+\mu)\rho_0}\right)\right\}^2\right]\frac{p_2}{\rho_1}\left\{1-\frac{\rho_1}{(1+\mu)\rho_0}\right\}^{-1}$$
$$\left\{1+\frac{p_1\triangle T}{T_1\langle p\rangle}\right\}\left[1-\frac{h_1^2}{2\langle p\rangle}\left\{1-(1+\mu)^2\rho_0^2\left[1+\frac{p_1}{p_2}\left\{(1+\mu)\rho_0-\rho_1\right\}\right.\right.\right.$$
$$\left.\left.\left.\left(1+\frac{\triangle T}{T_1}\right)\right]^{-2}\right\}\right]+\frac{h_1^2}{\rho_1}\left[1-\frac{p_2}{p_1}\left\{1+\frac{\langle p\rangle\rho_1}{p_1(1+\mu)\rho_0}+\frac{\triangle T}{T_1}\left(1-\frac{\rho_1}{\rho_0(1+\mu)}\right)\right\}^{-1}\right] \tag{14}$$

As a consequence of (1) and the gas law, (14) can be written as

$$\frac{\langle p\rangle}{\rho_0}+\left(\frac{\mu R}{M}+C\right)\triangle T=\frac{p_2}{2\rho_1\mu\rho_0^2}\left\{1+\frac{p_1}{\langle p\rangle}\frac{\triangle T}{T_1}\right\}\left[(1+\mu)^2\rho_0^2-\right.$$
$$\left.\frac{p_1^2}{p_2^2}\left\{(1+\mu)\rho_0+\frac{\langle p\rangle\rho_1}{p_1}+\frac{\triangle T\,\mu\rho_0}{T_1}\right\}^2\right]\left[1-\frac{h_1^2}{2\langle p\rangle}\left\{1-(1+\mu)^2\rho_0^2\left[1+\right.\right.\right.$$
$$\left.\left.\left.\frac{\mu p_1}{p_2}\left(1+\frac{\triangle T}{T_1}\right)\right]^{-2}\right\}\right]+\frac{h_1^2}{\rho_1}\left[1-\frac{p_2}{p_1}\left\{1+\frac{\langle p\rangle\rho_1}{p_1(1+\mu)\rho_0}+\frac{\triangle T}{T_1}\,\frac{\mu}{1+\mu}\right\}^{-1}\right] \tag{15}$$

where $R$ is the universal gas constant, $M$ the mean molecular weight of the gas and $C+C_{vg}$ has been replaced by $C$ without any appreciable loss of accuracy. Although the temperature rise across the shock, cannot be neglected for all purposes, the expressions relating pressure, density and velocity on the two sides of the shock assume a particularly simple form when $\triangle T\to 0$. The sacrifice of accuracy in the process, however, is very small. With this assumption, the equation (11) can now be written as,

$$u_1^2=\frac{p_2c_1^2}{p_1}\left[1-\frac{h_1^2}{2\langle p\rangle}\left\{1-(1+\mu)^2\rho_0^2\left[1+\frac{p_1^2}{p_2}\,\frac{(1+\mu)\rho_0}{c_1^2\rho_1}\right]^{-2}\right\}\right] \tag{16}$$

Similarly, the equation (10) can be approximated as,

$$\frac{\rho_2}{\rho_1}=\frac{p_2}{p_1}\left\{1+\frac{\langle p\rangle\rho_1}{p_1(1+\mu)\rho_0}\right\}^{-1} \tag{17}$$

and finally

$$\frac{u_2}{u_1} = \frac{\rho_1}{\rho_2} \tag{18}$$

Since the temperature rise across a normal shock wave in a perfect non-conducting gas tends to zero when $\gamma$, the ratio of specific heats tends to one and since a mixture becomes a perfect gas when $\mu \rightarrow \infty$, it may be anticipated that (16) and (17) when $\mu \rightarrow \infty$ and $h \rightarrow 0$ will be identical with the corresponding relations for a perfect non-conducting gas when $\gamma \rightarrow 1$. Referring to the following relations[3],

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1}\frac{u_1^2}{c_1^2} - \frac{\gamma-1}{\gamma+1} \tag{19}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma-1) + (\gamma+1)\frac{p_2}{p_1}}{(\gamma+1) + (\gamma-1)\frac{p_2}{p_1}} \tag{20}$$

we see that our anticipations are realised. The equations (16), (17) and (18) can be applied to consider the collision between two normal plane shocks moving in opposite directions. The temperature rise across shocks will be neglected to ensure that after collision, all the fluid in the regions between the resultant shocks have the same pressure, velocity, temperature and mean density. In the special case when the two colliding shocks are of the same strength, the problem reduces to the impact of a shock wave on a rigid wall.

## REFERENCES

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