SOLUTION OF THE EQUATIONS OF INTERNAL BALLISTICS FOR THE NON-HOMOGENEOUS LINEAR LAW OF BURNING

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In this paper, the fundamental differential equation of internal ballistics for the non-homogeneous linear law of burning for tubular and cord charges has been solved numerically and results presented graphically. The results for the usual homogeneous linear law of burning have been deduced as a particular case. Tables for maximum pressure and position at all burnt have been prepared. Finally conclusions have been drawn to show the effect of non-homogeneous term in the law of burning, on the maximum pressure and on the all burnt position.

The law of burning, which is in good agreement with the observed data is $r = \beta\,(p+p_1)$ where the positive constants β and p_1 depend upon the composition of the propellant and the initial temperature of firing. This law was first considered by Mansell and it was suggested by him that it represents more accurately the physical situation inside guns than the pressure-index law or the homogeneous law. But his only regret was that inspite of a great deal of mathematical ingenuity employed, the law was not amenable to analytic treatment. Corner[1] stated that this law was nearer to truth than the power law, but as it was inconvenient in interior ballistics, no ballistic theory used this law.

Kapur[2] and Kapur & Srivastava[3], discussed the ballistic theory with the above law and showed in particular that the solution of the equations could be reduced to the solution of the non-linear second order differential equations.

In the present paper, we have solved numerically the fundamental differential equation[2] of internal ballistics for the non-homogeneous linear law of burning for non-zero shot-start pressure.

DIFFERENTIAL EQUATIONS AND THEIR SOLUTION

The numerical solution of the differential equation of Kapur's first method for the non-homogeneous general linear law of burning is as follows :

Equation (25) of Kapur[2] is

$$\left[\frac{Y}{\zeta}\right]^{\frac{\gamma-1}{\gamma}}\left[1-q\,Y^{\frac{1}{\gamma}}\,\zeta^{\frac{\gamma-1}{\gamma}}\right]\times\left[-Y\zeta\zeta''(\zeta+\zeta_1)+Y\zeta'^2(\zeta+2\zeta_1)-\zeta\zeta'(\zeta+2\zeta_1)\right]$$

$$=\left[\frac{M\gamma\zeta'}{\nu^2(\zeta+\zeta_1)}+\frac{1}{2}\,q\,\zeta\,(\zeta+\zeta_1)(\zeta-Y\zeta')\right]$$

$$\times\left[1+\frac{1}{2}(\gamma-1)\,\frac{q}{M}\,\nu^2\,\frac{(\zeta+\zeta_1)^2(\zeta-Y\zeta')^2}{\gamma^2\zeta^4}\right] \qquad (1)$$

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For a tubular charge

$$\theta = 0, \quad \text{i.e. } z = 1 - f, \quad \frac{dz}{df} = -1, \quad \nu = 1, \quad q = 0,$$

so that (1) becomes

$$\left[\frac{Y}{\zeta}\right]^{\frac{\gamma-1}{\gamma}} \left[-Y\zeta\zeta''(\zeta+\zeta_1) + Y\zeta'^2(\zeta+2\zeta_1) - \zeta\zeta'(\zeta+2\zeta_1) \right] = \frac{M\gamma\zeta'}{\zeta+\zeta_1} \quad (2)$$

For a cord charge

$$z = 1 - f^2$$

$$\theta = 1 \text{ and } \frac{dz}{df} = -2f$$

$$\nu = 2 \text{ and } q = 1$$

so that (1) becomes

$$\left[\frac{Y}{\zeta}\right]^{\frac{\gamma-1}{\gamma}} \left[1 - Y^{\frac{1}{\gamma}} \zeta^{\frac{\gamma-1}{\gamma}}\right] \times \left[-Y\zeta\zeta''(\zeta+\zeta_1) + Y\zeta'^2(\zeta+2\zeta_1) - \zeta\zeta'(\zeta+2\zeta_1) \right]$$

$$= \left[\frac{M\gamma\zeta'}{4(\zeta+\zeta_1)} + \frac{1}{2}\zeta(\zeta+\zeta_1)(\zeta - Y\zeta') \right]$$

$$\times \left[1 + \frac{2(\gamma-1)}{M} \cdot \frac{(\zeta+\zeta_1)^2(\zeta - Y\zeta')^2}{\gamma^2\zeta^4} \right] \quad (3)$$

Equations (2) and (3) are the non-linear differential equations of the second order the numerical integration of which under suitable initial condition will determine the motion.

Initial conditions (at shot-start) for the integration of (2) and (3) are

$$\xi = 1, \quad \eta = 0, \quad \zeta = \zeta_0, \quad Y = \zeta_0 \text{ and } \zeta' = 1 \quad (4)$$

Solution of the equations

By numerical integration of equations (2) and (3) subject to initial conditions (4), we can evaluate ζ and ζ' as function of Y.

Let $\zeta = P(Y), \quad \zeta' = Q(Y)$

From equation (22) of Kapur & Srivastava[3] we have

$$\xi = \left(\frac{Y}{\zeta}\right)^{\frac{1}{\gamma}} = \left[\frac{Y}{P(Y)}\right]^{\frac{1}{\gamma}} = R(Y) \quad (5)$$

This determines ξ as a function of Y.

From equation (16) of Kapur & Srivastava[3], we have

$$\eta^2 = \frac{2M}{\gamma-1}\left(z - \zeta\xi\right) \quad (6)$$

Now from equations (21) and (22) of Kapur & Srivastava[3]

$$\frac{dY}{dz} = \xi^{\gamma-1} = \left(\frac{Y}{\zeta}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad z - z_0 = \int_{\zeta_0}^{\gamma} \left(\frac{\zeta}{Y}\right)^{\frac{\gamma-1}{\gamma}} dY$$

$$= \int_{\zeta_0}^{\gamma} \left(\frac{P(Y)}{Y}\right)^{\frac{\gamma-1}{\gamma}} dY = S(Y)\,(\text{say}) \tag{7}$$

Substituting it in (6) we have

$$\eta = \left[\frac{2M}{\gamma - 1} \left\{ z_0 + S(Y) - P(Y) R(Y) \right\} \right]^{1/2} = T(Y) \tag{8}$$

This determines the velocity at any instant during burning.

The numerical integration of (2) and (3) is to be continued till $S(Y) = 1 - \zeta_0$. If $Q(Y)$ vanishes at $Y = Y_1$ then $P(Y_1)$, $R(Y_1)$ and $T(Y_1)$ determine the maximum pressure, shot-travel upto the instant of maximum pressure and velocity at this instant. Again, let $S(Y_2) = 1 - \zeta_0$ then $Y = Y_2$ and $P(Y_2)$, $R(Y_2)$ and $T(Y_2)$ determine the pressure, shot-travel, and shot velocity at all-burnt and the muzzle velocity η_3 is given by

$$T(Y_3) = \eta_3 = \left[\frac{2M}{\gamma - 1} \left\{ 1 - P(Y_2) R(Y_2)^{\gamma} R(Y_3) \right\} \right]^{1/2} \tag{9}$$

where $R(Y_3)$ is ζ i.e. shot-travel upto the muzzle or the length of the barrel.

Equation (2) and (3) have been solved for $\gamma = 1\cdot25$ and

$M = 2; \zeta_0 = 0\cdot05, \quad \zeta_1 = 0\cdot00,\ 0\cdot02,\ 0\cdot04,\ 0\cdot06,\ 0\cdot08,\ 0\cdot10$

$M = 2; \zeta_0 = 0\cdot1, \quad \zeta_1 = 0\cdot00,\ 0\cdot02,\ 0\cdot04,\ 0\cdot06,\ 0\cdot08,\ 0\cdot10$

The equations have been solved on IBM 1620. Rung-Kutta method is used to solve the differential equations at an interval of 0·01, but for integration Weddle's rule is applied. The programming is in F-II language and calculations upto fourteen significant decimal places have been considered. The numerical results of calculations were obtained in 31 tables. The results are shown in the Fig. 1–4.

DISCUSSION

All the tables, available with the authors, are for variables in a dimensionless form. Using these, pressure, velocity and shot-travel can immediately be found out on multiplication by suitable scale factors.

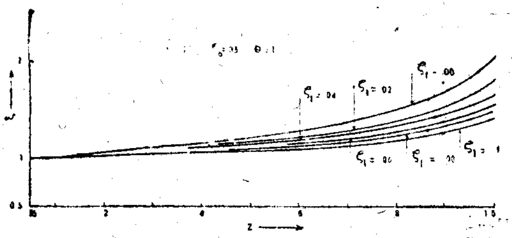

Fig. 1—Relation between Z and ξ for some values of ζ_1.

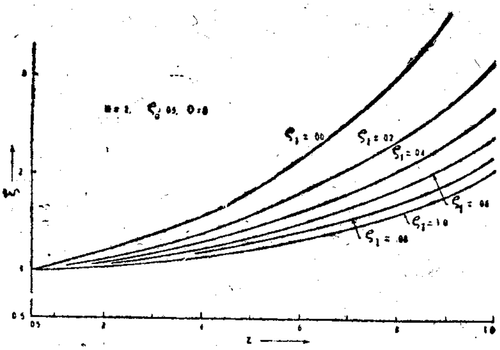

Fig. 3—Relation between Z and ξ for some values of ζ_1

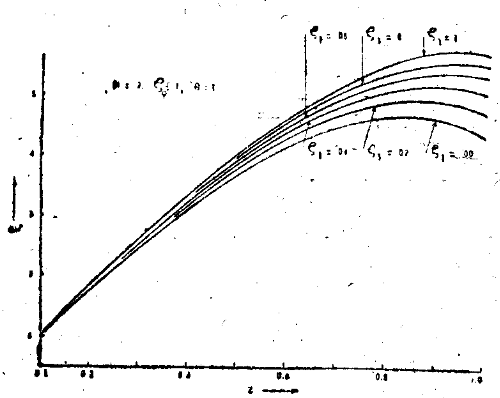

Fig. 2—Relation between Z and ξ for some values of ζ_1.

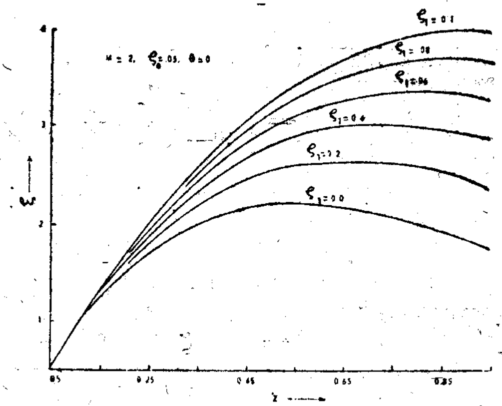

Fig. 4—Relation between Z and ξ for some values of ζ_1.

Following are some points of interest :

(*a*) *At all-burnt*—As the value of ζ_0 increases i.e. from homogeneous linear law to general linear law, shot-travel and velocity decrease, but pressure increases. As compared to the cord charge, the variation is more marked in the case of tubular charge. The effect of shot-start pressure is to decrease the shot-travel and velocity, but to increase the pressure.

(*b*) *Maximum pressure*—The maximum pressure is greater in case of powders which obey general linear law than in those of homogeneous linear law i.e. as ζ_1 increases, the value of maximum pressure also increases. The effect of shot-start pressure is to increases the value of maximum pressure.

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