A NOTE ON THE EQUIVALENT CHARGE METHOD IN THE THEORY OF COMPOSITE CHARGES

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(*Received* 26 *March* 68, *Revised* 26 *April* 68)

This note deals with finding the value of the equivalent form factor by taking into account the shot-start pressure. The methods used are (*i*) Corner's method (*ii*) Clemmow's method and (*iii*) Least square method.

The internal ballistics problem of composite charge can be solved either by the Equivalent Charge Method proposed by Clemmow [1,2] and Corner[3] or by the Direct Method as given by Venkatesan & Patni[4]. To use the existing tables of internal ballistics for calculating ballistics for composite charge, Clemmow and Corner gave the idea of reducing the composite charge consisting of two component charges to a single charge. They calculated the composition and the form which this equivalent charge should posses. In a recent paper, Kapur[5] extended the theory as given by Clemmow and Corner to the general case of n component charges.

Consider the equivalent charge to be such that at any instant the energy produced by it is the same as that produced by the component charges whereby the composition of the equivalent charge is deduced. The form factor for the equivalent charge is then calculated by any of the two different methods proposed by Clemmow and Corner. Knowing the composition and form factor for the equivalent charge, any one of the existing methods for a single charge can be used to calculate the internal ballistics of the gun. But the equivalence is not perfect in the sense that none of the methods takes into account the effect of shot-start pressure on the equivalent form factor. The complex process of band engraving is represented by the use of the shot-start pressure theory and this shot-start pressure has quite a considerable effect on the internal ballistics as established by Aggarwal[6]. Thus it is necessary to include the shot-start factor while calculating the form factor for the equivalent charge.

COMPOSITION OF THE EQUIVALENT CHARGE

In guns since generally we use only two charges having quadratic form functions, we will confine our attention only to such charges. As the energies produced by the equivalent and the composite charges are the same at any instant, we have

$$\frac{FC}{\gamma - 1} Z = \frac{F_1 C_1}{\gamma_1 - 1} Z_1 + \frac{F_2 C_2}{\gamma_2 - 1} Z_2 \qquad (1)$$

$$\text{or} \quad Z = \lambda_1 Z_1 + \lambda_2 Z_2 \qquad (2)$$

where

$$\lambda_1 = \frac{F_1 C_1/\gamma_1 - 1}{FC/\gamma - 1}$$

and $$\lambda_2 = \frac{F_2 C_2/\gamma_2 - 1}{FC/\gamma - 1}$$

Also, the equations for the rate of burning give

$$\frac{D_1}{\beta_1} \frac{df_1}{dt} = \frac{D_2}{\beta_2} \frac{df_2}{dt} = \frac{D}{\beta} \frac{df}{dt} = p^a$$

On integration, we have

$$\frac{f_{10} - f_1}{\beta_1'} = \frac{f_{20} - f_2}{\beta_2'} = \frac{f_0 - f}{\beta'} \qquad (3)$$

where f_{10}, f_{20} and f_0 are the fractions of web-remaining of the first, second and the equivalent charge respectively at the instant of shot-start pressure and

$$\beta_1' = \frac{\beta_1}{D_1}, \ \beta'_2 = \frac{\beta_2}{D_2}, \ \beta = \frac{\beta}{D}$$

From (3) it is obvious that if charge C_1 is to burn out first, we must have

$$\beta_1' f_{20} > \beta_2' f_{10}$$

Also, at the end of two stages of burning the fraction of equivalent charge remaining is

$$f_0 - \frac{f_{10}}{K_1}, \ f_0 - \frac{f_{20}}{K_2}$$

where

$$K_1 = \frac{\beta_1'}{\beta'}, \ K_2 = \frac{\beta_2'}{\beta'}$$

Since at the end of the second stage the equivalent charge is also burnt, we have

$$K_2 = \frac{f_{20}}{f_0}$$

i. e. $$\beta' = \frac{f_0}{f_{20}} \beta_2' \qquad (4)$$

which gives the value of β/D for the equivalent charge. Also at all-burnt stage, we have from (2)

$$\lambda_1 + \lambda_2 = 1$$

i.e. $$\frac{F\,C}{\gamma - 1} = \frac{F_1 C_1}{\gamma_1 - 1} + \frac{F_2\,C_2}{\gamma_2 - 1} \qquad (5)$$

meaning thereby that the energy generated by the equivalent charge is equal to the sum of the energies available from the two component charges.

Since at all-burnt stage the mass of the gas for the equivalent and the composite charges is the same,

$$C = C_1 + C_2 \qquad (6)$$

To a sufficient degree of accuracy the density of the equivalent charge can also be calculated from

$$\frac{1}{\delta} = \frac{\frac{C_1}{\delta_1} + \frac{C_2}{\delta_2}}{C_1 + C_2} \tag{7}$$

Thus (4), (5), (6) and (7) determine the characteristics of the equivalent charge in terms of the characteristics of the composite charge.

CALCULATION OF THF EQUIVALENT FORM FACTOR

From (2), when both the charges are burning, we have

$$Z = \lambda_1 (1 - f_1) (1 + \theta_1 f_1) + \lambda_2 (1 - f_2) (1 + \theta_2 f_2)$$

or

$$Z = (A - B f + C f^2) \tag{8}$$

where

$$A = \sum_{i=1}^{2} \lambda_i \left\{ Z_{i0} + K_i f_0 (1 - \theta_i + 2\theta_i f_{i0}) - \theta_i K^2_i f_0^2 \right\}$$

$$B = \sum_{i=1}^{2} \lambda_i K_i (1 - \theta_i + 2\theta_i f_{i0} - 2\theta_i f_0 K_i)$$

$$C = - \sum_{i=1}^{2} \lambda_i \theta_i K^2_i .$$

Again, when the first charge is burnt out and the second is still burning, we get

$$Z = \lambda_1 + \lambda_2 (1 - f_2) (1 + \theta_2 f_2)$$

or

$$Z = \lambda_1 + \lambda_2 (A' + B' f + C' f^2) \tag{9}$$

where

$$A' = \lambda_2 [Z_{20} + K_2 f_0 (1 - \theta_2 + 2\theta_2 f_{20}) - \theta_2 K_2^2 f_0^2]$$

$$B' = \lambda_2 K_2 (1 - \theta_2 + 2\theta_2 f_{20} - 2\theta_2 f_0 K_2)$$

$$C' = - \lambda_2 \theta_2 K_2^2$$

Now with the help of (8) and (9), we can find the value of the form factor by the methods given by Corner and Clemmow.

(*i*) *Corner's Method*—Following this method the value of θ is found by making the area under the curve $Z = (1 - f) (1 + \theta f)$ equal to the area under the $(Z - f)$ curves for the two component charges. Accordingly from (8) and (9), we have

$$\int_{f_0}^{0} (1-f)(1+\theta f)\,df = \int_{f_0}^{f_0-\frac{f_{10}}{K_1}} (A - Bf + Cf^2)\,df + \int_{f_0-\frac{f_{10}}{K_1}}^{f_0-\frac{f_{20}}{K_2}} [\lambda_1 + \lambda_2 (A' - B'f + C'f^2)]\,df$$

Integrating and simplifying for θ, we obtain

$$\theta = \frac{6K_2^2}{(2f_{20}-3)}\left[-\frac{1}{2K_2^2} + \frac{1}{2}\left(\frac{\lambda_1}{K_1}\frac{f_{10}^2}{f_{20}^2} + \frac{\lambda_2}{K_2}\right) - \frac{1}{2}\left(\frac{\lambda_1\theta_1}{K_1}\frac{f_{10}^2}{f_{20}^2} + \frac{\lambda_2\theta_2}{K}\right) + \frac{1}{3}\left(\frac{\lambda_1\theta_1}{K_1}\frac{f_{10}^3}{f_{20}^3} + \frac{\lambda_2\theta_2}{K_2}\right)\right] \quad (10)$$

Obviously if we put $f_{10} = f_{20} = 1$, we obtain the same result as that of Kapur for two component charges.

(*ii*) *Clemmow's Method*—Clemmow estimates the value of θ by ensuring that the curve $Z = (1-f)(1+\theta f)$ passes through the common point of the $(Z-f)$ curves for the two stages of burning and so obviously his method can be applied only when the composite charge consists of only two component charges. In our case, the common point of the form functions of the two stages of burning is

$$f = f_0 - \frac{f_{10}}{K_1}$$

$$Z = \lambda_1 + \lambda_2\left[A' - B'\left(f_0 - \frac{f_{10}}{K_1}\right) + C'\left(f_0 - \frac{f_{10}}{K_1}\right)^2\right]$$

Substituting these values in $Z = (1-f)(1+\theta f)$, we get

$$\lambda_1 + \lambda_2\left[A' - B'\left(f_0 - \frac{f_{10}}{K_1}\right) + C'\left(f_0 - \frac{f_{10}}{K_1}\right)^2\right] = \left(1 - f_0 + \frac{f_{10}}{K_1}\right)\left[1 + \theta\left(f_0 - \frac{f_{10}}{K_1}\right)\right]$$

Simplifying the above, we have

$$\theta = \frac{\lambda_1 K_1}{f_{10} + K_1(1-f_{20})} + \lambda_2\theta_2$$

$$= \frac{\lambda_1 K_1}{f_{10} + K_1(1-f_{20})} + (1-\lambda_1)\,\theta_2 \quad (11)$$

which agrees with Kapur's result in the particular case when $f_{10} = f_{20} = 1$.

(*iii*) *Least Square Method*—Yet another method for estimating the value of θ has been proposed by Kapur which is based upon the Least Square Method in Statistics. In this method we find the best fit of the form function for the equivalent charge to the different form functions for the stages of burning of the component charges. Accordingly, we have to find that value of θ which minimises the following :

$$\int_{f_0}^{f_3-\frac{f_{10}}{K_1}} [(1-f)(1+\theta f) - (A - Bf + Cf^2)]^2\,df + \int_{f_0-\frac{f_{10}}{K_1}}^{f_0-\frac{f_{20}}{K_2}} [(1-f)(1+\theta f) - \{\lambda_1 + \lambda_2(A' - B'f + C'f^2)\}]^2\,df$$

Following the method of differentiation under integration sign, we have

$$\int_{f_0}^{f_0-\frac{f_{10}}{K_1}} f(1-f)[(1-f)(1+\theta f)-(A-Bf+Cf^2)]df+\int_{f_0-\frac{f_{10}}{k_1}}^{f_0-\frac{f_{20}}{K_2}} f(1-f)[(1-f)(1+\theta f)-\{\lambda_1+\lambda_2(A'-B'f+C'f^2)\}]df=0$$

Integrating the above and after simplification we obtain the value of θ as follows :

$$\theta=\frac{\left(\frac{1}{4}\frac{f_{20}}{K_2}-\frac{1}{3}\right)}{\left(\frac{1}{2}\frac{f_{20}}{K_2}-\frac{1}{5}\frac{f_{20}^2}{K_2^2}-\frac{1}{3}\right)}+\sum_{i=1}^{2}\lambda_i A_i+\sum_{i=1}^{2}\lambda_i\ \theta_i\ B_i \tag{12}$$

where

$$A_i=\frac{f_{io}^2\left[\frac{f_{io}f_{20}}{3K_i^2}-\frac{1}{6}\frac{f_{ic}}{K_i^2}-\frac{1}{12}\frac{f_{io}^2}{K_i^3}+\frac{f_{20}}{2K_i}-\frac{f_{20}^2}{2Ki}\right]}{\left(\frac{1}{2}\frac{f_{20}}{K_2}-\frac{1}{5}\frac{f_{20}^2}{K_2^2}-\frac{1}{3}\right)}$$

$$B_i=f_{io}^2\left[\frac{1}{6}\frac{f_{io}}{K_i^2}+\frac{1}{12}\frac{f_{io}^2}{K_i^3}-\frac{1}{12}\frac{f_{io}^3}{K_i^3}-\frac{1}{30}\frac{f_{io}^3}{K_i^3}-\frac{2}{3}\frac{f_{io}f_{20}}{K_i}-\frac{1}{3}\frac{f_{io}f_{20}}{K_i^2}+\frac{2}{3}\frac{f_{io}f_{20}^2}{K_i}+\right.$$

$$\left.+\frac{1}{6}\frac{f_{io}^2f_{20}}{K_i^2}+\frac{1}{2}\frac{f_{20}^2}{K_i}-\frac{1}{2K_i}\right]\Big/\left(\frac{1}{2}\frac{f_{20}}{K_2}-\frac{1}{5}\frac{f_{20}^2}{K_2^2}-\frac{1}{3}\right)$$

$$i=1,\ 2.$$

Here again we observe that if we neglect the shot-start pressure, *i.e.* put $f_{10}=f_{20}=1$, we obtain the same result as given by Kapur for two component charges.

The above analysis can easily be extended to the case when the composite charge may consist of $n\ (>2)$ component charges.

ACKNOWLEDGEMENTS

The authors are extremely grateful to Dr. Kartar Singh, Director, Defence Science Laboratory, Delhi, for his interest and according permission to publish this work. Their thanks are due to Dr. R. R. Aggarwal for helpful discussions.

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