# ERRORS DUE TO MISALIGNMENT OF ROSETTES

## I. K. NARANG, S.S. SIVARAMAN & G. R. PRASAD

Instruments Research and Development Establishment, Dehradun

(Received 22 Nov. 67; Revised 29 March 68)

This paper presents an investigation of errors that are likely to creep in the evaluation of principal stresses when misaligned rosettes are used. It has been found that if the gauges forming the rosette are having orientation errors of the order of  $+2^{\circ}$ , the error in the evaluation of principal stresses by using normal rosette equations will be about 1 to 3 percent.

## GENERAL ROSETTE EQUATIONS

In any plane stress problem where a biaxial stress field exists with the directions of principal stresses unknown, three strain measurements are necessary to evaluate the principal stresses. It can (asily be shown! that strain measured in any direction  $\phi$  referred to a chosen x-axis is

$$\epsilon_{\phi} = \left(\frac{\epsilon_x + \epsilon_y}{2}\right) + \left(\frac{\epsilon_x - \epsilon_y}{2}\right)\cos 2\phi + \frac{\gamma_{xy}}{2}\sin 2\phi$$
 (1)

where

 $\epsilon_{\phi}$  = strain measured in any direction  $\phi$  (See Fig. 1)

 $\epsilon_x$ ,  $\epsilon_y$  = normal strains in the x, y directions respectively, and  $\gamma_{xy}$  = shearing strain in the xy plane.

The presence of three unknowns viz.  $\epsilon_x$ ,  $\epsilon_y$  and  $\gamma_{xy}$  necessitates three strain measurements. Normally these measurements are taken using Strain Gauge Rosettes.

Strain gauge rosettes are available commercially in discrete patterns like the Rectangular rosette, Delta rosette and the Tee-Delta rosette (see Fig. 2-4).

When the reference axis viz. x-axis is chosen along gauge 1, these patterns can then be defined by the orientations of the gauges with respect to the x-axis. These definitions are given in Table 1.

Any one of-the rosettes may be used in the m asurement of strain. We will then have strain values in three directions  $\epsilon_{\phi_1}$ ,  $\epsilon_{\phi^2}$  &  $\epsilon_{\phi_3}$  which, when successively substituted in (1) and solved, will give  $\epsilon_x$ ,  $\epsilon_y$  and  $\gamma_{xy}$ , Then the principal strains are obtained from the relations<sup>1</sup>

$$\epsilon_{max} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{1}{2} \sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}$$
 (2)

$$\epsilon_{min} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{1}{2} \sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}$$
 (3)

$$\gamma_{max} = \sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}$$
 (4)

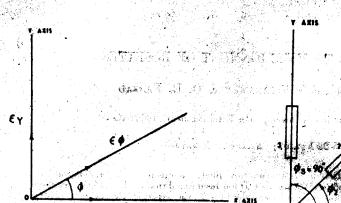
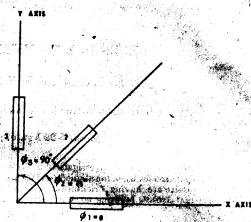
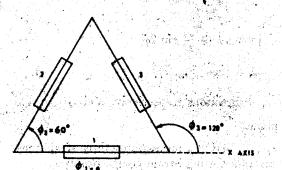
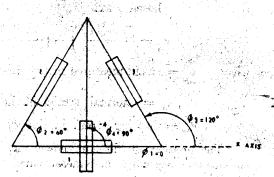


Fig. 1—Rosette Configuration with three gauges Fig. 2—Rectangular Rosette. —general pattern.





karawangi at silabah kili Fig. 3—Delta rosette.



A Gidel at

alkain - waki sa asalar abawa kiji

Fig. 4—Tee-Dolts resette.

TABLE 1

Bird and Charles of the fall o

terribe of a respect to the Energy and many branch are at a residence about the company to the company of The Committee of the contract place has been a self-affective of the contract of the contract of the contract of

	Type o	of rosette			Orien	tations	Ality
				φ,	<b>.</b>	ø,	<b>Ø</b> ₄
	Kectangular			. 0.	45°	90°	<del></del>
	Delta	MX Y.		0°	<b>60</b> °	120°	<del>-</del> %
**************************************	Tee-Delta		1.75.9	09	60°	120°	90*

The principal stresses are then obtained using the relations1.

$$\sigma_{max} = \frac{E}{1 - \mu^2} \left( \epsilon_{max} + \mu \epsilon_{min} \right) \tag{5}$$

$$\sigma_{min} = \frac{E}{1-\mu^2} \left( \epsilon_{min} + \mu \epsilon_{max} \right) \tag{6}$$

$$\tau_{max} = \frac{E}{2(1+\mu)} \gamma_{max} \qquad (7)$$

where E = Young's modulus of the material,

 $\mu =$  Poisson's ratio of the material.

The direction of principal stresses is given by

$$\phi = \frac{1}{2} \tan^{-1} \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} \tag{8}$$

The principal stress equations for rectangular and delta rosette are given in the Table 2.

MODIFIED EQUATIONS WHEN THEREARE ERRORS IN THE ALIGNMENT OF THE ROSETTES

Let us assume that there are errors in the alignment of the gauges forming the rosettes and modify the rosette equations listed in the Table 2 incorporating these errors.

CASE 1—Rectargular Rosette

Since we normally choose the axis of gauge 1 as the reference or the x-axis, the error in the alignment of gauge 1 can be taken as zero. Let  $\alpha$ , and  $\alpha_3$  be the errors in the alignments of gauges 2 and 3 respectively (see Fig. 5). Introducing these angles in the general equation (1) successively we get

$$\epsilon_{\phi_1} = \epsilon_1 = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\phi_1 + \frac{\gamma_{xy}}{2} \sin 2\phi_1$$
 (9)

$$\epsilon \left(\phi_2 + \alpha_2\right) = \epsilon_2 = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cdot \cos 2 \left(\phi_2 + \alpha_2\right) + \frac{\gamma_{xy}}{2} \sin 2(\phi_2 + \alpha_2) \tag{10}$$

$$\epsilon (\phi_3 + \alpha_3) = \epsilon_3 = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cdot \cos 2(\phi_3 + \alpha_3) + \frac{\gamma_{xy}}{2} \sin 2(\phi_3 + \alpha_3)$$
 (11)

for a rectangular rosette  $\phi_1=0^\circ$ ,  $\phi_2=45^\circ$  and  $\phi_3=90^\circ$ . Therefore (9), (10) and

(11) reduce to -

$$\epsilon_1 = \epsilon_x \tag{12}$$

$$\epsilon_2 = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \sin 2a_2 + \frac{\gamma_{xy}}{2} \cos 2a_2 \tag{13}$$

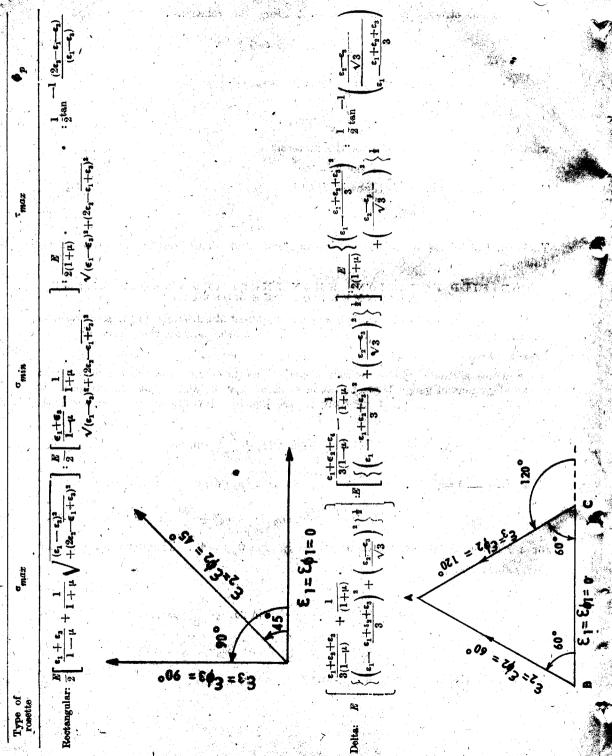
$$\epsilon_3 = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2a_3 - \frac{\gamma_{xy}}{2} \sin 2a_3 \tag{14}$$

Solving (12), (13) and (14) for  $\epsilon_x$ ,  $\epsilon_y$  and  $\gamma_{xy}$ , we get

$$\epsilon_x = \epsilon_1$$
 (15)

$$\epsilon_y = \frac{2\beta - \epsilon_1 A}{2} \tag{16}$$





$$\gamma_{xy} = \underbrace{\left[\frac{2\epsilon_2 - \left\{\frac{\epsilon_1 (B - A) + 2\beta}{B}\right\} + \left\{\frac{\epsilon_1 (B + A) - 2\beta}{B}\right\} \sin 2\alpha_2}{\cos 2\alpha_2}\right]}_{(17)}$$

Where

$$\beta = \epsilon_2 \sin 2a_3 + \epsilon_3 \cos 2a_2$$

$$A = \sin 2a_3 + \cos 2a_2 - \cos 2(a_2 - a_3)$$

$$B = \sin 2a_3 + \cos 2a_2 + \cos 2(a_2 - a_3)$$

With expressions for  $\epsilon_{\omega}$ ,  $\epsilon_{y}$  and  $\gamma_{\omega y}$  as given by equations (15), (16) and (17) we obtain the principal strains using the relationships vide equations (2) and (3). The principal stresses are then calculated using expressions given by equations (5), (6) and (7). The direction of principal stress is given by equation (8) used in conjunction with equations (15), (16) and (17).

## CASE 2—Delta Rosette

For a Delta rosette  $\phi_1=0^\circ$ ,  $\phi_2=60^\circ$  and  $\phi_3=120^\circ$  choose again the x-axis along the axis of gauge 1. (see Fig. 6). Let us assume  $a_2$  and  $a_3$  as the errors in the alignment of gauges 2 and 3 respectively ( $a_2=0$  as before). Proceeding in the same manner as was done for rectangular rosette but substituting  $\phi_1=0^\circ$ ,  $\phi_2=60^\circ$  and  $\phi_3=120^\circ$  we get the expressions for  $\epsilon_x$ ,  $\epsilon_y$  and  $\gamma_{xy}$  as:

$$\epsilon_x = \epsilon_1 \tag{18}$$

$$\epsilon_y = \frac{2\beta - \epsilon_1 A}{B}$$
 (19)

$$\gamma_{xy} = \frac{4}{\delta'} \left[ \frac{\epsilon_1}{2} \left\{ \left( 1 + \frac{A}{B} \right) - \frac{1}{2} \delta \left( 1 + \frac{A}{B} \right) \right\} + \frac{\beta}{B} \left( 1 + \frac{\delta}{2} \right) - \epsilon_3 \right]$$
 (20)

where

$$\begin{split} \beta &= \epsilon_2 \left[ \sqrt{3} \cos 2a_3 + \sin 2a_3 \right] + \epsilon_3 \left[ \sqrt{3} \cos 2a_2 - \sin 2a_2 \right] \\ A &= \sqrt{3} \left[ \cos 2a_3 + \cos 2a_2 - \cos 2 \left( a_2 - a_3 \right) \right] + \left[ \sin 2a_3 - \sin 2a_2 - \sin 2 \left( a_2 - a_3 \right) \right] \\ B &= \sqrt{3} \left[ \cos 2a_3 + \cos 2a_2 + \cos 2 \left( a_2 - a_3 \right) \right] + \left[ \sin 2a_3 - \sin 2a_2 + \sin 2 \left( a_2 - a_3 \right) \right] \end{split}$$

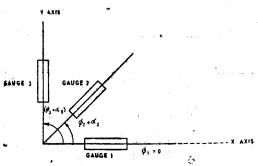


Fig. 5—Rectangular resette with orientation errors,

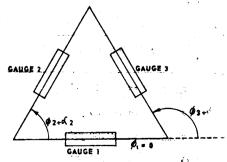


Fig. 6—Delta resette with orientation errors.

$$\delta = (\cos 2a_3 - \sqrt{3} \sin 2a_3)$$
and 
$$\delta' = (\sqrt{3} \cos 2a_3 + \sin 2a_3)$$

Using the expressions for  $\epsilon_x$ ,  $\epsilon_y$  and  $\gamma_{xy}$  as given by equations (18), (19) and (20) the principal strains and stresses are calculated using equations (2), (3) and (5), (6), (7) respectively. The direction of the principal stress is given by equation (8) used in conjunction with equations (18), (19) and (20).

## EXAMPLES

## CASE 1-Rectangular Rosette

(a) Let us assume that a rectangular rosette had been used with alignment errors of +2° in gauges 2 and 3. Let the three measured strains be

$$\epsilon_1 = 2000 \times 10^{-6}$$
 $\epsilon_2 = 500 \times 10^{-6}$ 
 $\epsilon_3 = 1000 \times 10^{-6}$ 
 $\epsilon_3 = \epsilon_3 = 000 \times 10^{-6}$ 

To start with let us calculate  $\beta$ , A & B. For  $a_2 = a_3 = +2^{\circ}$  and for the given strain values in this example

$$\beta = 1032 \cdot 5 \times 10^{-6}$$
 $A = 0.0674$ 
 $B = 2.0674$ 

Then following previous mentioned procedure we get :

$$\sigma_{max} = 87645 \ psi$$
 $\sigma_{min} = 38085 \ psi$ 
 $\tau_{max} = 24780 \ psi$ 

and

$$\phi_p = -30^{\circ} 07'$$

These values can be compared against the values that would have been obtained if the normal rosette equations, as shown in Table 2 were used. Table 3 shows this comparision.

TABLE 3

 From m	odified equations	From normal rosette equations
 max	= 87645 psi	o <sub>max</sub> = 90090 psi
o <sub>min</sub>	= 38085 psi	o <sub>min</sub> = 38490 psi
<sup>T</sup> max	= 24780 psi	τ <sub>max</sub> = 25800 psi
	= -30° 07′	$\phi_p = -31^{\circ} 43'$

(b) We assume an error of -2 in the alignment of guages 2 and 3. As before let the measured strains be  $\epsilon_1 = 2000 \times 10^{-6}$  in, per in.  $\epsilon_2 = 500 \times 10^{-6}$  in. per in. and  $\epsilon_3 = 1000 \times 10^{-6}$  in. per in.

The final result in this case along with the results obtained from the normal rosette equations, is given in Table 4.

- (e) Let us take the case where the error in the alignment of guage 2 is  $-2^{\circ}$  and the error in the orientation of gauge 3 is  $+2^{\circ}$  i.e.  $a_2=-2^{\circ}$  and  $a_3=+2^{\circ}$ . Assuming the same values for the strains as before we get the results as shown in Table 5.
- (d) Let us now assume that the error in the alignment of guage 2 is  $+2^{\circ}$  and the error in the orientation of gauge 3 is  $-2^{\circ}$  i.e.  $a_2 = +2^{\circ}$  and  $a_3 = -2^{\circ}$ . The results are shown in Table 6.
- (e) For the case where there is no error in the alignment of the gauge 2 but there is an error of  $+2^{\circ}$  in gauge 3 the results are given in Table 7.
  - (f) Let  $a_2 = 0^{\circ}$  and  $a_3 = -2^{\circ}$ . Results are shown in the Table 8.
  - (g) Let  $\alpha_2 = +2^{\circ}$  and  $\alpha_3 = 0^{\circ}$ . Results are shown in Table 9.
  - (h) Let  $a_2 = -2^{\circ}$  and  $a_3 = 0$ . Results are shown in Table 10.1

TABLE 4

	From mod	ified equations	From normal rosette equations
	o <sub>max</sub>	= 92820 psi	o <sub>max</sub> = 90090 pri
,	omin	= 38910 psi	o <sub>min</sub> = 38490 psi
	<sup>τ</sup> max	= 26940 psi	T <sub>max</sub> == 25800 psi
. ,	lacklacklacklacklacklack	= -33° 19′	

TABLE 5

	From mod	lified equations	From normal rosette equations
<del></del>	o <sub>max</sub>	= 89018 psi	max = 90090 psi
	$\sigma_{min}$	= 36495 psi	o <sub>min</sub> = 38490 psi
	<sup>T</sup> reax	= `26265 psi	τ <sub>max</sub> = 25800 psi
	$oldsymbol{\phi}_{oldsymbol{p}}$	= -30° 58′	$\phi_p = -31^{\circ} 43'$

# TABLE 6

From modified equations	From normal resette equations
\$10m mounted education	From normal rospore equations
₹ <sub>max</sub> = 91313 nsi	<sub>max</sub> = 90090 psi
omin = 40214 pei	o <sub>min</sub> = 38490 psi
mac = 25551 psi	τ <sub>max</sub> = 25800 psi
-38° 34′	$\phi_p = -31^{\circ} 43'$
TABLE	7
From modified equations	From normal resette equations
o <sub>max</sub> = .28290 psi	-
o <sub>min</sub> , = 37350 psi	o <sub>min</sub> = 38490 psi
*max = 25470 psi	$\tau_{max}$ = 25800 psi
<b>Ø</b> <sub>p</sub> == 30° 32′	<b>♦</b> <sub>p</sub> = −31° 43′
Table	8
From modified equations	From normal rosette equations
a <sub>man</sub> = 91155 psi	o <sub>max</sub> = 90090 psi
99765 pej	o <sub>min</sub> = 38490 psi
- 26070 psi	$ au_{max} = 25800 \ psi$
<b>♦</b> <sub>p</sub> · = −32° 56′	$\phi_p = -31^{\circ} 43'$
TABLE	9
From modified equations	From normal rosette equations
max = 89400 psi	o <sub>max</sub> = 90090 psi
omin = 39150 pei	σ <sub>min</sub> = 38490 psi
™ax = 25128 Pei	τ <sub>max</sub> = 25800 psi

TABLE 10

From me	odified equations		From normal resette equations	
$\sigma_{max}$	= 90804 psi		max = 90090 psi	
$\sigma_{min}$	= 37710 pai		omin = 38490 mer	
$\sigma_{max}$	= 26565 psi		*max = 25800 psi	
$\phi_p$	= -32° 08'	e de la companya de l	$\phi_p = -31^{\circ} 48^{\circ}$	

## CASE 2—Delta Rosette

(a) Assuming now that a Delta rosette is used with errors  $a_2 = a_3 = +2^{\circ}$  and assuming that we obtain the three strain readings as

$$\epsilon_1 = 2000 \times 10^{-6}$$
 $\epsilon_2 = 500 \times 10^{-6}$ 
 $\epsilon_3 = 1000 \times 10^{-6}$ 

and then following the procedure mentioned before, we get

$$\sigma_{max} = 69828 \ psi$$
 $\sigma_{min} = 29622 \ psi$ 
 $\tau_{max} = 20088 \ psi$ 
 $\phi_p = -7^{\circ} \ 40'$ 

These values can be compared against the values that would have been obtained with the use of normal Delta rosette equations. Table 11 shows this comparision.

- (b) Assuming the same strain readings as in (a) but with the errors  $a_2 = a_3 = -2^\circ$ , we obtain the results as shown in Table 12.
  - (c) Case where  $a_2 = -2^{\circ}$ ;  $a_3 = +2^{\circ}$ .

(d) Case where  $a_2=+2^{\circ}$ ;  $a_3=-2^{\circ}$ . Results are shown in Table 14.

(4) Cubb (12010 - 4) - 1 - 1

(e) Case where  $a_3 = +2^{\circ}$ ;  $a_2 = 0^{\circ}$ . Results are shown in Table 15.

(f) Case where  $a_3 = -2^{\circ}$ ;  $a_2 = 0^{\circ}$ . (g) Case where  $a_3 = 0^{\circ}$ ;  $a_2 = +2^{\circ}$ .

Results are shown in Table 17.

Results are shown in Table 16.

Results are shown in Table 13.

(h) Case where  $a_3 = 0^{\circ}$ ;  $a_2 = -2^{\circ}$ .

Results are shown in Table 18.

# TABLE 11

	From modified equations	From normal resette equations
•	a <sub>max</sub> = 69828 psi	o <sub>max</sub> = 70350 psi
	o <sub>min</sub> = 29622 pei	o <sub>min</sub> = 29870 psi
	т <sub>тах</sub> = 20088 ре <u>і</u>	τ <sub>max</sub> = 20340 psi
	<b>\$</b> <sub><b>p</b></sub> = −7° 40′	$\phi_p = -9^{\circ} 34'$

#### TABLE 1

	From mo	difled	l equations			From nor	mal r	osette equations
	omax		71025 psi		-	o <sub>max</sub>	=	70350 psi
,	o <sub>min</sub>	**	29595 psi		· ,·	<sup>o</sup> min	-	29670 psi
	$^{ au}_{max}$	-	20715 psi			<sup>T</sup> max		20340 psi
	$\phi_p$	=	—11° 25′	* ,	•	<b>•</b> <sub>p</sub>	=	9° 34′
	•					-		

# TABLE 13

	From 1	nodified	equations	F	rom nor	mal r	osette equations	
	omax	==	69527 psi		o <sub>max</sub>	_	70350 psi	
	$\sigma_{min}$	pos	27413 psi		$\sigma_{min}$	. =	29670 psi	
	$\tau_{max}$	=	21057 psi		$^{ au}max$	, sta	20340 psi	•
	$\phi_p$		-8° 53′		<b>,</b>	=	—9° <b>34</b> ′	

## TABLE 14

From modified equations		From normal	rosette equations
o <sub>max</sub> = 71130 psi		o <sub>max</sub> =	= 70350 psi
omin = 31590 psi		σ <sub>min</sub> =	= 29670 psi
τ <sub>max</sub> = 19770 psi	n de la companya de La companya de la co	<sup>T</sup> max =	= 20340 psi
= -10° 19′		φ <sub>p</sub> =	=, -9°.34'

# NARANG et al : Errors due to Micaligrment of Rosettes

#### TARVE IS

			TABLE 15	
	From mo	dified equations		From normal resette equations
	o <sub>max</sub>	= 69660 psi		o <sub>max</sub> = 70350 psi
	o <sub>min</sub>	= 28590 psi		omin = 29670 pei
	<sup>T</sup> max	= 20535 psi		max = 20840 psi
e e e e e e e e e e e e e e e e e e e	9,	<b>8° 20′</b>		φ <sub>p</sub> = -9° 34′
1			TABLE 16	agad selentra Riptopher (b. 1821) 1980 - Paritha Allia de Statist
	From mo	dified equations		From normal rosette equations
	omax	= 71070 psi		max = 70350 pei
Section 1995 Schrift and Sec	omin	⇒ 30660 psi		omin == 29670 psi
	THAT	= 20205 psi		Tmax = 20340 psi
	$\phi_p$	= -10° 51'		<b>p</b> = -9° 34′
			Table 17	
, <del></del>	From m	odified equations		From normal rosette equations
	omax	= 70470 psi		6 <sub>max</sub> = 70350 pei
	$\sigma_{min}$	= 30630 psi		omin = 29670 psi
AMILE ALT	max	= 19920 psi		max = 20340 pei
	$oldsymbol{\phi}_{oldsymbol{p}}$	= —8° 59'	্ৰ সংস্কৃতিস্কৃত্ৰ	ф <sub>р</sub> = —9° 34'
	•		TABLE 18	
	From m	odified equations		From normal resette equations
	omax	= 70425 psi		o <sub>max</sub> = 70350 pei
	$\sigma_{min}$	= 31125 psi		o <sub>min</sub> = 29670 psi
•	<sup>†</sup> max	= 19650 psi		$\tau_{max} = 20340 \text{ psi}$

## CONCLUSIONS

It is noted that in case of Rectangular rosette, for orientation errors of  $\pm 2^{\circ}$  in gauges 2 and 3, the errors in the evaluation of principal stresses using normal rosette equations vary between about 1% to 3% for different possible combinations of errors. In case of Delta rosette, for the same orientation errors, the errors in the evaluation of principal stresses using the normal equations vary between 0·1% to 1·3%. This suggests that the case of Delta rosette bears a distinct advantage over the Rectangular rosette. Actually this should be expected as Delta rosette gives the maximum possible angle between gauge axes and thus minimises the error in the computation of the principal strains. As for the direction of the principal strains, the error is close to the orientation error itself in most of the cases. In some of these cases this error is quite less and in any case the error in the direction of principal strains is not greater than the orientation error.

A study of the examples discussed in this paper also suggests that in certain cases the resultant errors in the principal stresses lie on the conservative side, and in some other cases on the non-conservative side. Taking the case of Rectangular rosette for the example discussed here, the maximum stress actually exists where misaligned rosette with error of +2° in gauges 2 & 3 is less than the values that are to be obtained from normal rosette equations. However, when the error is —2°, the actual maximum stress is greater than the one that is obtained from normal equations. It is quite clear that in the former case the error is conservative and as such is not detrimental as far as the design is concerned but this will result in the wastage of material. However, in the latter case where the error in not conservative it may lead to a design failure.

By studying cases (e) and (f) in comparison with cases (g) and (h) under Rectangular rosette, it can be seen that the orientation error in gauge 3 is more dominant than the error in gauge 2, so far as the errors in the principal stresses are concerned. These points hold equally good for Delta rosette. This suggests that proper care should be exercised in checking the error in gauges which are remote from the chosen reference gauge viz the gauge that is chosen to be along the x-axis.

In cases where 2% error in the principal stresses can be tolerated the normal rosette equations can be used directly. However, where accuracy is essential the modified equations may be used when errors in the alignment of the rosette gauges are known.

## ACKNOWLEDGEMENTS

The authors are grateful to Shri S. S. Dharmayya, Assistant Director, for his keen interest and encouragement in preparing this paper. Thanks are also due to the Director for permission to publish this paper.

## REFERENCE

1. Perry, C. C. & Lissner, H. R., The Strain Gauge Primer; (McGraw-Hill Book Co., Inc., New York) Second Edition, 1962.