ON PROPAGATION OF ONE-DIMENSIONAL SMALL AMPLITUDE WAVES IN RADIATING, VISCOUS AND HEAT CONDUCTING GAS

S. G. TAGARE

Indian Institute of Science, Bangalore

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In this paper, effect of radiation, heat-conduction and viscosity on propagation of one-dimensional small amplitude waves is investigated. It is shown that there are three distinct modes of propagation viz. (*i*) Radiation-induced mode, (*ii*) Modified gas-dynamic mode and (*iii*) Coupled heat-conduction and viscous mode.

The dispersion relation is solved both asymptotically and numerically. For very small values of ω, the asymptotic solution predicts the speed of propagation of disturbance as zero, a_S (isentropic sound velocity) and $0 \cdot 336$ times the isothermal sound velocity. For very large values of ω, the high frequency waves propagate with characteristic speeds of the seventh order operation.

The problem of the propagation of small amplitude waves in a radiating gas has been examined by various authors such as Vincenti & Baldwin[1] and Lick[2]. In both these papers authors have neglected radiation pressure p_R and radiation energy density E_R in comparison with gas pressure p_G and gas energy density E_G. In fact, radiation pressure and energy become important only at very high temperature $10^7 K$ while radiation as a mode of heat transfer through radiation flux may have to be considered at much lower temperatures $\sim 10^3 K$. Prasad[3] has considered the same problem by considering radiation energy density as comparable with gas pressure and gas energy density.

But all these authors have neglected viscosity and heat conductivity. In this paper we assume that the temperature is of the order of $10^7 K$. We also assume that gas is neutral, radiating, viscous and heat-conducting.

Prasad's[3] equations of motion, without viscosity and heat-conduction terms, have been used here. An interesting feature of the approximate equations of motion of Prasad[3] is that these equations form a hyperbolic system of equations with distinct characteristics. The outermost characteristics carry radiation induced waves, which propagate with velocity $\frac{c}{\sqrt{3}}$ where c is the speed of light in vacuum, and they determine the range of influence and domain of dependence. However, when we take the viscosity and heat—conduction into account we find from our accoustic equation that out of the seven characteristics, four of them are coincident with the x-axis and these coincident characteristics represent waves propagating with infinite velocity. Thus there is some inconsistency in our equations of motion. This is due to the fact that we have taken into account the effect of viscosity and heat conduction by introducing diffusive terms as in the Navier-Stokes equations.

FORMULATION OF THE PROBLEM

Let us consider the equations for one-dimensional flow, parallel to x-axis, of a viscous, heat-conducting and radiating fluid where all physical quantities are independent of y, z. Then we have:

$$p_R = \frac{1}{c} \int I \mu^{*2} \, d\omega \tag{1}$$

$$E_R = \frac{1}{c} \int I \, d\omega \tag{2}$$

and

$$F = \int I \mu^* \, d\omega \tag{3}$$

where $\mu^* = \cos \theta$, I is the specific intensity of radiation making an angle θ with x-axis, F is the radiation flux in positive x-direction and $d\omega$ is an element of solid angle. Also we have :

$$E_G = \frac{p_G}{(\gamma - 1) \rho} \, ; p_G = R \rho T \tag{4}$$

where ρ is density of a fluid, T is temperature and R is the gas constant. The equations of continuity, momentum and energy are :

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0 \, , \tag{5}$$

$$\rho \left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} \right) u + \frac{\partial}{\partial x} (p_G + p_R) - \frac{4\mu}{3} \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} \right) = 0 \tag{6}$$

and

$$\rho \left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} \right) \left(E_G + \frac{E_R}{\rho} \right) + (p_G + p_R) \frac{\partial u}{\partial x} - \frac{4\mu}{3} \left(\frac{\partial u}{\partial x} \right)^2 - K \frac{\partial^2 T}{\partial x^2} + \frac{\partial F}{\partial x} = 0 \tag{7}$$

where E_G is gas energy density per unit mass, E_R is radiation energy density per unit volume, u is the fluid velocity and μ and K are coefficients of viscosity and heat-conduction which are assumed to be constant.

We shall make the assumption that the source function for radiation is

$$B = \frac{\sigma}{\pi} T^4 \tag{8}$$

so that the equation of radiative transfer is

$$\frac{1}{c} \frac{\partial I}{\partial t} + \mu^* \frac{\partial I}{\partial x} = \alpha (B - I) \tag{9}$$

where α is volume absorption coefficient and σ is Stefan's constant.

By Milne-Eddington approximation, we get from equations (1) to (3) and (9)

$$\left(\frac{\partial^2 F}{\partial x^2} - \frac{3}{c^2} \frac{\partial^2 F}{\partial t^2} \right) = 4 \pi \alpha \frac{\partial B}{\partial x} + \frac{6\alpha}{c} \frac{\partial F}{\partial t} + 3 \alpha^2 F \tag{10}$$

$$\frac{1}{c} \frac{\partial F}{\partial t} + c \frac{\partial p_R}{\partial x} + \alpha F = 0 \tag{11}$$

and

$$E_R = 3 \, p_R \tag{12}$$

The nine equations (4) to (8) and (10) to (12) involve nine unknowns and thus are sufficient to solve any problem of one-dimensional R.G.D. provided correct initial and boundary conditions are given. Radiation travels with velocity of light in the medium, whereas equation (10) shows that $\frac{dx}{dt} = \pm \frac{c}{\sqrt{3}}$ are two characteristics of these equations. Thus it is true that the front of any radiation induced wave in R.G.D. travels with velocity c, the Milne-Eddington approximation shows that the front of the radiation induced waves travels with velocity $\frac{c}{\sqrt{3}}$ as discussed by Prasad[3].

LINEARIZATION OF EQUATIONS AND DISPERSION RELATION

Equations (5) to (9) may be simplified by the usual process of linearization. We assume that there is a uniform equilibrium state characterized by

$$u = 0\,,\; p_G = p_{G0}\,,\; T = T_0\,,\; F = 0\,,\; p_R = p_{R0} = \frac{4\sigma}{3c} T_0^4\,,\; E = E_{R0} = 3p_{R0} \qquad (13)$$

where p_{G0}, ρ_0 , T_0 satisfy the equation of state.

The perturbations about this constant state are defined by

$$u = u'\,,\; \rho = \rho_0 + \rho'\,,\; p_G = p_{G0} + p'_G\,,\; F = F'\,,\; p_R = p_{R0} + p'_R\,, \qquad (14)$$

so that

$$T' = T - T_0 = T_0 \left(\frac{p'_G}{p_{G0}} - \frac{\rho'}{\rho_0} \right),\; E_R' = E_R - E_{R0} = 3\, p_R'\,. \qquad (15)$$

Substituting (14) and (15) in (4), (6), (7), (8), (10), (11), (12) and retaining only the first order terms in small quantities u', ρ', ... etc., we obtain :

$$\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial u'}{\partial x} = 0\,, \qquad (16)$$

$$\rho_0 \frac{\partial u'}{\partial t} + \frac{\partial}{\partial x}(p'_G + p'_R) - \frac{4\mu}{3}\frac{\partial^2 u'}{\partial x^2} = 0\,, \qquad (17)$$

$$\frac{\partial}{\partial t}\left\{ (p'_G + p'_R) + (3\gamma - 4)\, p'_R \right\} - \frac{\gamma p_{G0} + 4(\gamma - 1) p_{R0}}{\rho_0} \cdot \frac{\partial \rho'}{\partial t}$$
$$+ (\gamma - 1)\frac{\partial F'}{\partial x} - \frac{K(\gamma - 1)}{R\rho_0}\left(\frac{\partial^2 p'_G}{\partial x^2} - \frac{p_{G0}}{\rho_0}\frac{\partial^2 \rho'}{\partial x^2} \right) = 0\,, \qquad (18)$$

$$\frac{\partial^2 F'}{\partial x^2} - \frac{3}{c^2}\frac{\partial^2 F'}{\partial t^2} = \frac{16\sigma\alpha T_0^3}{R\rho_0}\left(\frac{\partial p'_G}{\partial x} - \frac{p_{G0}}{\rho_0} \cdot \frac{\partial \rho'}{\partial x} \right) + \frac{6\alpha}{c} \cdot \frac{\partial F'}{\partial t} + 3\alpha^2 F'\,, \qquad (19)$$

$$\frac{1}{c}\frac{\partial F'}{\partial t} + c\frac{\partial p'_R}{\partial x} = -\alpha F'\,. \qquad (20)$$

From (17) we can assume that there exists a velocity potential ϕ such that :

$$u' = \frac{\partial \phi}{\partial x}\,,\; p'_G + p'_R = \left(-\rho_0 \frac{\partial \phi}{\partial t} + \frac{4\mu}{3}\frac{\partial^2 \phi}{\partial x^2} \right), \qquad (21)$$

From (16), we get

$$\frac{\partial \rho'}{\partial t} = -\rho_0 \frac{\partial^2 \phi}{\partial x^2}, \tag{22}$$

Substituting (21) and (22) in (18), (19) and (20) and eliminating F' and p'_R from three equations, we get :

$$\left[\frac{4\mu K(\gamma-1)}{R\rho_0}\left(\frac{\partial^2}{\partial t^2}-\frac{c^2}{3}\frac{\partial^2}{\partial x^2}\right)\frac{\partial^5\phi}{\partial x^4\,\partial t}\right]+\left[\frac{-3K(\gamma-1)}{Rc^2}\left(\frac{\partial^2}{\partial t^2}-\alpha_T^2\frac{\partial^2}{\partial x^2}\right)\right.$$

$$\left(\frac{\partial^2}{\partial t^2}-\frac{c^2}{3}\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2\phi}{\partial x^2}-\frac{4\mu}{c^2}\left\{\frac{\partial^2}{\partial t^2}-\left(\frac{\dfrac{R\rho_0}{3K(\gamma-1)}+\dfrac{2\alpha}{c}}{\dfrac{R\rho_0}{K(\gamma-1)\,c^2}}\right)\frac{\partial^2}{\partial x^2}\right\}\left.\frac{\partial^4\phi}{\partial x^2\,\partial t^2}\right]$$

$$+\left[\frac{3\rho_0}{c^2}\left(\frac{\partial^2}{\partial t^2}-\frac{c^2}{3}\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2}{\partial t^2}-a_5^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial\phi}{\partial t}-\frac{4\mu a_1^2}{c}\left(\frac{\partial^2}{\partial t^2}-\frac{c^2}{3}\frac{\partial^2}{\partial x^2}\right)\frac{\partial^3\phi}{\partial x^2\partial t}\right.$$

$$\left.-\frac{8\mu\alpha}{c}\left(\frac{\partial^2}{\partial t^2}-\frac{2\mu\alpha cK(\gamma-1)+3\,\alpha_T^2\,\rho_0\,K(\gamma-1)}{4\mu R\rho_0}\cdot\frac{\partial^2}{\partial x^2}\right)\frac{\partial^3\phi}{\partial x^2\partial t}\right]$$

$$+\left[a_1^2\,\rho_0\left(\frac{3\gamma-4}{(\gamma-1)c^2}\cdot\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2}{\partial t^2}-\alpha_T^2\frac{\partial^2}{\partial x^2}\right)\phi\right.$$

$$-\frac{3\alpha^2K(\gamma-1)}{R}\cdot\left\{1+\frac{R}{K(\gamma-1)}\cdot\frac{4\mu}{3}\left(1+\frac{a_1^2}{c\alpha}\right)\right\}$$

$$\times\left(\frac{\partial^2}{\partial t^2}-\frac{a_T^2}{1+\dfrac{R}{K(\gamma-1)}\cdot\dfrac{4\mu}{3}\left(1+\dfrac{a_1^2}{c\alpha}\right)}\cdot\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2\phi}{\partial x^2}$$

$$\left.+\rho_0\left(\frac{6\alpha}{c}+\frac{a_1^2}{(\gamma-1)c^2}\right)\left(\frac{\partial^2}{\partial t^2}-a_5^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2\phi}{\partial t^2}\right]$$

$$+\left[3\rho_0\left(\alpha^2+\frac{a_1^2\alpha}{c}\right)\left(\frac{\partial^2}{\partial t^2}-a_s^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial\phi}{\partial t}\right]=0, \tag{23}$$

where

$a_T^2 = \dfrac{p_{G0}}{\rho_0}$ is an isothermal sound speed ,

$$a_5^2 = \frac{\gamma p_{G0} + 4(\gamma-1)\,p_{R0}}{\rho_0},$$

$$a_1^2 = \frac{16\,\sigma\,\alpha\,T_0^3(\gamma-1)}{R\rho_0},$$

$$\beta = \frac{p_{G0}}{p_{G0}+p_{R0}},$$

and

$$a_s^2 = \frac{a_5^2\left(3\alpha^2+\dfrac{a_1^2\alpha}{c(\gamma-1)}\right)+a_T^2\cdot\dfrac{a_1^2\alpha}{c(\gamma-1)}\cdot(3\gamma-4)}{3\left(\alpha^2+\dfrac{a_1^2\alpha}{c}\right)}$$

is the isentropic sound velocity.

The left hand side of equation (23) is grouped by five square brackets. Each bracket contains a linear homogeneous differential operator with constant co-efficients in x and t and the orders of these operators are seven, six, five, four and three. If we denote these operators by P_7, P_6, P_5, P_4 and P_3, we can write (23) as

$$P_7\{\phi\} + P_6\{\phi\} + P_5\{\phi\} + P_4\{\phi\} + P_3\{\phi\} = 0, \tag{24}$$

We shall define the solutions $\{\phi\}$ satisfying $P_n(\phi) = 0$ ($n = 1, 2, \ldots\ldots, 6, 7$) as nth order wave.

Let us take $L = \dfrac{1}{\alpha}$, called the mean free path of radiation, as a characteristic length in the Flow field and we define the non-dimensional quantities :

$$\bar{x} = \frac{x}{L} = \alpha x, \quad \bar{t} = \frac{a_T t}{L} = a_T t\alpha, \quad \bar{a}_5^2 = \frac{a_5^2}{a_T^2}, \quad \bar{a}_s^2 = \frac{\alpha_s^2}{a_T^2};$$

$$\bar{c} = \frac{c}{a_T}, \quad \bar{a}_1^2 = \frac{a_1^2}{a_T\ \alpha} \text{ and } \bar{\phi} = \frac{\alpha\phi}{a_T}, \tag{25}$$

We also introduce non-dimensional parameters Prandtl number P_r and Reynold number R_e as :

$$P_r = \frac{\mu R\gamma}{K(\gamma - 1)}, \quad R_e = \frac{a_T \rho_0}{\alpha\mu}. \tag{26}$$

Substituting (25) and (26) in equation (23) we get :

$$L_1 D^4 D' \bar{\phi} - \left\{ \frac{3}{4} R_e L_1 L_2 D^2 \bar{\phi} + \frac{R_e P_r}{\gamma}\left(D'^2 - \left(\frac{\bar{c}^2}{3} + \frac{2\bar{c}\gamma}{R_e P_r}\right) D^2 \right) D^2 D'^2 \bar{\phi} \right\}$$

$$+ \left\{ \frac{3 R_e^2 P_r}{4\gamma} L_1 L_3 D' \bar{\phi} - \frac{P_r \bar{a}_1^2 R_e}{\gamma} L_1 D^2 D' \bar{\phi} - \frac{2 P_r \bar{c} R_e}{\gamma} \right.$$

$$\left. \left(D'^2 - \frac{\gamma\left(3 + \frac{2\bar{c}}{R_e}\right)}{4 P_r} D^2 \right) D^2 D' \bar{\phi} \right\} + \left\{ \frac{R_e^2 P_r \bar{a}_1^2 \bar{c}^2}{4\gamma} \right.$$

$$\left(\frac{3\gamma - 4}{(\gamma - 1)\bar{c}^2} D'^2 - D^2 \right) L_2 \bar{\phi} + \frac{R_e^2 P_r \bar{c}^2}{4\gamma}\left(\frac{6}{\bar{c}} + \frac{\bar{a}_1^2}{(\gamma - 1)\bar{c}^2} \right) L_3 D'^2 \bar{\phi}$$

$$\left. - \frac{3R_e \bar{c}^2}{4\sigma}\left(1 + \frac{4}{3}\frac{P_r}{\gamma}\left(1 + \frac{\bar{a}_1^2}{\bar{c}}\right)\right)\left(D'^2 - \frac{1}{1 + \frac{4}{3}\frac{P_r}{\gamma}\left(1 + \frac{\bar{a}_1^2}{\bar{c}}\right)} D^2 \right) D^2 \bar{\phi} \right\}$$

$$+ \left\{ \frac{3R_e^2 P_r \bar{c}^2}{4\gamma}\left(1 + \frac{\bar{a}_1^2}{\bar{c}}\right)\left(D'^2 - \bar{a}_s^2 D^2 \right) D' \bar{\phi} \right\} = 0, \tag{27}$$

where

$$L_1 \equiv D'^2 - \frac{\bar{c}^2}{3} D^2; \quad L_2 \equiv D'^2 - D^2; \quad L_3 \equiv D'^2 - \bar{a}_s^2 D^2$$

$$D' \equiv \frac{\partial}{\partial t}, \quad D \equiv \frac{\partial}{\partial x}.$$

TABLE 1

WAVE VELOCITY AND DAMPING DISTANCE VERSUS FREQUENCY

$\beta = 0.9,\ R_e = 10^4,\ T = 10^7.\ \bar{C} = 0.7351 \times 10^3,\ \bar{a}_s^2 = 1.73639;\ a_5^2 = 1.962963,\ \bar{a}_1^2 = 0\cdot6934218 \times 10^3$

$\bar{\omega}$	$\bar{k}_{1R}$	$\bar{k}_{1I}$	$\frac{\bar{\omega}}{\bar{k}_{1R}}$	$\bar{k}_{2R}$	$\bar{k}_{2I}$	$\frac{\bar{\omega}}{\bar{k}_{2R}}$	$\bar{k}_{3R}$	$\bar{k}_{3I}$	$-\frac{\bar{\omega}}{\bar{k}_{3I}}$
$Lt\ \bar{\omega} \to 0$	0	$-0\cdot198\times10^4$	$0\cdot336\times10^0$	0	0	0	0	0	$\bar{a}_s = 0\cdot131872\times10^1$
10^{-4}	$0\cdot297\times10^{-3}$	$-0\cdot198\times10^4$	$0\cdot337\times10^0$	$0\cdot923\times10^{-3}$	$-0\cdot920\times10^{-3}$	$0\cdot108\times10^0$	$0\cdot714\times10^{-4}$	$-0\cdot102\times10^{-6}$	$0\cdot140\times10^1$
10^{-3}	$0\cdot297\times10^{-2}$	$-0\cdot198\times10^4$	$0\cdot337\times10^0$	$0\cdot296\times10^{-2}$	$-0\cdot287\times10^{-2}$	$0\cdot338\times10^0$	$0\cdot718\times10^{-3}$	$-0\cdot121\times10^{-3}$	$0\cdot139\times10^1$
10^{-2}	$0\cdot297\times10^{-2}$	$-0\cdot198\times10^4$	$0\cdot337\times10^0$	$0\cdot889\times10^{-2}$	$-0\cdot866\times10^{-2}$	$0\cdot112\times10^1$	$0\cdot724\times10^{-1}$	$-0\cdot112\times10^{-2}$	$0\cdot138\times10^0$
10^{-1}	$0\cdot297\times10^0$	$-0\cdot198\times10^4$	$0\cdot337\times10^0$	$0\cdot997\times10^{-1}$	$-0\cdot418\times10^{-2}$	$0\cdot1\times10^1$	$0\cdot217\times10^{-1}$	$-0\cdot199\times10^{-1}$	$0\cdot460\times10^1$
5×10^{-1}	$0\cdot149\times10^1$	$-0\cdot198\times10^4$	$0\cdot336\times10^0$	$0\cdot5\times10^0$	$-0\cdot469\times10^{-2}$	$0\cdot1\times10^1$	$0\cdot469\times10^{-1}$	$-0\cdot462\times10^{-1}$	$0\cdot106\times10^2$
10^0	$0\cdot316\times10^1$	$-0\cdot197\times10^4$	$0\cdot316\times10^0$	$0\cdot998\times10^0$	$-0\cdot995\times10^{-2}$	$0\cdot1\times10^1$	$0\cdot661\times10^{-1}$	$-0\cdot655\times10^{-1}$	$0\cdot151\times10^2$
5×10^0	$0\cdot148\times10^2$	$-0\cdot198\times10^4$	$0\cdot337\times10^0$	$4\cdot9\times10^0$	$-0\cdot248\times10^{-1}$	$0\cdot102\times10^1$	$0\cdot834\times10^{-1}$	$-0\cdot822\times10^{-1}$	$0\cdot599\times10^2$
10^1	$0\cdot297\times10^2$	$-0\cdot198\times10^4$	$0\cdot336\times10^0$	$8\cdot36\times10^0$	$-0\cdot104\times10^0$	$0\cdot119\times10^1$	$0\cdot209\times10^0$	$-0\cdot207\times10^0$	$0\cdot477\times10^2$
10^2	$0\cdot906\times10^2$	$-0\cdot632\times10^4$	$0\cdot110\times10^1$	$7\cdot32\times10^1$	$-0\cdot133\times10^3$	$0\cdot113\times10^1$	$0\cdot676\times10^0$	$-0\cdot638\times10^0$	$0\cdot148\times10^3$
10^3	$0\cdot201\times10^4$	$-0\cdot288\times10^4$	$0\cdot496\times10^0$	$7\cdot46\times10^2$	$-0\cdot136\times10^3$	$0\cdot113\times10^1$	$0\cdot279\times10^1$	$-0\cdot158\times10^1$	$0\cdot358\times10^3$
10^4	$0\cdot657\times10^4$	$-0\cdot809\times10^4$	$0\cdot152\times10^1$	$5\cdot46\times10^3$	$-0\cdot181\times10^4$	$0\cdot181\times10^1$	$0\cdot234\times10^1$	$-0\cdot327\times10^1$	$0\cdot426\times10^4$
$Lt\ \bar{\omega} \to \infty$	∞	∞	∞	∞	∞	∞	∞	$-\sqrt{3} = -1\cdot732$	$\frac{\bar{c}}{\sqrt{3}} = 0\cdot42445\times10^3$

Substituting $$\bar{\phi} = e^{i(\bar{\omega}\bar{t} - \bar{k}\bar{x})} \tag{28}$$

in (27), we get the dispersion relation as :

$$\bar{k}^6\left(\frac{\bar{c}^2}{4}R_e + \bar{i\omega}\frac{\bar{c}^2}{3}\right) + \bar{k}^4\left[\left\{-\frac{3}{4}R_e\left(1+\frac{\bar{c}^2}{3}\bar{\omega}^2\right) - \frac{3}{5}R_e\left(\frac{\bar{c}^2}{3}+\frac{10}{3}\frac{\bar{c}}{R_e}\right)\bar{\omega}^2\right.\right.$$

$$\left.\left. + \frac{3}{4}R_e\bar{c}^2 + \frac{3}{20}R_e^2\bar{a}_1^2\bar{c}^2\right\} + i\left\{-\bar{\omega}^3 + \bar{\omega}\left(\frac{3}{20}R_e^2\bar{c}^2\bar{a}_5^2 + \frac{\bar{a}_1^2\bar{c}^2R_e}{5} + \frac{3}{2}\bar{c}R_e + \bar{c}^2\right)\right\}\right]$$

$$+\bar{k}^2\left[\left\{\frac{27}{20}R_e\bar{\omega}^4 - \frac{3}{20}R_e^2\bar{a}_1^2\bar{c}^2\left(1+\frac{3}{2\bar{c}^2}\right)\bar{\omega}^2 - \frac{9}{10}R_e^2\bar{c}^2\,\bar{a}_5^2\bar{\omega}^2\right.\right.$$

$$\left. - \frac{9}{40}R_e^2\bar{a}_1^2a_5^2\bar{\omega}^2 - \frac{27}{20}R_e\bar{c}^2\,\bar{\omega}^2 - \frac{3}{5}R_e\bar{c}\,\bar{a}_1^2\,\bar{\omega}^2\right\}$$

$$+ i\left\{-\left(\frac{3}{20}R_e^2\bar{c}^2 + \frac{9}{20}R_e^2\bar{a}_5^2 + \frac{3}{5}R_e\;\bar{a}_1^2 + \frac{6}{5}\;\bar{c}R_e\right)\bar{\omega}^3\right.$$

$$\left.\left. + \bar{\omega}\,\bar{a}_s^2\cdot\frac{9}{20}R_e^2\bar{c}^2\left(1+\frac{\bar{a}_1^2}{\bar{c}}\right)\right\}\right] + \left[\left(\frac{9}{20}R_e^2\bar{a}_1^2 + \frac{9}{10}R_e^2\bar{c}\right)\bar{\omega}^4\right.$$

$$\left. + i\left\{\frac{9}{20}R_e^2\bar{\omega} - \frac{9}{20}R_e^2\bar{c}^2\left(1+\frac{\bar{a}_1^2}{\bar{c}}\right)\bar{\omega}^3\right\}\right] = 0\,. \tag{29}$$

Equation (29) is a dispersion relation in non-dimensional wave-number $\bar{k}$ and non-dimensional frequency $\bar{\omega}$. For a given value of $\bar{\omega}$, equation (29) is a sixth degree polynomial in $\bar{k}$. The roots of this polynomial equation are of the form $\pm\bar{k}_1, \pm\bar{k}_2$ and $\pm\bar{k}_3$, where $\bar{k}_1, \bar{k}_2, \bar{k}_3$ are complex quantities. The positive and negative signs before each value suggest the possibility of propagation in positive and negative directions of x-axis and the numerical values (see Table 1) and asymptotic values for small $\bar{\omega}$ and large $\bar{\omega}$ of $\bar{k}$ are such that real and imaginary parts of any of these six roots have opposite signs so that whenever we follow any wave (propagating in positive or negative direction of x-axis) its amplitude always decreases. Thus, in general, three distinct modes of propagation are possible. In the absence of heat-conduction and viscosity, we have only two distinct modes of propagation viz. (*i*) radiation induced waves, (*ii*) modified gas-dynamic waves. The three distinct modes of propagation are (*i*) heat-conduction and viscosity induced waves, (*ii*) radiation induced waves and (*iii*) modified gas-dynamic waves. If we take $R_e \to \infty$ keeping P_r to be constant then we find that equation (29) reduces to the equation of Prasad[3].

If $\bar{k} = \bar{k}_R + i\bar{k}_I$, then the velocity of propagation is $\dfrac{\bar{\omega}}{\bar{k}_R}$ and damping distance is $\dfrac{1}{|\bar{k}_I|}$

RESULTS AND DISCUSSIONS

Here we are solving a dispersion relation (29) for various values of $\bar{\omega}$. We will get three distinct values of wave numbers $\bar{k}_1, \bar{k}_2, \bar{k}_3$ as a function of $\bar{\omega}$ corresponding to three distinct modes of propagation. A numerical result is obtained by using a method of Van A. Mcauley[4]. The results are given in Table 1.

For very small $\bar{\omega}$ and for very large $\bar{\omega}$ equation (29) is solved asymptotically for $\bar{k}$ as a function of $\bar{\omega}$.

Case (*i*)—When $\bar{\omega}$ is very small

Substituting

$$\bar{k}^2 = x_0 + x_1 \bar{\omega} + 0(\bar{\omega}^2) \tag{30}$$

in (29), we get

$$x_0 = 0,\ 0,\ -\left(3 + \frac{3}{5} R_e\, \bar{a}_1^2\right)$$

and for $x_0 \neq 0$, we get

$$x_1 = -i\left[3\left(\frac{R_e\, \bar{a}_5^2}{5} + \frac{2}{\bar{c}}\right) + \frac{9\, R_e}{5\, \bar{c}^2\left(3 + \frac{3}{5} R_e\, \bar{a}_1^2\right)^2} - \frac{9\, R_e\, \bar{a}_5^2\left(1 + \frac{\bar{a}_1^2}{\bar{c}}\right)}{5\left(3 + \frac{3}{5} R_e\, \bar{a}_1^2\right)}\right]$$

$$\therefore\ \bar{k}_1 = \pm\left[-\frac{\bar{\omega}}{\left(3 + \frac{3}{5} R_e\, \bar{a}_1^2\right)^{\frac{1}{2}}}\left\{\frac{3}{2}\left(\frac{R_e\, \bar{a}_5^2}{2} + \frac{2}{\bar{c}}\right) + \frac{27}{20}\, \frac{R_e}{\bar{c}^2\left(3 + \frac{3}{5} R_e\, \bar{a}_1^2\right)^2}\right.\right.$$

$$\left.\left. - \frac{27}{20}\, \frac{R_e\, \bar{a}_5^2\left(1 + \frac{\bar{a}_1^2}{\bar{c}}\right)}{\left(3 + \frac{3}{5} R_e\, \bar{a}_1^2\right)}\right\} + i\left(3 + \frac{3}{5} R_e\, \bar{a}_1^2\right)^{\frac{1}{2}}\right] \tag{31}$$

Thus $\bar{k}_{1R} = A\bar{\omega}$, where A is a constant depending on R_e, $\bar{a}_1^2$, ..., and $\bar{k}_{1I} = \left(3 + \frac{3}{5} R_e\, \bar{a}_1^2\right)$ and is independent of $\bar{\omega}$.

Thus damping distance for first mode of propagation is independent of $\bar{\omega}$ and wave velocity A is also independent of $\bar{\omega}$ upto the first order in $\bar{\omega}$. This mode of propagation is due to heat-conduction and viscosity.

When $x_0 = 0$, we will get two remaining modes by assuming that

$$\left.\begin{aligned} \bar{k}_2^2 &= x_1 \bar{\omega} + 0(\bar{\omega}^2) \\ \text{and}\quad \bar{k}_3^2 &= x_1 \bar{\omega}^2 + 0(\bar{\omega}^3) \end{aligned}\right\} \tag{32}$$

and substituting in (29) and equating coefficient of lowest power of $\bar{\omega}$ to zero.

$$\therefore\ \bar{k}_2^2 = -i\,\bar{\omega}\,.\,\frac{\frac{3}{5} R_e\, \bar{a}_5^2\left(1 + \frac{\bar{a}_1^2}{c}\right)}{1 + R_e\,.\,\frac{\bar{a}_1^2}{5}} \tag{33}$$

and

$$\bar{k}_3^2 = \frac{\bar{\varpi}^2}{\bar{a}_s^2}. \tag{34}$$

From (33), $\bar{k}_2$ is a complex number of type :

$$\bar{k}_2 = \pm (a\sqrt{\varpi} - i\, a\sqrt{\varpi}), \tag{35}$$

where a is some numerical constant which can be obtained by putting numerical values of $\bar{a}_5^2$, R_e, $\bar{a}_1^2$, $\bar{c}$. The velocity $\bar{v}_2$ of second mode of propagation is

$$\bar{v}_2 = \frac{\bar{\varpi}}{\bar{k}_{2R}} = \frac{\bar{\varpi}}{a\sqrt{\bar{\varpi}}} = \frac{\sqrt{\bar{\varpi}}}{a} \to 0 \text{ as } \bar{\varpi} \to 0.$$

Damping distance $= \dfrac{1}{a\sqrt{\bar{\varpi}}} \to \infty$ as $\bar{\varpi} \to 0$.

Hence there is no damping to a wave corresponding to second mode of propagation and wave velocity is zero.

From (34) we have

$$\bar{k}_3 = \pm \frac{\bar{\varpi}}{\bar{a}_s}. \tag{36}$$

Hence there is no damping to third mode of propagation when $\bar{\varpi}$ is very small, and

$$v_3 = \frac{\bar{w}}{\bar{k}_{3R}} = \bar{a}_s$$

Thus a wave corresponding to third mode of propagation travels with isentropic sound velocity.

Case (*ii*)—When $\bar{\varpi}$ is very large

Putting

$$\bar{k}_1^2 = x_1 \bar{\varpi} \tag{37}$$

in dispersion relation (29) and equating coefficient of highest power of $\bar{\varpi}$ to zero, we get

$$\left.\begin{aligned} \bar{k}_1 &= \pm (a\sqrt{\bar{\omega}} - ia\sqrt{\bar{\omega}}) \\ \text{and} \quad \bar{k}_2 &= \pm (b\sqrt{\bar{\omega}} - ib\sqrt{\bar{\omega}}) \end{aligned}\right\}. \tag{38}$$

These are two distinct wave numbers corresponding to two distinct modes of propagation. In both these modes of propagation

$$\bar{v}_1 = \frac{\bar{\omega}}{\bar{k}_{1R}} = \frac{\sqrt{\bar{\omega}}}{a} \text{ and } \bar{v}_2 = \frac{\bar{\omega}}{\bar{k}_{2R}} = \frac{\sqrt{\bar{\omega}}}{b}.$$

$$\therefore\ \bar{v}_1 \to \infty \text{ and } \bar{v}_2 \to \infty \text{ as } \bar{\omega} \to \infty .$$

Damping distance for two modes of propagation is $\dfrac{1}{a\sqrt{\bar{\varpi}}}$ and $\dfrac{1}{b\sqrt{\bar{\varpi}}}$.

Damping distance tends to zero as $\bar{\omega} \to \infty$.

To obtain third mode of propagation for very large value of $\bar{\omega}$, let us put

$$\bar{k}^2 = \frac{3}{\bar{c}^2} \bar{\omega}^2 + \bar{k}_1 \bar{\omega} \tag{39}$$

in dispersion relation (29). The coefficient of $\bar{w}^7$ becomes zero and coefficient of $\bar{w}^6$ equated to zero gives us

$$\bar{k}_1 = - i \frac{6}{\bar{c}} .$$

$$\therefore \; \bar{k}^2 = \frac{3}{\bar{c}^2} \; \bar{\omega}^2 \left(1 - i \frac{2\bar{c}}{\bar{\omega}} \right)$$

$$\therefore \; k_R = \frac{\sqrt{3}}{\bar{c}} \, \bar{\omega} \text{ and } \bar{k_I} = - \sqrt{3}$$

$$\therefore \; \bar{v}_3 = \frac{\bar{\omega}}{\bar{k}_R} = \frac{\bar{c}}{\sqrt{3}}$$

is the limiting value of the velocity of third mode of propagation. The corresponding damping distance for this mode of propagation is

$$\frac{1}{|\bar{k}_I|} = \frac{1}{\sqrt{3}}.$$

In Table 1 we have shown the variation of wave-velocity and damping for three distinct modes of propagation for $R_e = 10^4$ and various values of $\bar{\omega}$.

We note the following points from Table 1 and limiting cases :

(*i*) The waves corresponding to the roots $\bar{k}_2$ and $\bar{k}_3$ for $\bar{\omega} \to 0$ propagate with the characteristic speeds of the third order wave operator. The damping of the two waves for $\bar{\omega} \to 0$ is zero.

(*ii*) The high frequency waves ($\bar{\omega} \to \infty$) propagate with characteristic speeds of the seventh order operator.

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