A NEW DEFERRED SENTENCING SCHEME

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(*Received* 12 *Dec.* 67 : *Revised* 23 *April* 67)

A new deferred sentencing scheme resembling double sampling scheme has been suggested from viewpoint of operational and administrative convenience. It is recommended particularly when the inspection is destructive. The O.C. curves of the scheme for two sample sizes of 5 and 10 have been given.

In 1947, Anscombe *et al*[1] developed certain deferred sentencing schemes for lot by lot inspection. In such schemes the decision about the acceptance or rejection of a lot is not taken immediately after inspecting a lot but is delayed till some more lots are also inspected. They have discussed, mainly three such schemes.

In the first scheme, the decision about acceptance or rejection of a lot can be taken only when a large number of lots have been inspected. This scheme, involving, as it does, a lot of storage space and delay in despatch of accepted material, may not be administratively convenient. Moreover, the sample size, *i.e.* one item from each lot remains fixed whatever the inspection results might be. In the second scheme, this drawback has been sought to be remedied by increasing the sample size as soon as the item inspected in the previous lot was found to be defective. Here, a lot is divided into several parts and one item from each part is taken. It may not, however, be always possible to divide the lot conveniently into a number of parts as required by the scheme. Moreover, in the above two schemes the fraction defective is assumed to be infinitesimal. In the third scheme the fraction defective is not assumed to be infinitesimal but the number of lots to be inspected before a decision could be reached about the acceptance or rejection of a lot is four or five. Moreover, the number of samples to be taken from each lot is forty or twenty, which may be too much in the case of destructive testing unless the lot size is very large.

Keeping these schemes in view, we have evolved an inspection scheme which is quite simple to operate and is also administratively convenient. It calls for a second sample to be taken from a lot either when a defective occurs in the first sample or when the next lot is found rejectable. However, the decision about its acceptance or rejection is taken when the next consecutive lot has been inspected except when it is found to have more than one defective in two samples combined in which case it is rejected straightaway. In case we come across a rejectable lot we are not sure whether the process average fraction defective has worsened when the lot under question was under production or previous to it. We then look back and try to give the previous lot stricter inspection.

SCHEME

Let us visualise some production process which is producing certain items continuously. Let these items be grouped into lots of some fixed number of items and be presented for inspection in the order of production. Then we consider the following double sampling deferred sentencing inspection plan in which all the samples are of equal size, consisting of n (say) items.

Inspect a sample from the first lot.

I. No defective, inspect a sample from the second lot. If it contains

(*a*) no defective, accept the first lot,

(*b*) one defective, inspect another sample from the second lot and if it contains

(1) no defective, accept the first lot,

(2) one or more defectives, reject the second lot and also inspect a second sample from the first lot and if it contains

(*i*) no defective, accept the first lot,

(*ii*) one or more defectives, reject the first lot.

Also, start afresh from the third lot onwards.

(*c*) more than one defective, reject the second lot and proceed as in *b* (2) above.

II. One defective, inspect another sample from the first lot itself and if it contains

(*a*) no defective, inspect a sample from the second lot and if it contains

(1) no defective, accept the first lot,

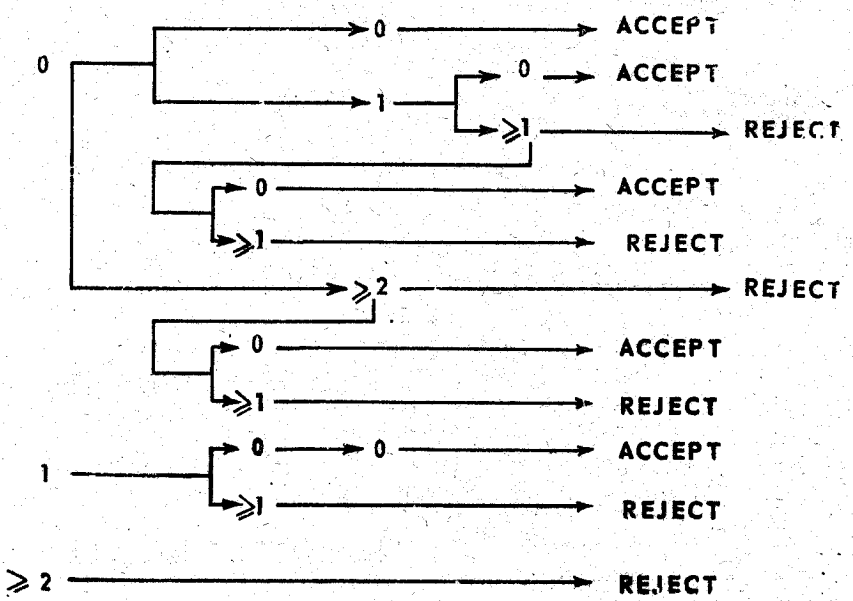
(2) one or more defectives, reject the first lot.

(*b*) one or more defectives, reject the first lot. Start afresh from the second lot onwards.

III. More than one defective, reject the first lot. Start afresh from the second lot onwards.

The above scheme can be represented diagramatically as follows:

First lot	Second lot			Decision on	
Number of Defectives	Number of Defectives			First lot	Second lot
First Sample	Second Sample	First Sample	Second Sample	First lot	Second lot



OPERATING CHARACTERISTICS OF THE SCHEME

From the diagramatic representation of the scheme, the probability of acceptance (P_A) of a lot can be easily derived and is given by

$$P_A = (1 + p)(q^{2n} + npq^{3n-1})$$

and the average amount of inspection (AI) per lot is given by

$$AI = n + nq^{n-1}(1 - q^n)\{q + np(1 + q^n)\}$$

where p is the process average fraction defective.

By definition:

$AI = n$. Probability samples size in n

$+2n$. Probability that sample size is $2n$.

Now probability that sample size is n

$$= q^{2n} + q^n \cdot npq^{n-1} \cdot q^n + 1 - q^n - npq^{n-1}$$
$$= q^{2n} + npq^{3n-1} + 1 - q^n - npq^{n-1}$$

Obviously,

probability that sample size is $2n$

= 1—probability that sample size is n

$$= q^n + npq^{n-1} - q^{2n} - npq^{3n-1}$$

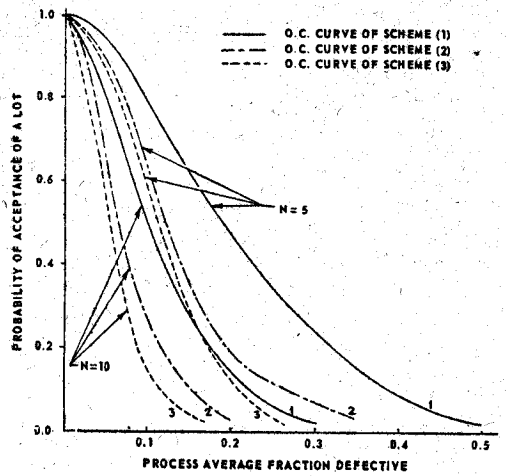
Hence,

$$AI = n\{q^{2n} + npq^{3n-1} + 1 - q^n - npq^{n-1}\} + 2n\{q^n + npq^{n-1} - q^{2n} - npq^{3n-1}\}$$
$$= nq^{2n} + n^2pq^{3n-1} + n - nq^n - n^2pq^{n-1} + 2nq^n + 2n^2pq^{n-1} - 2nq^{2n} - 2n^2pq^{3n-1}$$
$$= n + nq^n - nq^{2n} + n^2pq^{n-1} - n^2pq^{3n-1}$$
$$= n + nq^{n-1}\{q - q^{n+1} + np - np\,q^{2n}\}$$
$$= n + nq^{n-1}\{q(1 - q^n) + np(1 - q^{2n})\}$$
$$= n + nq^{n-1}(1 - q^n)\{q + np(1 + q^n)\}$$

From the above calculations, we have verified that if the calculations are done with the help of the above formula then the values given in the tables are obtained.

DISCUSSION AND CONCLUSION

In the case of destructive testing, the sample size has to be kept necessarily low. Hence, previous information that is available must be made use of in improving the operating characteristics of the plan. Our results show that, while deferring sentence accomplishes the function of utilising previous information, the more removed the previous lot from the one under actual test, the less is the extra information gained in the form of improving the O.C. curve. Administrative convenience should thus be the guiding factor about the number of lots to be considered for deferred sentencing.



We have presented in Fig. 1, the O.C. curves of three schemes, viz.,

(1) Non-deferred double sampling scheme: Acceptance and rejection numbers being 0 and 2 for first sample and 1 and 2 for two samples combined.

(2) Double sampling scheme with deferred sentencing upto the inspection of the next lot.

(3) Double sampling scheme with deferred sentencing upto the inspection of the next two lots.

Fig. 1—O.C. curves of various schemes.

The average sample sizes inspected per lot and the LTPD (the process average fraction defective for which the probability of acceptance is 0·1) of the schemes (1) and (2) have been given in Table 1 for $n = 5$ and $n = 10$.

TABLE 1

AVERAGE AMOUNT OF INSPECTION (AI) AND LOT TOLERANCE PROPORTION DEFECTIVE (LTPD) FOR VARIOUS SCHEMES

P	$n = 5$				$n = 10$			
	Scheme (1)		Scheme (2)		Scheme (1)		Scheme (2)	
	AI	LTPD	AI	LTPD	AI	LTPD	AI	LTPD
0·01	5·2		5·2		10·9		11·0	
0·02	5·5		5·5		11·7		11·7	
0·03	5·7		5·8		12·3		12·3	
0·04	5·8	0·39	6·0	0·25	12·8	0·23	12·8	0·15
0·05	6·0		6·3		13·1		13·2	
0·10	6·6		7·3		13·9		15·6	
0·15	7·0		7·8		13·5		14·9	
0·20	7·0		7·9		12·7		13·6	
0·25	6·2		6·5		11·2		11·3	

It is clear that whereas there is only marginal increases in the number of items inspected per lot when deferred sentencing is adopted, the gain in protection is considerable in the form of steeper O.C. curve. It may be mentioned that the value of n, *i.e.* the number of items inspected per lot is to be decided taking the lot size and protection desired into consideration.

Further, for facilitating the use of the suggested scheme the average amounts of inspection and lot tolerance proportion defective for $n = 4$, 6 and 8 have been given in Table 2.

TABLE 2

AVERAGE AMOUNT OF INSPECTION (AI) AND LOT TOLERANCE PROPORTION DEFECTIVE (LTPD) FOR SUGGESTED SCHEME FOR $n = 4$, 6 AND 8

P	$n = 4$		$n = 6$		$n = 8$	
	AI	LTPD	AI	LTPD	AI	LTPD
0·01	4·2		6·4		8·7	
0·02	4·3		6·7		9·3	
0·03	4·5		7·1		10·0	
0·04	4·7	0·30	7·5	0·21	10·5	0·16
0·05	4·8		7·8		11·0	
0·10	5·6		9·0		12·5	
0·15	6·1		9·5		12·4	
0·20	6·3		9·4		11·7	
0·25	6·4		8·9		10·8	

ACKNOWLEDGEMENTS

The authors are grateful to Dr. Kartar Singh, Director, Defence Science Laboratory, for his kind permission to publish this paper and to the referee for some useful suggestions.

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